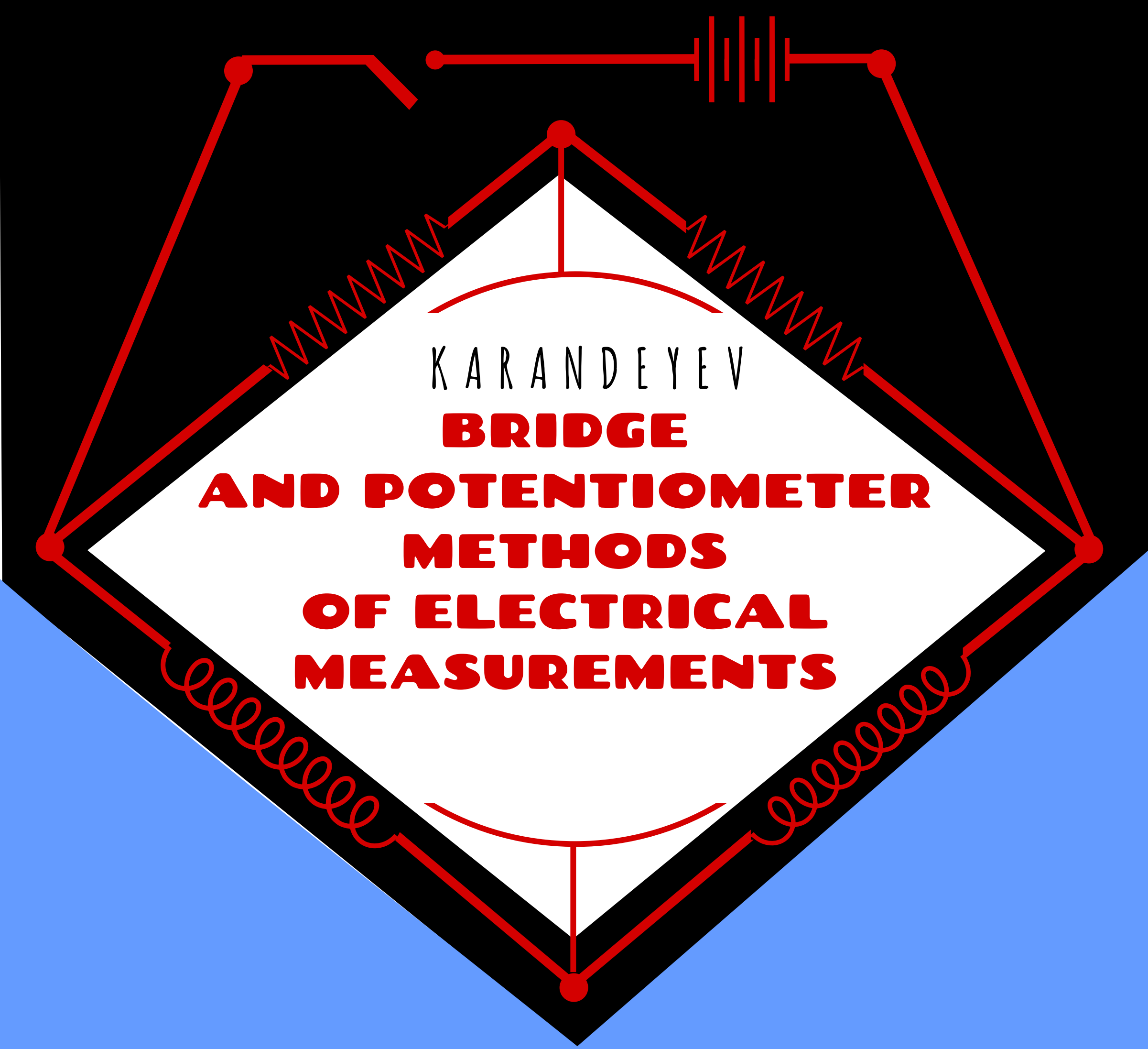

KARANDEYEV

BRIDGE AND POTENTIOMETER METHODS OF ELECTRICAL MEASUREMENTS

PEACE
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К. Б. КАРАНДЕЕВ

СПЕЦИАЛЬНЫЕ МЕТОДЫ ЭЛЕКТРИЧЕСКИХ ИЗМЕРЕНИЙ

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K. KARANDEYEV

BRIDGE AND POTENTIOMETER METHODS OF ELECTRICAL MEASUREMENTS

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INTRODUCTION

Measurement is essentially a comparison of any given quantity with another quantity of the same kind chosen as a standard or a unit.

One cannot stress too strongly the importance of measurements to present-day science and technology. Indeed, no physical experiment is conceivable without a sufficiently accurate technique of measurement. In some cases, the development of such a technique may constitute the main handicap, although it may, on the other hand, yield valuable results in its own right. A suitable example is provided by the experimental technique devised for light velocity determination. Later it formed the basis of the interference methods of length measurements. Today, these methods, among the most accurate and perfect, are widely employed quite apart from their original purpose.

As for importance of measurements to engineering, it will suffice to recall that the interchangeability of parts—the fundamental principle of modern technology—would be impossible without sophisticated and perfect measuring facilities. It will be no exaggeration to say that the quality of measuring tools and instruments is a very accurate index of technological progress in any industry.

True of any field of science and technology, this is especially true of electrical engineering and electrical physics which have now expanded to include many new applications.

The trend towards electrification has affected—in a straightforward manner and on a large scale—measuring techniques and instruments themselves. Owing to their perfection and convenience, ever wider use has of late been made of electrical methods of measurement in which the unknown quantity is converted into an electrical quantity functionally related to the former, and then the electrical quantity is measured directly. Such electrical methods of measuring nonelectrical quantities have won general recognition.

What has been said would seem enough to show the importance of electrical methods of measurement. Present-day progress in science and technology, however, especially the ever greater emphasis placed on process automation, underlines this importance still more.

The reader apparently knows that any automatic control system depends for its operation on reliable information about the state

of the controlled plant or process. This information is obtained by sensing elements which are, in fact, measuring instruments. Therefore, progress in automatic control involves the perfection of measuring elements.* In most cases, these measuring elements are based on an electrical method of measurement. Thus the development and study of measuring techniques in general and of electrical methods of measurement in particular are obviously of paramount importance.

Before we take up electrical methods of measurement in detail, however, we should define the very concept and the ground to be covered.

By "method of measurement" is meant the manner in which the extent or magnitude of the unknown quantity is ascertained, i.e., the various techniques by which any given quantity is measured. Obviously, from a technical point of view, any given method will involve a definite assortment of physical instruments (basic and auxiliary) and experimental techniques.

This general definition is very broad. In fact, it embraces any measurement of any quantity, including an electrical one. Therefore, we should limit the ground we are to cover. In our discussion we shall be solely concerned with the bridge and potentiometer methods of measurement. The two methods are capable of both direct and indirect measurement of practically all electrical quantities. The particular feature about them is that they belong to the balance class of methods and, as such, involve certain auxiliary operations.

Until quite recently the bridge and potentiometer methods were almost exclusively used in laboratory research. For some time past, however, they have found a broader field of application, mainly owing to material advance in the measurement of nonelectrical quantities by electrical methods and in industrial process control. As a result, the bridge and potentiometer methods which are the basis of most measuring circuits have come to be used under shop and field conditions.

An apt example is supplied by electrical extensometers employing resistance strain gauges. The circuits of these extensometers are based on the bridge and potentiometer methods.

The requirements of reliability and convenience in service necessitate continuous improvement in the basic and auxiliary equipment and the use of sufficiently accurate automatic instruments with which measurements are taken by these two methods.

* It should be noted that the measuring elements employed in automatic control systems may perform independent functions in nonautomated processes or plants.

In automatic bridges and potentiometers, the measuring circuit is balanced by an automatic arrangement rather than by a human operator. The result can then be either read directly from a scale,* or recorded on a chart, or transmitted as a signal to the input of an automatic regulating unit. In the latter case and also in more sophisticated control systems, indications are often processed by auxiliary logical or numerical units.

This brings us to the more involved automatic measuring systems. In a typical case, this is an automatic computer with capabilities for multipoint measurements and the processing of indications to a preset program. Output information may be in the form of tables, curves, or static characteristics giving the whole lot of data about the performance of the plant or process. Alternatively, only some of the performance characteristics may be displayed (say, the efficiency of a motor, the one-minute output of a blast furnace, the rate of a chemical reaction, etc.). In recent years such data acquisition systems have been coming into wide use both in industry and in research; groundwork has been done for a general theory of such systems.

Automatic measuring systems are practicable and attractive for at least two reasons: the job of the human operator is made easier and more accurate results are obtained.

Practically no physical effort is required of the human operator in taking measurements (leaving out some auxiliary operations, such as the carrying of the parts to be measured). His is mental work, although it may involve different levels of mental effort, depending on circumstances. Nevertheless, it may be very tiring. The repetitive operations of a checker may be more tiring because of the monotony than the job of a worker having to perform more complicated but interesting and diverse operations. Consequently, automatic measurement is a very important aspect of the overall problem of automation in mental work which is now receiving an ever greater attention.

In addition to the natural desire to make the task of the human operator easier, there is another, purely utilitarian reason for automation in measurement. This reason lies in the limitations of the human operator. Above all, this applies to the ability of the human operator to receive and process information, including indications of instruments. It is a well known fact that the information-handling capacity of the human nervous system is fairly limited, although it may be increased by special training. Automatic systems are superior to man in gathering and processing information, al-

* The advent of automatic systems seems to necessitate a revision of the delineation between directly and indirectly reading varieties of instruments.

though they are inferior to him as far as flexibility, adaptability to a changing situation and reliability of perception are concerned. Furthermore, machine does not grow tired; it can work in environments harmful to man and will not respond to many disturbances not provided for by the program (such as unexpected loud sounds, flashes, etc.). This makes the substitution of an automatic device for a human operator where essential and complex measurements are involved not only practicable but also essential.

Problems of automation in measurement will not be taken up in the book. It should be remembered, however, that the balancing of a measuring circuit is an auxiliary technical operation. Therefore the underlying principle and features of the method as such remain the same both in the manual and the automatic variety of the technique. In most cases, the fundamental relationships derived for manual balancing are applicable to automatic operation.

To sum up, our terms of reference will include the operating principles, basic features and techniques (including auxiliary apparatus) of the two important electrical methods of measurement: the bridge method and the potentiometer method, assuming that the measuring circuit is balanced by hand.

Chapter One

BASIC DEFINITIONS AND CONCEPTS

1-1. Classification of Methods of Measurement

A given quantity can be measured in a variety of ways, by different methods, techniques or their combinations. It is essential, therefore, to begin with the basic definitions and concepts of measurement and with a classification of methods of electrical measurement. Such classifications are many, but none has been accepted as an all-embracing one. We shall take what serves our purpose best.

Before we lay down the main points of such a classification, we must obtain a clear picture of what we call measurement. According to Malikov, "*measurement is a procedure consisting essentially in a comparison of a given quantity with another quantity of the same kind chosen as a unit through a physical experiment.*"

This involves two concepts: the *objective of measurement* (the sought-for quantity), i.e., the quantity whose extent or magnitude are to be ascertained in the final analysis; and the *object of measurement* (the quantity being measured), i.e., an auxiliary quantity which lends itself to measurement readily, but plays no part of its own and only serves to determine the sought-for quantity.

The above definition entails three more concepts: *measurement, the measuring operation*, and *the method of measurement*. The concept of *measurement* covers all that is necessary in order to ascertain the extent or magnitude of the unknown; *the measuring operation* applies to the actual comparison irrespective of the way it is carried out; and, finally, *the method of measurement* defines the manner in which a given physical experiment is carried out and is identified with a certain definite circuitry, apparatus, experimental techniques, etc. It is obvious that the method of measurement is an adjunct to the more general concept of measuring operation.

It may seem at first glance that measurement and a measuring operation coincide. Although this may be so in certain instances,

this is not so generally. Far from every sought-for quantity can be measured directly (for example, temperature, humidity, the distance between inaccessible points, etc.). As a result, the objective and the object of measurement are different—a factor which determines the difference between measurement and a measuring operation. The first step in a measurement is to single out the objective of measurement. Then from an analysis of the nature of this quantity, the object of measurement is selected. The measuring operation yields an observation (or an indication) which, after appropriate reduction and processing (if they are necessary), is presented as the final result. In other words, measurement generally starts with the establishment of the sought-for quantity and ends with the presentation of the result, incorporating as an integral part the measuring operation which, in turn, starts with the selection of the quantity to measure and ends with the indication. In few particular cases, where the objective and the object of measurement (the sought-for quantity and the measurand) are the same, the concepts of measurement and measuring operation formally coincide.

Measurements may be direct, indirect and cumulative, depending on the physical relationship between the sought-for quantity and the measurand. For example, the resistance of a four-arm bridge is a case of direct measurement; the resistivity of a conductor will be measured indirectly (on the basis of the direct measurement of its resistance and geometrical dimensions); and the temperature coefficient of resistance involves cumulative measurement (a series of direct measurements of a specimen's resistance at different temperatures).

We may now go over to an analysis of the measuring operation and a classification of methods of measurement. By definition, the measurement of any given quantity is a comparison of this quantity with a standard through a physical experiment, by one technique or another. Thus, explicitly or implicitly, a measuring operation involves:

(a) un unknown;
(b) a standard;
(c) a comparison apparatus (an instrument or a test set);
(d) a measuring technique.

The above list suggests three approaches to the delineation of measuring operations:

(1) according to the way the unknown quantity is compared with a standard;

(2) according to the manner in which the physical experiment of comparison (the method of measurement, in its narrow sense) is conducted;

(3) according to the measuring technique employed.

These approaches leave us with the following specific groups of methods.

Two fundamental ways can be envisioned for the comparison of the unknown quantity with a standard. In the first, a standard is always present and employed in the work, serving as a means of direct comparison. Arbitrarily, this may be called *concurrent comparison.* In the second, the continuous presence of a standard is not mandatory; it comes into and out of the work, being used for calibration and testing in dubious cases. This may be referred to as *consecutive comparison.*

Each of the two groups may be further subdivided into two subgroups. In concurrent comparison, it may happen that the whole of a standard is directly compared with the whole of the unknown. This may be termed *direct concurrent comparison.*

Sometimes, this may cause certain inconvenience, such as when the unknown is too big or two small. Then it will be more attractive to combine the unknown and a standard into a functional group and to obtain the total indication (in an algebraic sense) for this group through another standard. This calls for the use of at least two standards at a time. The unknown quantity is then found from the immediate result of the measurement and from the standard included in the functional group. This device is used in the measurement of large capacitances which are connected in series with a standard or, conversely, of low capacitances connected in parallel. This is also true of resistance measurements.

Another instance where this device proves helpful is when the absolute magnitude of the unknown is within the specified limits, but it is essential to reduce the error of measurement. The goal is easy to attain where the unknown is a vector such as e.m.f., voltage, etc. Then, its combination with a standard of opposite sign will give a differential reading. Obviously, if the difference is small, the error in its determination will affect the result but little, which mainly depends on the precision of the standard. This "differential method" is very common in testing instrument transformers, standard cells, etc.

Last but not least, a case may crop up where the nature of the observable quantity has to be changed to some extent. By way of example, one cannot measure the inductance of a coil directly on a capacitance bridge because of the phase difference. If, however, the coil is connected with a standard capacitor so as to produce a negative phase angle for the combination, the inductance can be measured and calculated. This idea is, incidentally, embodied in resonance methods. Irrespective of the actual embodiment, however, they may be all referred to as *combination (group) comparison.*

Two extreme realisations of consecutive comparison will do in our discussion. In one, a standard is actually present in the work, but as a substitute for the unknown, and the system is restored to balance by adjusting the standard which must of necessity be adjustable. This very common method, analogous to classical weighing on a balance known to be maladjusted is resorted to where doubts exist as to the reliability of the measuring apparatus. This may be called *consecutive comparison by substitution.*

In the other, a standard is solely used for calibration and testing and is never present in the actual work. This is *consecutive comparison by calibration* and it naturally applies to direct-reading instruments.

Such are the main four groups which may, in our opinion, be singled out according to the manner of comparison of the unknown with a standard. Of course, they may well be further subdivided, which is hardly necessary.

Here we come to the next aspect of delineation—the manner in which the physical experiment is conducted. Again, our examination will be limited to the bridge and potentiometer methods.

By the bridge method is meant the measurement (or reduction to zero) of the difference between two voltage drops produced by a single source across an electrical circuit consisting of at least two parallel branches. In this case, at balance (or quasi-balance) the indication is independent of the supply voltage.

By the potentiometer method is meant the measurement (or reduction to zero) of the difference between two independent voltage drops due to different sources. In this case, the indication, even at balance, is dependent on the supply voltages.

For the reason that the bridge has only one source of supply it is suitable for the measurement of circuit constants and also of currents and voltages on the basis of the functional relationships between them and the circuit constants. The potentiometer method, on account of its character, can be employed for the direct measurement of e.m.f.s., voltages and currents and for the indirect measurement of circuit constants.

Finally, there is the third aspect of delineation—the measuring technique. According to it, methods of measurement may be classed into four groups:

(1) according to the mode of control: *automatic* and *manual*;

(2) according to indication: *null-type* and *deflection-type.*

The above groups may form a variety of combinations, as will be seen from the chart of Fig. 1-1.

We have deliberately left out one more aspect of delineation: according to whether or not indications require some mathematical

reduction. We consider this aspect to be a minor one and, in any case, not affecting the fundamental nature of a given method of measurement.

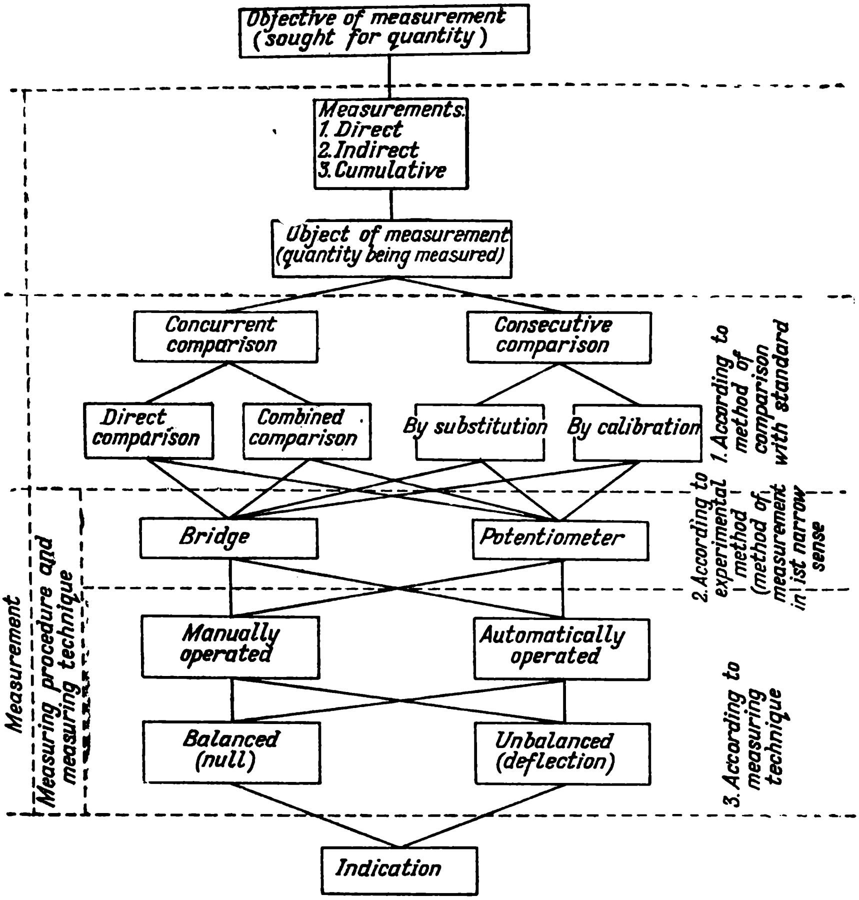

Fig. 1-1

Undoubtedly, other classifications, nomenclatures and arrangements are possible. The one presented above, however, serves our purpose best and will be adhered to hereafter.

1-2. General Remarks on Sensitivity

The value of a measuring method or a measuring instrument is materially dependent on its degree of responsiveness to changes in the measured variable. The smaller the change detectable by a me-

thod or an instrument, the greater its responsiveness, or *sensitivity*. This important property is designated by the letter S with an appropriate subscript and expressed as a numeric.

The relevant USSR State Standard defines sensitivity as the ratio of instrument deflection (linear or angular) to the change in the measured variable that has caused this deflection. The same Standard defines the *threshold of sensitivity* as the smallest change in the measured variable which produces effective response of the instrument.

It must be qualified that the above definition applies more to ordinary deflection-type instruments. As for instruments of high sensitivity based on the null (or balance) method, the investigator is mainly interested in small deflections from zero due to as small changes in the measured variable from balance. Therefore, the ratio of the deflection to the change or, rather the limit of the ratio with the change tending to zero, is usually referred to as balance-point or simply balance sensitivity.* These niceties in nomenclature are, however, often omitted. Therefore, when it comes to bridge or potentiometer measuring systems, the term "sensitivity" should be understood to mean balance sensitivity, even though it is not thus specified.

Thus, the mathematical expression for the response of a measuring device will be

$$\alpha = f(x),$$

where α is the deflection of the instrument and x is the measured variable. Then, by definition, the sensitivity of the device will be

$$S = \Delta\alpha/\Delta x.$$

In a general case, for any kind of the relation $\alpha = f(x)$, the sensitivity at any point of the scale will be

$$\underset{x=a}{S} = \lim_{\Delta x \to 0} |\Delta\alpha/\Delta x| = d\alpha/dx = (d/dx)\, f(x). \tag{1-1}$$

Thus, if the kind of the function is known, Eq. (1-1) will give balance sensitivity when $\alpha = 0$.

So far we have treated sensitivity as the quantity relating a change in a variable (the cause) and the deflection of an instrument (its effect). In fact, this quantity gives a measure of the sensitivity of a given measuring system as a whole (S_t). Ordinarily, any measuring system will consist of at least a measuring circuit and detector, each of which has its own sensitivity. The two components are

* As a matter of record, the relevant State Standard specifies only sensitivity and threshold of sensitivity. All other terms, including balance sensitivity, are not officially established.

linked up by some intermediate quantity which is the output of the measuring circuit and the input of the detector. Since we are solely concerned with electrical circuits, this intermediate quantity may be current, voltage or power at the circuit output. With the current I_D taken as this intermediate quantity, the sensitivities of the circuit and of the detector will be given by

$$S_c = \Delta I_D / \Delta x;$$
$$S_D = \Delta \alpha / \Delta I_D.$$

Consequently, we could use three sensitivities in our analysis. But such multiplicity is redundant, since the three bear a certain relation to each other.

$$S_t = \Delta\alpha/\Delta_x = (\Delta\alpha/\Delta I_D)(\Delta I_D/\Delta x) = S_D S_c. \qquad (1\text{-}2)$$

Furthermore, the sensitivity of the detector, S_D, remains the same, since it depends on the type of instrument chosen by the experimentor. Then the sensitivity of the measuring system can be adjusted by varying the sensitivity of the circuit, S_c.

As a result, all the problems related to sensitivity can be reduced to the sensitivity of the circuit. This consideration will subsequently govern our choice of circuit arrangements and constants with a view to obtaining maximum sensitivity.*

As already noted, the sensitivity of a circuit may be given in terms of output voltage, output current or output power. A more recent trend has been towards a more discerning approach. The maximum output voltage is sought only where the resistance (or impedance) of the detector is very high (tending to infinity in the limit). This is true of a.c. bridges whose detector is usually connected via a valve amplifier of high impedance. As for galvanometer-based systems operating without an amplifier, the resistance is ordinarily very low, and a maximum should be sought for the relationships:

$$I_D = f(R_x) \text{ or } P_D = \varphi(R_x),$$

where R_x is the unknown.

In any of the three cases, the sensitivity is the ratio of a change in a certain output (I_D, V_D or P_D) to the absolute change, ΔR, in the unknown. Very often, however, it is more important to know the fractional change:

$$\varepsilon = \Delta R/R,$$

and not the absolute change, ΔR. Instead of the absolute threshold sensitivity this involves the relative threshold sensitivity which

* For simplicity, the symbol S, if not otherwise specified, will stand for circuit sensitivity.

is often expressed as a percentage. Then the relative current sensitivity S_I, the relative voltage sensitivity S_V and the relative power sensitivity S_P will be:

$$S_I = \Delta I_D/\varepsilon;\ S_V = \Delta V_D/\varepsilon;\ S_P = \Delta P_D/\varepsilon.$$

Use in most cases is made of these sensitivities. As often, the reciprocal of sensitivity, termed the instrument constant, enters into calculations, especially where galvanometers are involved.

Whatever the form of sensitivity, it is not at all easy technically to determine the best possible conditions for operation of the measuring circuit. The thing is that the design equations are rather unwieldy and contain a number of terms each of which can be regarded as an independent variable determining its own extremum. For simplification, it is customary, therefore, to impose certain constraints which, incidentally, have a definite physical meaning. Usually these constraints are imposed on the type, power and internal resistance of the energy source. Very often a limit is fixed for the power dissipated in a circuit component or components. This is done, however, not so much in order to simplify the analysis as to make it more realistic. Where the problem has to be solved without simplifications, the result can be obtained by the classical methods of network theory.

In conclusion, mention should be made of the relationship between the sensitivity of the system and the errors due to environmental factors. For Eq. (1-1) we defined the sensitivity as the ratio of instrument response to the change in the measured variable. Although this was not stated, it was presumed that the change in the measured variable was due to some useful function. It is obvious, however, that a similar response may be caused by an undesired change in some other variable or circuit constant. The only difference between the two responses is that the former is desired and the latter undesired. Let us designate the former the functional sensitivity and the latter the parasitic sensitivity.

Summing up, if the response of a given system is a function of many independent variables

$$\alpha = f(x_1, x_2, x_3 \ldots$$

then the total or complete differential of the response will be the sum of the partial differentials of the individual variables:

$$\Delta\alpha \approx (\partial f/\partial x_1)\,\Delta x_1 + (\partial f/\partial x_2)\,\Delta x_2 + (\partial f/\partial x_3)\,\Delta x_3 \ldots\ldots$$

One of the partial derivatives (say, $\partial f/\partial x_1$) will be the functional sensitivity, and the remaining ones will be parasitic sensitivities. This suggests a very important conclusion, namely that the

sensitivity of a circuit must not be divorced, as often is the case, from the errors due to the instability of its constants. In principle, it would be right to calculate measuring systems so as to maximize their functional sensitivity and to minimize their parasitic sensitivities.

The above considerations necessitate a revision of the established view on the sensitivity of measuring systems and a more cautious approach to the sensitivity requirements. That sensitivity is an important characteristic is only too obvious. It must not be smaller than the least adjustable portion of the variable. It would seem that the higher the sensitivity the better the measuring system, and so every effort ought to be made to improve it. In some applications, however, quite the opposite may prove true. There exist many ways and means of improving sensitivity, both general (such as the use of amplifiers) and specific (adjustment of operating conditions, circuit constants, etc.). These methods may tell differently on the various partial sensitivities of the circuit. If both the functional and parasitic sensitivities are raised in proportion, the magnitude of errors (the accuracy of the system) will remain unchanged. A case can, however, be imagined where the magnitude of errors increases (the accuracy of the system is impaired) due to an inadvertent increase in one or several parasitic sensitivities.

Another point to remember is that higher sensitivity requirements lead to extra problems in the maintenance of operating conditions and to higher costs. This is why higher sensitivity should be strived for only where warranted.

Chapter Two

PRIMARY AND SECONDARY STANDARDS OF ELECTRICAL UNITS

2-1. General

The units of the various physical quantities, including electrical ones, find a concrete, physical embodiment in *standards*. They represent the respective units to the highest degree of precision attainable at the existing state of science and engineering (metrological accuracy). Primary standards serve as the fundamental basis for the calibration of secondary standards, the latter for the calibration of working or laboratory standards, and working or laboratory standards are employed to standardize and calibrate laboratory and commercial instruments.

The value of a given unit is passed on from top to bottom—from a more to a less precise device. The proper sequence of standardization and calibration, which is effected through regular tests and checks of standards and instruments, is vitally essential to the maintenance of precise measures and weights in a country. In the Soviet Union this is ensured through special "test charts".

Figure 2-1 shows the chart for the testing of standard cells (used as e.m.f. and voltage standards). Referring to the chart, the comparison procedure starts with a primary standard, follows through a "transfer" standard, reference standards, secondary standards of the 1st and then the 2nd category, laboratory standards of various accuracy classes, and winds up with commercial instruments.

Leaving out the realization and maintenance of primary standards, we shall discuss the basic design features and uses of secondary and laboratory (or working) standards of which standards of resistance, capacitance, inductance and e.m.f. are most important to electrical measurements.

Although secondary and laboratory (working) standards of electrical units are fundamentally identical in design (coils, capacitors,

cells), they differ in application. Secondary standards are solely used as reference standards for laboratory (working) standards, being never employed for routine measurements. Laboratory standards,

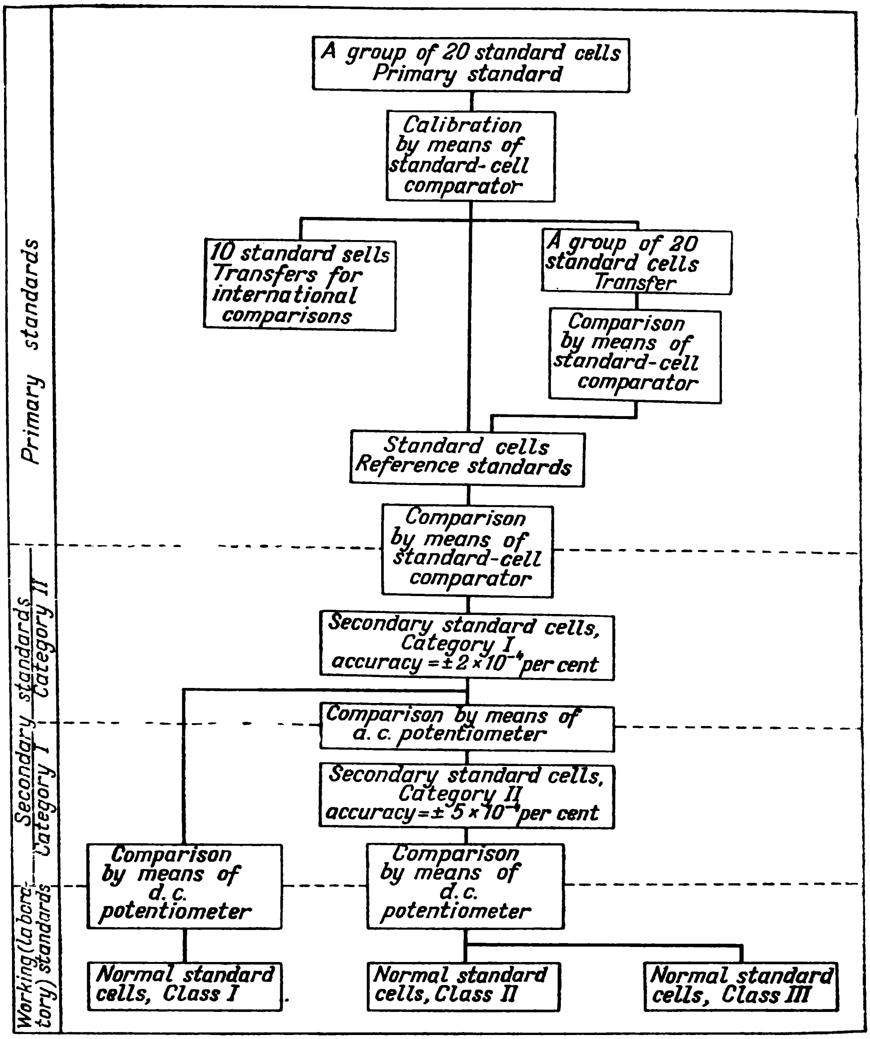

Fig. 2-1

on the other hand, are used for general laboratory work, including routine measurements. Accordingly, greater care must be exercised in verifying and keeping the former than the latter.

2-2. Standards of Resistance

Standards of electrical units can be fixed and variable. Resistance standards of the fixed variety are standard resistance coils and of the variable type resistance boxes. In some types of bridges and potentiometers use is made of another type of the variable standards of resistance (known as slide-wires).

The basic requirements that resistance standards, and indeed any standard, must meet are (a) permanence—there should be as little variation in resistance with time as possible; and (b) accuracy—their actual resistance must be as close to the nominal one as possible.

Additionally, resistance standards for d.c. circuits should have a low thermo-e.m.f. against copper, while those for a.c. circuits should have residual capacitance and inductance of an order of magnitude which would not affect the resistance or result in a large phase angle between the current and the applied voltage. Also, resistance standards should have a small temperature coefficient of resistance in order that the correction for temperature variation may be small, and a high resistivity in order that the standard may be reasonably compact.

Early work on materials for resistance standards began about a century ago Today, quite a gamut of alloys have been obtained for the purpose. The properties of some of them are summarized in Table 2-1.

Table 2-1

Alloy	Constituent metals	Resistivity, ohms·mm²/m	Thermo-e.m.f. against copper, μV/deg.	Temperature coefficient, per °C×10⁻⁵
Manganin	Cu, Mn, Ni	0.44	3.0	1-2
Constantan	Cu, Ni	0.48	40.0	1
Carma	Cu, Al, Cr, Ni	1.30	2.0	2
Ivenom	Cu, Al, Cr, Ni	1.30	2.0	2
Alloy A	Cu, Mn, Al	0.45	0.2	0.1

Laboratory standards of resistance for d.c. work are currently made of manganin wire or ribbon. The resistance-temperature relationship of manganin and of the other alloys listed in Table 2-1 is a complicated one. In most cases, it is given with sufficient accuracy by

$$R_t = R_{20}[1 + \alpha(t - 20) + \beta(t - 20)^2], \qquad (2\text{-}1)$$

where R_t=resistance at ambient temperature;
R_{20}=resistance at 20°C.

The temperature coefficient α of manganin is $(1\text{-}2)\times10^{-5}$ and changes sign as the temperature increases. The coefficient β lies anywhere between -0.3×10^{-6} and -0.8×10^{-6}.

Constantan, which is nearly as good as manganin, cannot be used in standards of resistance for d.c. work since it has a high thermo-e.m.f. against copper (up to 49 μV/deg.).

Of late, wide use has been made of Carma, Ivenom and alloy A. Alloy A has a very low thermo-e.m.f. against copper and a very low temperature coefficient. Carma and Ivenom, which are made up of the same components but in different proportions, have about the same thermo-e.m.f. against copper and temperature coefficient as manganin. However, their resistivity is three times that

of manganin—a definite advantage over the latter. Also, Carma and Ivenom are stronger mechanically and can be drawn into very thin wire.

For better permanence, finished standard coils are given multiple annealing (heating and cooling) during manufacture. The number and duration of these cycles and the anealing temperature depend on the material, wire diameter, etc.

Under the relevant USSR Standard, resistance coils have resistance values of 1×10^n, where the exponent n can be anywhere from —5 to +5, depending on the coil type.

Resistance coils are available in three accuracy classes: 0.01, 0.02 and 0.05 *, the numerals giving the percentage accuracy of the coils (i. e., the discrepancy between the actual and nominal resistance value of a coil). The nominal watt load on coils ranges from 0.1 to 3 watts, and the maximum load from 1 to 30 watts, depending on the accuracy class and nominal resistance value. Class 0.01 coils are immersed in transformer oil and hermetically sealed in a container fitted with a mechanical stirrer as a precaution against overheating.

Resistance coils are adjusted and their actual values measured at +20°C.

Resistance coils for use with alternating currents must have as low residual inductance and interturn capacitance as practicable. For all the precau-

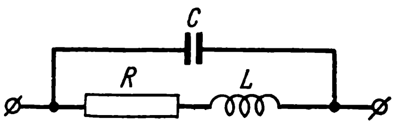

Fig. 2-2

tions, however, every coil has some residual reactance. The effect of residual inductance and capacitance of a coil can be described in terms of the time constant τ. Working from the equivalent circuit given in Fig. 2-2 and neglecting second-order quantities, the time constant is given by

$$\tau=(L/R)-CR, \tag{2-2}$$

where L and C are the residual inductance and capacitance of the coil and R is the d.c. resistance of the coil.

Special methods of coil winding (bifilar, two-layer, Chaperon's series-opposing, Ayrton-Perry parallel-opposing) make it possible to reduce the time constant to a negligibly small value. Depending on the nominal resistance of a coil, its time constant will at present be anywhere between 1×10^{-8} and 5×10^{-6} sec.

For convenience of connection, resistance coils are placed in boxes with their terminals brought to copper blocks or studs on the box cover. The blocks are connected in the desired manner (and, consequently, the desired resistance is brought in circuit) by means of plugs or dial switches using rotary laminated fingers. The total resistance can, in most cases, be varied continuously and without a break in the circuit. The accuracy of resistance boxes is somewhat lower than that of the individual resistance coils. Under the relevant USSR Standard, resistance boxes are available in accuracy classes 0.02, 0.05, 0.1 and 0.2

Plug-type boxes have the advantage of a lower and more constant contact resistance in comparison with boxes using the dial switch, but the latter is much more convenient. Indeed, existing designs of the dial switch are as good as plug

* Coils for d.c. work are made in the three classes while for a.c. work only of class 0.02.

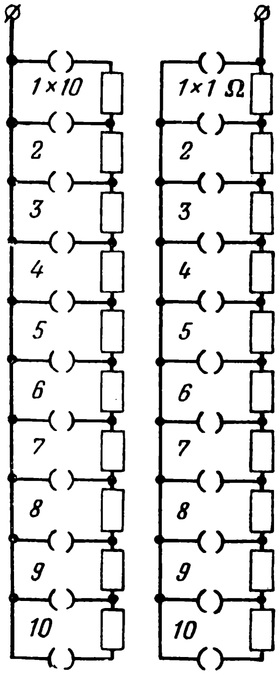

Fig. 2-3

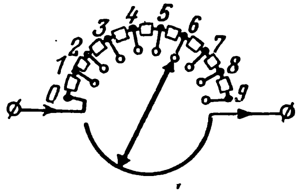

Fig. 2-4

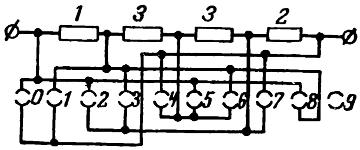

Fig. 2-5

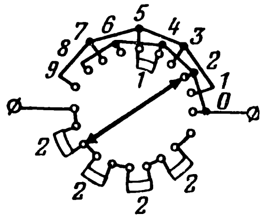

Fig. 2-6

contacts. As far as the connections inside resistance boxes are concerned, preference should be given to decade boxes, especially to those fitted with dial switches.

In an ordinary resistance box each section (the units, the tens, etc.) contains several (usually, four) resistors differing in value so that any resistance within a given section can be obtained (for example, 1, 1, 3, 5; 1, 2, 2, 5). The value of the resistance put in circuit is then found by adding the values of each section. If not careful enough, the operator is prone to errors, a feature nonexistent in *decade resistance boxes* which conform to the usual decimal system of numbers very closely. In a decade resistance box, each section having 10 times the value of the preceding one is divided into 10 equal parts (or, which is the same, the resistors of equal value are connected in series, such as in Figs. 2-3 and 2-4). By means of a ten-position dial switch the box can be set to any desired value within its range.

In comparison with conventional resistance boxes, the decade arrangement involves a greater number of resistors, and so the cost may be higher. This increase, however, is not prohibitive, since the individual resistors are identical. Furthermore, with more sophistication in connections, the number of resistors in a decade can be reduced to five or even four (of different value, of course), or to as many as there are in a conventional box. Such arrangements using plugs and dial switches are shown in Figs. 2-5 and 2-6.

2-3. Standards of Capacitance

Alternating-current measurements involve the wide use of secondary and laboratory standards of capacitance which are embodied in fixed and variable standard capacitors.

The principal requirements that standard capacitors must meet are:

(1) Permanence; there should be as little variation in capacitance with time as possible.

(2) Low temperature coefficient.

(3) Small loss angle and high insulation resistance.

(4) Independence of capacitance from the supply frequency and waveform as far as possible.

The best possible results are obtained with standard capacitors using air as the dielectric. Unfortunately, standard air capacitors have very low capacitance (because of very low ε) and can only be used where values in the range 0.001-0.01 μF are involved. Fixed standard air capacitors also find use in high-voltage testing and measurements. Such capacitors are either of the parallel-plate or the concentric-cylinder type.

Use is often made of *variable standard air capacitors* which bear a very close resemblance to common air capacitors used in radio engineering. Such a capacitor is a combination of two sets of plates, one movable and the other fixed. The parts of a variable standard air capacitor can be insulated from each other by fused quartz, special grades of glass or porcelain. The maximum capacity is usually in the neighbourhood of 0.001 μF (sometimes, although seldom, as high as 0.005 μF), and the minimum capacity from 2 to 10 per cent of the maximum. Variable standard air capacitors are often fitted with a micrometric arrangement and a vernier scale so that readings can be taken to a high degree of accuracy. They can be adjusted accurate to within 0.05 to 0.1 per cent. The capacitance-temperature coefficient is 0.003-0.005 per cent/°C. The value of tan δ is less than 1×10^{-4}. It should be noted that the loss angle of a variable capacitor markedly increases with increasing capacitance.

Variable standard capacitors can also be of the concentric cylinder type. Generally, such a capacitor consists of two concentric cylinders, with the capacitance being varied by moving one (usually, inner) cylinder with respect to the other. It is obvious that such a system can only have a low maximum capacitance, since its geometrical dimensions cannot be large. Variable standard air capacitors of the concentric cylinder type have not found any appreciable use.

As far as air capacitors are concerned, it should be remembered that the capacitance across the terminals is due not only to the plates but also due to the terminals and other structural elements with respect to nearby earthed objects. Since the basic capacitance of an air capacitor is low, this stray electrostatic coupling may be a serious source of errors. The situation is aggravated still more by the fact that the relative position of the nearby objects can change (for example, the experimentor's hand moves to and away from the capacitor during adjustment), also changing the stray capacitances, and the errors become uncertain.

In order to shield the condenser from external electrostatic effects and to render more definite the stray capacitances, guard plates or guard rings are employed around the capacitor, and a separate terminal is provided for the guard (hence the name "three-terminal capacitors"). This arrangement also helps to keep possible sources of errors to a minimum in composite measuring systems (see Chapter 9).

As laboratory standards, *capacitors having mica* as the dielectric instead of air are used. * They are available as separate standard capacitors of fixed values (from 0.001 to 1.0 μF) and as capacitance boxes. Although inferior to air capacitors in terms of electric properties, mica standard capacitors are much more convenient and compact. Preference should be given to air capacitors, however, where very low values of capacitance are involved (under 0.001 μF).

The electric characteristics of present-day standard mica capacitors are as follows:

Permanence (over several years)... 0.01-0.03 per cent

Adjustment accuracy:

$C = 0.01\text{-}0.1$ μF . . . . . ± 0.01-0.02 per cent

$C = 0.001\text{-}0.01$ μF . . . . . ± 0.05-0.1 per cent

Temperature coefficient of

capacitance . . . . 0.01-0.02 per cent/°C

$\tan\delta$ 0.0001-0.0005

Insulation resistance:

$C = 0.01\text{-}0.1$ μF . . . . . $10^4\text{-}2\times10^4$ megohms

$C = 0.001$ μF . . . . $10^3\text{-}2\times10^3$ megohms

Standard capacitors may be arranged into capacitance boxes which are available in accuracy classes 0.05, 0.1, 0.2, 0.5 and 1. For single-decade boxes, the accuracy class index stands for the percentage accuracy of a given box. For multiple-decade boxes, the accuracy is given by

$$\delta = al + m\,(C_d/C) \text{ per cent},$$

where

δ=accuracy of the box;

a=accuracy class index;

m=number of decades;

C=capacitance in circuit;

C_d=capacitance of a step in the lowest decade.

The initial capacitance (for boxes with $C_{max}=1$ μF) is anywhere between 30 and 50 pF. The maximum working voltage for mica capacitors should not exceed 500 V (d.c. or maximum instantaneous for a.c.).

Like resistance boxes, capacitance boxes use either plugs or dial switches, the latter being more convenient.

The standard capacitors described above are only suitable for work at relatively low voltages—not over several hundred volts. Cases are, however, often where much higher voltages are involved, such as in testing high-voltage cables, transformers and other high-voltage equipment. For the most part, the measurements and tests are based on standard capacitors. This has led to the construction of several types of standard capacitors for working voltages as high as 800-900 kV.

Naturally, standard capacitors for high-voltage work are only available in the fixed variety with a very low capacitance of the order of 50 to 100 pF. The

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* For some time past, the trend has been towards making capacitors with dielectrics in the form of synthetic films (polystyrene, fluorinated plastics, etc.) and ceramic materials such as special grades of porcelain and silica products. Mica capacitors, however, still remain superior to them, which is the reason why they are used as laboratory standards of capacitance. It is not unlikely, on the other hand, that further progress in synthetics will result in successful capacitors with solid dielectric other than mica. In some cases, they are already being used as low-class commercial standards.

most common type is the concentric cylinder capacitor incorporating special features so as to avoid brush discharges, to reduce leakage currents, etc.

The dielectric may be air under normal pressure. Because of the low dielectric strength of air, however, the air gap between the electrodes has to be increased as the working voltage increases. In the long run, high-voltage capacitors may grow out of all proportions. This is the reason why they are mainly built 100-150 kV and lower, although there are units for 300-kV service,

An appreciable increase in breakdown voltage and a decrease in overall dimensions can be achieved by using compressed gas (nitrogen, carbon dioxide) instead of air as the dielectric. Filled with gas compressed to 10-12 atm, these capacitors are fairly permanent standards. The main source of error is the natural gas leakage which brings down the pressure (a variation of one atmosphere in pressure causes the capacitance to change approximately 0.06 to 0.1 per cent) and also increases the chance of a breakdown. As a precaution, each compressed-gas capacitor is fitted with a pressure gauge.

Finally, there is one more type of standard capacitor for high voltage work which uses ceramics as the dielectric. Such capacitors, currently made only for working voltages up to 10 kV, are very compact and convenient in service. They are subject, however, to greater errors in comparison with air or compressed-gas capacitors. On account of this, they may only be used in portable testing sets of lower accuracy classes where small size and low weight are essential and the magnitude of errors is of minor consequence.

2-4. Standards of Inductance

In addition, standards of inductance are to meet two specific requirements. These are that the resistance of an inductance standard must be very small in comparison with its inductance and the latter should be independent of the supply frequency and current as far as possible.

A very important point in the construction of a good inductance standard is the material for its bobbin or core. The material should be nonconducting, nonmagnetic and should not change in dimensions with time or with variations in temperature and humidity. These requirements are met in good measure by white marble, porcelain and fused quartz. Prior to winding, the bobbin, which is porous, is impregnated with paraffin. The coil is wound with copper wire. Inductance standards for radio frequencies are wound with wire consisting of a large number of insulated strands; this makes the resistance and inductance of thecoil independent of the supply frequency. The leads of the coil are brought to terminals.

The size and shape of the core (or bobbin) are chosen so as to produce a square winding and mean radius which is at least three times the side of this square. The same design is used for standards of mutual inductance.

Standards of self-inductance are made with nominal values from 0.0001 to 1 henry, and standards of mutual inductances from 0.001 to 0.1 henry. They are adjusted accurate to 0.1 to 0.05 per cent. It should be noted that the resistance of the inductance standard cannot be neglected. In this, inductance standards differ materially from capacitance standards whose resistive component (i. e., the loss angle) may in most cases be ignored.

Another point to remember when using inductance standards, especially at radio frequencies, is that the distributed capacitance of the coil and the dielectric loss in the insulation of the wire may be sources of appreciable errors. Because of this, the effective inductance and resistance of large standards (i. e., coils with a great number of turns) tend to vary with frequency.

The dependence of the inductance and resistance of a coil on frequency can be determined analytically by replacing the distributed parameters with

lumped ones. Denoting the lumped capacitance (connected in parallel with the coil) by C and neglecting the smaller terms, the effective values of L and R will be given by:

$$L \approx L_0 (1+\omega^2 CL_0);$$
$$R \approx R_0 (1+2\omega^2 CL_0),$$

where L_0 and R_0 are the nominal inductance and resistance at a very low frequency (in effect, d. c. values).

Designating the lumped leakage resistance (also taken as connected in parallel with the coil) by R_1, we obtain:

$$L = R_1^2 L_0 / [(R_0+R_1)^2+\omega^2 L_0^2];$$
$$R = R_1 [R_0(R_0+R_1)+\omega^2 L_0^2] / [(R_0+R_1)^2+\omega^2 L_0^2].$$

It is possible to calculate the simultaneous effect of these two factors, but the expressions derived for the purpose are rather unwieldy.

From the expressions for L it follows that C and R_1 have an opposite effect on the inductance of the coil. Practically, it is much easier to make a good insulation than to reduce the capacitance of the coil. As a result, the capacitive effect is predominant, and in all inductance coils the effective inductance L increases with increasing frequency.

On account of this, standard inductance coils are certified on the basis of the effective inductance at a fixed frequency (or frequencies). It is not advised to use standard inductance coils at other frequencies, since their effective inductance may go up by as much as 30 to 50 per cent.

A more precise analysis (without the above assumptions) of the effect the distributed capacitance and leakage resistance of a coil have on its inductance is rather involved. It is more reliable and accurate to measure the effective inductance and resistance of an inductance standard than to calculate the errors.

In most cases inductance standards are fixed standards. Sometimes they are arranged as inductance boxes fitted with a suitable switch to obtain the necessary combinations of inductance coils. In certain applications, use is made of variable inductance standards, or inductometers, which are also known as variometers. For all the diversity of design, an inductometer is essentially a combination of two coil systems (each of which may consist of one or several coils), one being stationary, while the other rotates within it. In fact, this arrangement gives a variable mutual inductance. When the two systems are connected in series, however, the total inductance of the variometer, given by $L=L_1+L_2\pm 2M$, will vary too. Travel of the mobile system is indicated by a suitable reading arrangement. When appropriately connected, such variometers can, therefore, serve as standards of both self- and mutual inductance, for which reason they are fitted with double scales. It stands to reason that variable inductance standards (boxes and inductometers) are prone to greater errors than fixed standards. Fixed standard inductance coils (with nominal values from 0.0001 to 1.0 henry) are intended for use at frequencies up to 1,000 c/s. The inductance of coils of 0.01 to 1.0 henry is adjusted until it is within ± 0.3 per cent of the nominal value. For standard inductance coils of 0.001 and 0.0001 henry the accuracy of adjustment is within ± 0.1 per cent. The actual value of inductance is given in the certificate accurate to within ± 0.1 per cent of the nominal value.

A typical inductance box with a dial switch will have three decades: $10 \times 10\ \mu H$, $10 \times 1\ \mu H$ and $10 \times 0.1\ \mu H$, and a variometer for continuous adjustment of the inductance between 0.06 and 0.15 μH. The first two decades are adjusted accurate to within ± 0.5 per cent of the nominal values, and the $10 \times 0.1\ \mu H$ decade to within ± 1 per cent. The coil sections brought out of circuit are re-

placed by an equivalent resistance, i. e., a resistor having the resistance of the disconnected section. Therefore, the total resistance of the resistance box as a whole remains unchanged (being of the order of 50 ohms) at any inductance setting.

2-5. Standards of E.M.F.

The generally accepted standards of e.m.f. are Weston cells which may be saturated or normal, and unsaturated.

In a normal cell the positive electrode is mercury, the negative electrode is amalgamated cadmium (10-12.5 per cent cadmium, 90-87.5 per cent mercury). The depolarizer is a pasty mixture of mercurous sulphate and cadmium sulphate. The electrolyte is a saturated solution of cadmium sulphate. To ensure saturation of the electrolyte, cadmium sulphate crystals are added to it. The connections to an external circuit are made by platinum wires sealed into the glass container which is the shape of an H. Diagrammatically, the Weston cell

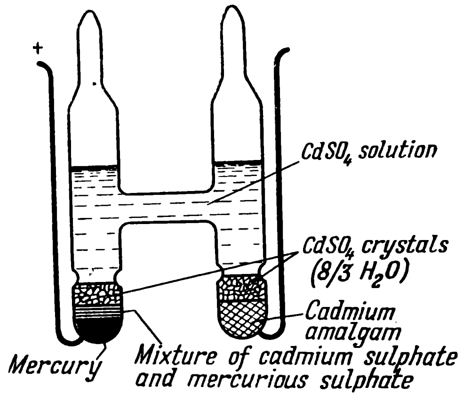

Fig. 2-7

is shown in Fig. 2-7. There are also other types of standard cells due to other investigators.

When carefully made to the relevant standards, normal cells will agree with each other within a few parts in 100,000 (their actual e.m.f. lies anywhere between 1.01850 and 1.01870 V, as measured at +20°C). The internal resistance of a cell is 500 to 1,000 ohms, and the maximum load is 1 μA.

A very important advantage of the Weston cell is its relatively low and quite definite temperature coefficient. The following formula has been internationally adopted for determining the e.m.f. as a function of temperature:

$$E_t = E_{20} - 0.0000406\,(t-20) - 0.00000095\,(t-20)^2 + 0.00000001\,(t-20)^3.$$

This formula applies, if both branches of a cell are at the same temperature. This is because the branches have fairly great temperature coefficients which cancel each other since they are of opposite signs.

In an unsaturated Weston cell, the electrolyte is a solution of cadmium sulphate which is saturated at 4°C and unsaturated at room temperature. The reproducibility and permanence of unsaturated cells are lower than those of saturated cells. The e.m.f. of a new unsaturated cell is 1.0185-1.0195 V.

The advantages of unsaturated cells are a relatively low internal resistance of the order of 300 ohms and an extremely low temperature coefficient which may be neglected in many cases. This makes unsaturated cells very convenient for work at varying temperature.

According to accuracy in e.m.f. determination and permanence, standard cells are divided into three classes. The first two classes cover normal cells and the third class embraces unsaturated cells. Their classification is given in Table 2-2.

Table 2-2

Class	E.m.f. range, abvolts		Max. current, A	Max. drop in e.m.f. over year, abvolt
	from	to		
Class I . . .	1.01850	1.01870	1×10^{-6}	50×10^{-6}
Class II . .	1.0185	1.0187	1×10^{-6}	100×10^{-6}
Class III . .	1.0185	1.0195	10×10^{-6}	300×10^{-6}

The following precautions must be observed when using standard cells:

(1) Care should be taken in moving a standard cell, since any appreciable shaking up of the chemicals in the cell tends to produce variations of e.m.f. After a cell has been carried over large distances, it should be allowed to settle overnight prior to use.

(2) Standard cells should never be exposed to sunlight or other strong source of light and heat.

(3) For storage purposes, a dry location should be selected, having a fairly uniform temperature of about $+18°$ to $+22°C$ for class I cells and $+10°$ to $+30°C$ for class II cells.

(4) No current greater than 1 μA should be permitted to pass through class I and II cells. Class III cells will stand up to a short-time passage of up to 10 μA current.

When the above rules are observed, standard cells are sufficiently reliable and precise standards of e.m.f. for null methods of d.c. measurement.

Chapter Three

DIRECT-CURRENT AND ALTERNATING-CURRENT NULL DETECTORS

3-1. Types of Null Detectors

Any null-method network incorporates a null detector, i. e., a device which indicates when the network is at balance. In effect, this device indicates when there is no voltage between two characteristic points of the network(or there is no current in the branch between these two points). Hence the name "null detector".

From the above definition it follows that null detectors are not expected to measure a voltage or a current. Consequently, they may often have no graduated scale at all. Their reading arrangement must only be capable of demonstrating the presence, or otherwise, of a particular effect related to the voltage across the terminals of the detector. Quite a variety of reading arrangement exist, sometimes differing markedly from the usual pointer and dial.

From the purpose of null detectors, another specific feature of theirs is obvious. As will be realized, a null detector of greater sensitivity will sense the state of balance better than one of lower sensitivity. Hence arises the requirement for high sensitivity, especially near balance. This is why instruments with a square-law scale (electrodynamometers, rectifier instruments) are unsuitable as null detectors, since the actuating forces in them decrease rapidly near zero.

Obviously, there are detectors for both d.c. and a.c. work. The former are practically all moving-coil pointer or reflecting galvanometers of well known design and performance. Their prevalence is due to the fact that existing makes of galvanometers do their job quite well. It is only in few cases that their sensitivity has to be enhanced by suitable expedients and devices. In the subsequent discussion (see Sec. 3-2) we shall leave out the well known theory,

operating principle and design of moving-coil galvanometers and shall take up only two points: the matching of a moving-coil galvanometer used as a null detector with the measuring circuit and means of improving galvanometer sensitivity.

A different picture emerges in the case of a.c. null detectors where the preference for a particular type, so characteristic of d.c. detectors, is nonexistent. In part, this is because a variety of conflicting factors have to be taken care of (above all, frequency constraints). Also, there is no a. c. detector which would be as versatile and perfect as the moving-coil galvanometer. So, among the instruments employed as a.c. detectors are ordinary and tuned telephones, several designs of the vibration galvanometer, reflecting electrodynamometers with d.c. excitation, and various systems using rectifiers, valves, etc. Some of them deserve a more detailed discussion.

To begin with, it should be noted that an a.c. null detector is easier to select for sensitivity and matching with the measuring circuit than is a d.c. instrument. The thing is that in a.c. work the experimentor can always use a matching amplifier between the output of the measuring network and the detector. This is an elementary problem for modern electronics (see Sec. 4-5). It would seem that amplifiers might as well be used in d.c. work. However, suitable amplifiers are less perfect, reliable and convenient than a.c. amplifiers—a feature which limits their use only to cases where they are indispensable (in conjunction with bridges having a very high output resistance). In d.c. potentiometer work, amplifiers are not practically used.

While the choice of an a.c. null detector of the desired sensitivity is a fairly simple matter, a.c. networks impose several limitations which are not so simple to go around.

In the first place, a.c. measurements, especially at higher frequencies, are subject to errors due to leakage effects (most often, stray capacitive coupling). This necessitates the use of sophisticated shielding—a thing rarely occurring in d.c. measurements. Indeed, in the few instances where d.c. work calls for shielding, it can be accomplished by simple methods. This fully applies to the respective null detectors, so a.c. detectors should have appropriate provisions incorporated in their design (this is discussed in greater detail in Chapter 9).

Secondly, a.c. detectors must be sensitive to both the amplitude and phase of output voltage. This requirement, which has become important in recent years, stems from the fact that phase-sensitive detectors appreciably simplify and speed up the balancing operation in many cases. What is more important is that the balancing can be done to a plan rather than on the off-chance, as is the case with amplitude-sensitive detectors.

A third requirement is that a.c. detectors should be frequency-selective. This feature is embodied in phase-sensitive networks, vibration galvanometers and some other types.

The detectors in most common use for a.c. bridge measurements in the first half of this century were the telephones and the vibration galvanometer. The telephones in which the absence of a current is indicated by the cessation of the note are a very simple null detector for audio frequency, cheap and sufficiently sensitive (up to 10^{-8} A). Unfortunately, they suffer from serious drawbacks, namely that the exact moment when the note ceases will be determined differently by different operators; the sensitivity is heavily dependent on the frequency of the supply, varying by a factor of 2,000 in the audio range; and, finally, errors due to harmonics in the supply waveform are fairly great. By far the most unpleasant inconveniency, however, is that the telephone receiver has to be applied to the ear: this introduces a relatively great (and varying) capacitance between the measuring circuit and the experimentor's body and is not safe, especially in high-voltage work. The leakage current through this stray capacitance produces what may be called the head effect, and the minimum note appears blurred and difficult to determine. The head effect can be controlled but this calls for auxiliary facilities and complicates the procedure. At present, telephone detectors are used but seldom.

Vibration galvanometers were at one time the most commonly used tuned detectors for low frequencies. Their attraction lies in the fact that they can be tuned to resonance at the fundamental frequency so that response to harmonics will be greatly minimized. Unfortunately, vibration galvanometers are too involved and expensive, the minimum blur of the light spot is difficult to determine, the frequency range is rather narrow, and the sensitivity is relatively low (their current constant is of the order of 10^{-7} A/div.). This is the reason why vibration galvanometers have now dropped out of use, except in stationary applications.

Both the telephones and the vibration galvanometer are purely amplitude-type detectors, insensitive to the phase of the supply. This shortcoming has been avoided in reflecting galvanometers of the excited-field electrodynamometer type. The operating principle is the electrodynamic action between the current in a movable coil (or coils) and the current in two or more fixed coils between which the moving coil is suspended. If the current carried to the fixed coils is I_0, the actuating force of the electrodynamometer will be

$$M = kI_xI_0 \cos(\widehat{I_x, I_0}).$$

Consequently, when I_0=const, the response will be proportional to both I_x and the cosine of the phase angle between the unknown current and the reference current energizing the fixed coils.

The amplitude sensitivity of the detector (which is proportional to the number of turns and the magnitude of the supply current) can be made fairly high. Purely technical difficulties in using electrodynamometers (levelling, protection from mechanical vibration, light-spot indication with all the accompanying inconveniencies) coupled with high cost could not stimulate interest in them, and they are not manufactured in quantity.

The requirements of simplicity, reliability and low cost are, on the other hand, readily met by a.c. detectors based on valves and transistors, with a moving-coil instrument or a cathode-ray tube as the presentation instrument. They are most promising and least known. A more detailed account will be given in Sec. 3-3.

3-2. The Moving-coil Galvanometer

Existing galvanometers can detect currents down to 10^{-11} A and voltages down to 10^{-8} V, which is a sufficiently good performance for precision measurements.

Moving-coil galvanometers operate on the same principle as commercial moving-coil instruments. They, however, incorporate some additional features of design intended to enhance their sensitivity.

One such feature is a coil suspended by a flexible filament or between two filaments held taut by a bronze spring. This feature reduces friction practically to zero and makes it possible to obtain a very low restoring torque. Another feature is light-spot indication capable of detecting the smallest deflection of the coil.

The induction in the air gap of the magnetic circuit is made as high as possible so as to obtain a high sensitivity and efficient damping due to the eddy currents induced in the coil frame. True, the latter necessitates reduction of the induced current by including an additional resistance in the circuit under measurement. The external resistance at which critical damping is attained (the external critical damping resistance) is a characteristic next important after sensitivity. The optimum damping time is obtained with a condition near to critical (the galvanometer is said to be slightly underdamped). Ordinarily, the external critical damping resistance of current-sensitive galvanometers is several times their internal resistance. Therefore, when the measured source has a low resistance (r_i), the sensitivity has to be reduced, because a series resistance R_s is connected in the measuring circuit (Fig. 3-1) for optimum damping. If, on the other hand, the resistance of the measured source (r_i)

is in excess of the critical value, optimum damping is possible with a small sacrifice in sensitivity by shunting the galvanometer with a resistance R_{sh} (Fig. 3-2) of a value close to critical. Finally, when the resistance of the measured source varies within broad limits, damping close to critical can be obtained by putting in both resistances of approximately critical value, one shunt and the other series.

Another salient feature of galvanometers is a great free period (which sometimes is up to over ten seconds). Since the free period of a galvanometer is inversely proportional to the square root of the

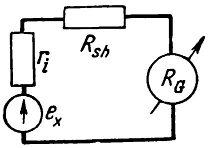

Fig. 3-1

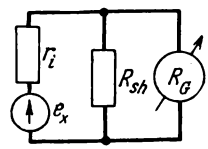

Fig. 3-2

restoring torque, reduction in the torque of the suspension for the sake of better sensitivity has an adverse effect on the speed of response of the galvanometer.

Thus, the desire to obtain maximum sensitivity conflicts with convenience in use and the speed of response. The way out is to obtain the maximum sensitivity (maximum power output) of the entire network and to match the galvanometer to the circuit for maximum power input into the galvanometer. If this is not done, a more sensitive galvanometer will have to be used to the detriment of mechanical properties and speed of response.

The sensitivity of a galvanometer can be enhanced in a variety of ways. Direct methods by improvement in design are least promising because of spontaneous zero drift and fluctuations due to a number of physical causes. Indirect methods are more attractive and efficient. In fact, an indirect method improves the sensitivity of the system consisting of a galvanometer and auxiliary (say, reading) devices, and not of the galvanometer proper. In the final analysis, however, much weaker currents and lower voltages can be detected than by the detector alone, and this is exactly what is sought.

Leaving out valve-type instruments, these indirect methods involve a combination of an amplifier and a relatively coarse moving-coil instrument, and can be broken down into four groups:

(1) Optical methods and apparatus for increasing the resolving power of the reading device.

(2) Electrical methods and apparatus for increasing the resolving power of the reading devices (in most cases based on photoelectric and thermoelectric effects). This group uses a second galvanometer and no feedback.

(3) Methods of increasing the deflection of the coil with small current. In one way or another, these methods employ positive feedback which increases the deflection of the galvanometer.

(4) Methods (usually photoelectric) of amplifying the galvanometer current by the compensation principle (i. e., with negative feedback).

The apparatus in the first two groups are fairly simple but operate in a narrow range of the measured quantities, i. e., with the coil of the primary galvanometer deflecting through a small angle. Since we are interested in galvanometers as null detectors, the small deflection of the coil is of no consequence, and such apparatus can be used as current or voltage null detectors.*

The last two groups (using feedback) can give better results over a broader range of the measured quantities (such as Soviet-made photoelectric amplifiers). These instruments are, however, expensive and their use as null detectors is hardly warranted. In our discussion we shall leave them out and shall only outline the operating principles of the apparatus in the first two groups.

The wide use of reflecting galvanometers could not but stimulate attempts to improve their sensitivity through optical systems of a greater resolving power. First introduced in 1826, the light spot and mirror system has been improved with the aid of optical *multipliers* which increase the final deflection of the light spot for the same deflection of the coil. The simplest of them is based on the multiple reflection of the light beam from two nonparallel mirrors before it reaches the scale. It is obvious that the final deflection will be proportional to the number of reflections. This method is employed in many makes of galvanometers, but the number of reflections has to be limited because of the considerable light absorption, and so the gain in sensitivity is ordinarily low.

There are also a variety of methods for increasing the final deflection using a single-reflection arrangement. By far the most effective system is probably one using a curvilinear (notably cylindrical) fixed auxiliary mirror. The path of the light beam in such a multiplier is shown in Fig. 3-3. The light beam reflected from the galvanometer mirror is incident upon an auxiliary cylindrical mirror of radius r at a distance R from the galvanometer, to be reflected from it at an angle θ.

* There are, however, instruments belonging to the second group which are primarily intended for the measurement of small currents and low voltages, such as the Kozyrev photoelectric-optical amplifier.

Let the angle between the tangent at the point where the light beam is incident upon the cylindrical mirror and the light beam itself be φ. When the incident beam turns through an angle $\Delta\alpha$, the reflected beam will be turned through an angle $\Delta\theta$, so that $\Delta\theta \gg \Delta\alpha$. With this arrangement, it is possible to increase the apparent

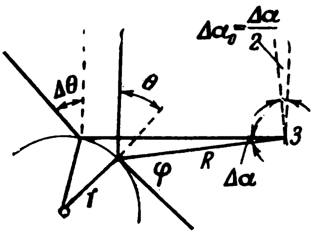

Fig. 3-3

angle of deflection several hundred times. With $R=100$ cm, $r=1$ cm, $\varphi=45°$, and $\alpha=2\alpha_0$, where α_0 is the deflection of the primary galvanometer, the final deflection will be

$$\Delta\theta \simeq 800\alpha_0.$$

A major drawback of this system is that there is no linear relationship between $\Delta\theta$ and $\Delta\alpha_0$ but this is of no consequence for null detectors.

Other systems have also been developed by different authors, including the principle of interference, but they have not found practical application.

The methods based on photoelectric and thermoelectric effects (the second group of methods) are used more frequently. In them, the light beam from the primary galvanometer is made to fall upon a suitable sensing element. The current generated in the sensing element varies in proportion to the deflection of the mirror in the primary galvanometer, resulting in an amplified movement of the light spot of the secondary galvanometer. Such is the fundamental principle which is modified in various layouts, the difference usually being in the type of sensing element used and in the associated circuit components and optical system.

The system using a differential thermocell was described as early as 1925. When the light spot from the primary galvanometer falls equally on the two junctions of a differential thermocell, the junctions are heated equally*, and the resultant thermo-e.m.f. will

* It is obvious that the thermocell must be very sensitive, since the thermal energy of the galvanometer light beam is small.

be equal to zero. An extremely small movement of the primary galvanometer disturbs the balance, and a current begins to flow in the external circuit, proportional to the amount of out-of-balance and of a sign dependent on the direction in which the primary galvanometer has deflected. The resultant thermoelectric current is measured by a secondary galvanometer whose deflection is nearly proportional (both in magnitude and direction) to the deflection of the primary galvanometer. If the cell and secondary galvanometer are sufficiently sensitive, an amplification of a hundred times can be obtained.

The thermoelectric principle can be embodied in a portable detector of the taut-suspension pointer type of a fairly high sensitivity. The pointer carries an auxiliary heater wound with a fine nichrome wire current to which is conducted by torqueless filaments. The movement of the pointer and, consequently, of the heater relative to a differential thermopile gives rise to a thermoelectric current which is measured by a secondary galvanometer also of the pointer type in most cases.

This principle has been successfully employed in the T-316 null detector which is part of the P-316 portable bridge (Sec. 5-6). The series-parallel thermopile employed in the bridge consists of 80 hot junctions and gives an amplification of the order of a hundred times, so that the total current constant is 5×10^{-8} A/division.

Thermoelectric amplifiers are sufficiently simple and convenient. Unfortunately, they suffer from a noticeable thermal lag. Even in instruments of short periodicity such as the T-316, the secondary galvanometer has a free period of 2.5 to 3 seconds, which is, of course, far too much. The periodicity can be reduced by using separate vacuum thermocells made up of very fine wires. This, however, makes the entire instrument too involved and calls for a special differential optical system.

It should be stressed that the inherent time lag of thermocells cannot be eliminated completely by any device. Since in many cases this is an important requirement, it is vital to have some other system free from this shortcoming. The problem can be solved by the use of photoelectric cells.* The underlying principle is the same as with thermocells, the only difference being that they are replaced by photoelectric cells.

It stands to reason that any form of photoelectric cell can be used, but the photovoltaic cell holds out especially great promise. Photovoltaic cells can be easily arranged into a differential relay and they require no auxiliary power supply. The fact that photovoltaic

* For some time past an increasing use has been made of radioisotopes for the detection of small deflections. A radioisotope is applied to the pointer, and the radiation is measured by a differential method. It is doubtfull, however, whether such instruments are practicable because of their complexity.

cells are difficult to use in conjunction with an amplifier is immaterial, since their current is of a sufficiently great magnitude without amplification.

3-3. Valve-type A. C. Null Detectors

Ever wider use is being made of valve-type a.c. null detectors, especially at audio and medium high frequencies. They can be employed in quite a variety of applications, namely to measure the extremum of the unknown voltage, to establish the equality between the absolute values of two voltages, to determine the presence of a phase shift between them, to establish the equality between phasors, etc. Each of these applications employs a special type of null detector, such as amplitude-sensitive, differential, phase-sensitive, vectorial, etc., which merit a separate analysis. We shall limit ourselves to amplitude null detectors (including phase-sensitive) intended to sense the least voltage across a portion of the measuring circuit. This type of detector is especially widely used in bridge and potentiometer work.

A valve-type null detector consists essentially of an a.c. voltage amplifier and a presentation device. The unknown voltage is fed to the input of the amplifier and, after amplification, is coupled out into the presentation device which converts it into an audio, light or mechanical signal. The null detector should have high sensitivity, high overload capacity, frequency selectivity, and ability to indicate the signal level objectively. The requisite resolving power is usually obtained by providing an adequate number of amplifying stages and by using selective amplifiers.

For objective indication and convenience in operation, preference is given to presentation devices which give visual indication. In manually adjusted systems, use is ordinarily made of a pointer instrument, a cathode-ray tube or an electron-tube tuning indicator. Simple and cheap makes of null detectors mostly use electron-tube tuning indicators, also known as magic eyes (for example, the Soviet-made Type 6E5C). They are, however, inferior to more complicated arrangements in terms of sensitivity and accuracy, although they are more reliable and convenient in service.

Before it is applied to the control grid of the 6E5C valve, the a.c. voltage must be rectified. The rectifier at the output of the amplifier does not usually entail any difficulties, since the magic eye, which acts as the rectifier load, has a high input impedance. Fig. 3-4 shows a typical arrangement using a magic eye.

C_1 and R_1 in this circuit make up a shunt-fed detector. R_2 and C_2 filter the rectified voltage. R_3 controls the negative potential at the grid, thereby controlling the initial opening of the dark sector.

Figure 3-5 relates the angle of opening to the control-grid potential of the indicator.

For maximum balance sensitivity (i. e., sensitivity near balance) it is essential that the angle of opening of the indicator account for

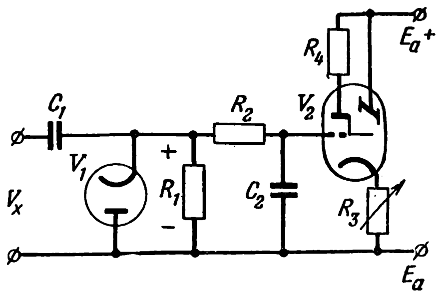

Fig. 3-4

as great a fraction of the initial angle as possible. Therefore, bias is so selected that the angle of the dark sector is close to zero in the absence of the signal.

The major drawback of magic-eye null detectors is the small size of the fluorescent target which cannot give precise indication. Indeed,

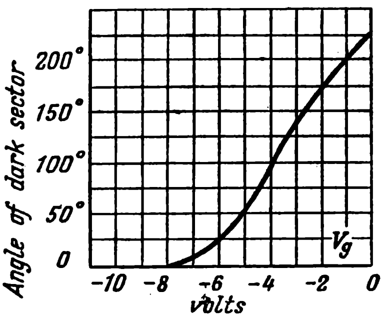

Fig. 3-5

if the detector is too sensitive, the range is very small: even the slightest unbalance of the basic measuring circuit causes the magic eye to open fully. Conversely, the sensitivity is intolerably reduced if the range is broad. A much better performance can be obtained with two magic eyes included in the measuring network, one for coarse adjustment of the measuring network proper and the other for fine adjustment at high sensitivity (it goes into action when the input voltage approaches zero).

The advantages of valve-type null detectors are objective indication, high overload capacity, and a broad frequency range.

The next common presentation device after the magic eye is the cathode-ray tube. The unknown voltage is applied to the vertical deflection (Y) plates. When no voltage is applied to the horizontal deflection (X) plates, the screen of the tube shows a straight line. Changes in the length of this line give a measure of changes in the amplitude of the output voltage. When this voltage is reduced to zero, the line contracts to a point. The resolving power of the cathode-ray tube is sufficiently high even with a beam of medium brightness but well focused.

Unfortunately, this otherwise convenient method suffers from a serious drawback. The spot on the screen where the beam is focused burns out and loses luminosity fairly quickly. It is more advantageous, therefore, to connect the cathode-ray tube so that the pattern on the screen varies in some manner without changing into a point. In the null detectors used in a.c. bridges, for example, the cathode-ray tube is often so connected that the Y-plates are fed with the signal voltage and the X-plates with the supply voltage of the bridge. As the unknown voltage is reduced to a minimum, the axis of the ellipse observed on the screen gradually rotates, and the ellipse contracts to a straight line which tends to take up a horizontal position, as the input signal decreases. This arrangement makes the null detector sensitive to both the amplitude and phase of the signal and provides for a longer service life of the cathode-ray tube.

Cathode-ray tubes do not require rectified voltage, have a high overload capacity and can operate over a broad frequency range.

By far the most common presentation device in amplitude-sensitive null detectors is, however, a moving-coil instrument with a semiconductor, valve or, though seldom, mechanical rectifier. Where valve rectifiers are employed, they are most often of the diode type. If the instrument is of high sensitivity, the rectifying diode is usually arranged for shunt feed and is connected to the anode of the output valve via a capacitor. The desired input resistance of the rectifier is obtained by inserting a multiplier resistor in series with the moving-coil instrument. Where necessary, the instrument can also be shunted by a resistor of the value equal to the external critical damping resistance (Fig. 3-6).

The zero shift due to initial-velocity current which is likely to occur in diode rectifiers is eliminated by the inclusion of another diode.

Wide use is made in presentation devices of cuprox and, especially, germanium rectifiers. Because of the low permissible reverse voltage, cuprox rectifiers are arranged either into full-wave or half-

wave bridge networks with opposing connected cells (Figs. 3-7 and 3-8).

Germanium and silicon diodes have a high reverse voltage and may be arranged into the same networks as valve diodes. The absence of initial-velocity currents makes them more advantageous than valve rectifiers.

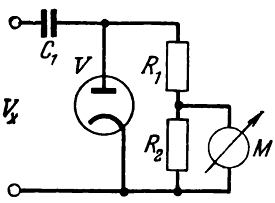

Fig. 3-6

The circuits examined above are not sensitive to phase reversal (i. e., a 180-degree change of phase) — a feature which limits their use and also makes it impossible to balance an a.c. circuit as it is done on direct current, where the passage through balance causes the pointer to deflect in an opposite direction. To make rectifier-type circuits sensitive to phase reversal, they are made controlled.

Among the simplest controlled rectifiers are mechanical rectifiers whose contacts are closed at regular intervals by what is called

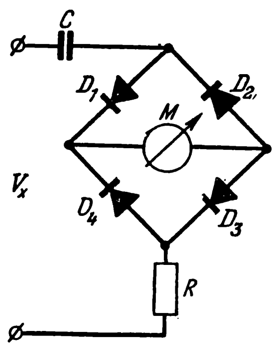

Fig. 3-7

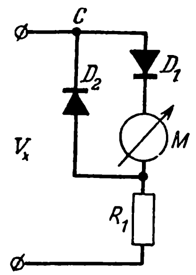

Fig. 3-8

a reference or control voltage. Since the conduction of a controlled rectifier changes in synchronism with the signal, the phase reversal of the signal causes the rectified current to reverse too. The current in the instrument of a mechanical rectifier is given by

$$I = kV_x \cos \varphi, \tag{3-1}$$

where k = circuit constant;

V_x = signal voltage;

φ = phase angle between reference voltage and signal.

The presentation device using a mechanical rectifier reads zero when $V_x=0$ and $\varphi=\pm\pi/2$. To make sure that the zero indication is

obtained when $V_x=0$, a simple phase-shifting network is incorporated in the circuit. It shifts the phase of the reference voltage through some angle (preferably, close to 90°). If the current in the instrument remains zero with a change in the phase of the reference voltage, then $V_x=0$.

The initial phase displacement of the reference voltage must be very close to $\varphi=0$. Then the sensitivity of the null detector will be the highest. A characteristic feature of bridge and potentiometer methods of measurement is that, while the circuit is being balanced, the phase of the signal varies between zero and 360°. If the sensitivity were to be constant, the phase of the reference voltage would have to be varied continually. It stands to reason that such continuous phase control is impossible to realise. Instead, the sensitivity is checked at regular intervals by taking the phase shift between the signal and the reference voltage while the measuring circuit is balanced. This, naturally, makes the measuring procedure somewhat complicated. The phase-shifting network must therefore be as simple as possible, such as the one shown in Fig. 3-9. When R is brought in circuit, the control current is approximately in phase with the supply voltage of the bridge. When C is brought in circuit, the current is shifted through an angle close to $\pi/2$. Instead of alternately bringing R and C in circuit, use may be made of an arrangement of two controlled rectifiers in which the reference voltages are in quadrature, and of two instruments. Then the rectifier currents will be proportional to the resistive and reactive components of the signal. When both instruments read zero simultaneously, the measuring circuit is at balance.

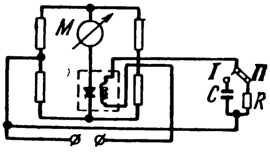

Fig. 3-9

Mechanical rectifiers have an exceptionally high coefficient of rectification, independent of the value of the rectified current, and enable the sensitivity of the moving-coil instrument to be utilized to the utmost. A disadvantage of mechanical rectifiers consists in the moving parts necessitating continuous watch on them. Because of considerable inertia, mechanical rectifiers can only be used at low frequencies. Where the signal frequency is in excess of several hundred cycles, use has to be made of controlled rectifiers built around semiconductor devices or valves.

Such rectifiers require a fairly high control voltage of rectangular or sine waveform. This voltage periodically shifts the operating point to the steeper portion of the volt-ampere characteristic, and the sensitivity of the controlled rectifier remains unchanged even though the signal is reduced—a feature nonexistent in uncontrolled rectifiers.

One of the simplest controlled rectifier circuits is shown in Fig. 3-10. Assume the control voltage V_0 to be of rectangular waveform. Then one diode D_1 is conducting during one half-cycle and the other diode D_2 during the other. Since the circuit is symmetrical, equal currents flow through the diodes, producing equal voltage drops across equal resistors R_1 and R_2. Therefore, in the absence of the signal voltage V_x, the current in the instrument is zero. If the signal

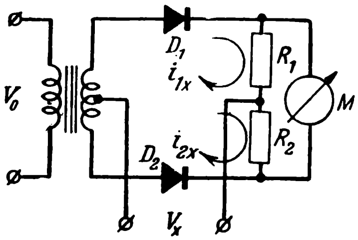

Fig. 3-10

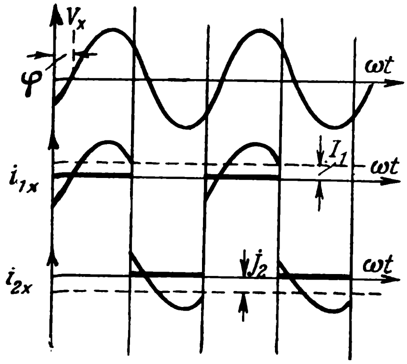

Fig. 3-11

voltage is shifted through an angle φ (Fig. 3-11) with respect to the control voltage, the average currents through R_1 and R_2 due to V_x will be

$$\left.\begin{aligned} I_1 &= \frac{1}{2}\pi \int_0^{\pi} (V_x/\Sigma R)\sin(\omega t+\varphi)\,d\omega t = (V_x/\pi\Sigma R)\cos\varphi; \\ I_2 &= \frac{1}{2}\pi \int_{\pi}^{2\pi} (V_x/\Sigma R)\sin(\omega t+\varphi)\,d\omega t = -(V_x/\pi\Sigma R)\cos\varphi. \end{aligned}\right\} \quad (3\text{-}2)$$

Here ΣR is the resistance of the circuit to the current flowing through the respective diode.

The instrument reads a current proportional to the difference in the voltage drops across R_1 and R_2. Since the voltage drops across these resistors due to the control voltage cancel out each other, then for $R_1 = R_2 = R$ we may write:

$$I = k_1(I_1R_1 - I_2R_2) = 2kR\,(V_x/\sum R)\cos\varphi = kV_x\cos\varphi. \quad (3\text{-}3)$$

If the control voltage is sinusoidal, the diodes D_1 and D_2 are fed with the sum and the difference of $\dot{V}_0$ and $\dot{V}_x$. The current flowing through the instrument remains proportional to the difference of the voltage drops across R_1 and R_2. The vectorial sum and difference

($\dot{V}_0+\dot{V}_x$) can be found from the vector diagram of Fig. 3-12 where the two voltages are shown shifted through a phase angle φ.

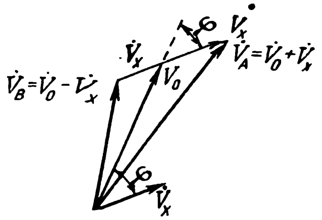

Fig. 3-12

The r.m.s. values of the voltages across D_1 and D_2 respectively are:

$$\left.\begin{aligned} V_A &= V_0\sqrt{1+2\cos\varphi\,(V_x/V_0)+(V_x/V_0)^2};\\ V_B &= V_0\sqrt{1-2\cos\varphi\,(V_x/V_0)+(V_x/V_0)^2}.\end{aligned}\right\} \qquad (3\text{-}4)$$

For a controlled rectifier to operate normally, it is essential that $V_0 \gg V_x$. Then $(V_x/V_0)^2 \approx 0$, and $2\cos\varphi\,(V_x/V_0) \ll 1$. From $\sqrt{1\pm\delta} \approx 1 \pm {}^1/_2\,\delta$ we obtain

$$\left.\begin{aligned} V_A &\approx V_0\,[1+\cos\varphi\,(V_x/V_0)];\\ V_B &\approx V_0\,[1-\cos\varphi\,(V_x/V_0)].\end{aligned}\right\} \qquad (3\text{-}5)$$

The current through the instrument is equal to the difference of these two voltages, or

$$I_M \approx kV_x\cos\varphi.$$

In other words, irrespective of the waveform of the control voltage (rectangular or sine), if a rectifier has linear dynamic characteristics, the deflection of the instrument will be proportional to the unknown voltage V_x, will be dependent on its phase and, when $V_0 \gg V_x$, will be independent of the magnitude of the control voltage.* When V_x goes through zero (which involves a phase reversal in bridge and potentiometer networks), the instrument deflects in an opposite direction.

In addition to a d.c. component, the current flowing through the instrument contains an a.c. component at a double frequency which is due to the reference voltage. Fairly large in magnitude, this component (a) overloads the instrument and (b) finds its way into the measuring circuit via the V_x terminals and produces undesirable effects there. To some degree, these drawbacks can be controlled

* As a matter of record, with a sinewave control voltage instrument deflection is not fully independent of the control voltage.

by interchanging the connections of V_0 and V_x (although this may introduce asymmetry in the operation of the transformer).

There are many types of controlled rectifiers using semiconductor devices. We shall take up the ring circuit, free from the disadvantages of the arrangement shown in Fig. 3-10 and widely used in practice. Fig. 3-13 shows the transformer variety of the ring circuit, while the one shown in Fig. 3-14 has no transformer.

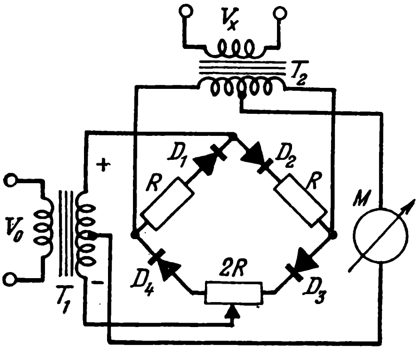

Fig. 3-13

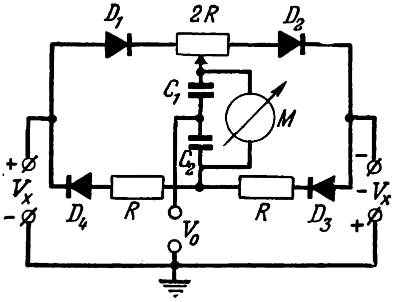

Fig. 3-14

In the circuit of Fig. 3-13, the diodes D_1 and D_4 may be regarded as making up a gating circuit. When the polarity of the voltage across the secondary of the transformer Tr_1 at a given moment is as shown in the figure, the diodes D_1 and D_4 are blocked and their conductivity is practically zero. It will be noted that the diodes D_2 and D_3 also make up a similar gating circuit. The only difference is that they are reverse-connected. As a result, when the left-hand circuit is conducting, the right-hand one is non-conducting, and vice versa. When D_1 and D_4 are conducting, while D_2 and D_3 are blocked, the indicating instrument will be connected to the left-hand half of the secondary of transformer Tr_2 via the diodes D_1 and D_4 and via both halves of the Tr_1 secondary. When the polarity of the control voltage is reversed, the indicating instrument is connected to the right-hand half of the Tr_2 secondary. Thus, the circuit shown in Fig. 3-13 is a controlled full-wave rectifier.

In the circuit of Fig. 3-14, the control voltage makes conducting the diodes D_2/D_4 or D_1/D_3 in turn. During each half-cycle the signal voltage (V_x and $-V_x$) produces a current which flows through the conducting diodes, charges the respective capacitor (C_1 or C_2), and completes circuit through the source of V_0 at the terminal V_x. At the same time, the charging current of one of the capacitors (i. e., half

the signal current flowing through the diode) flows through the indicating instrument. During the other half-cycle, the charging current again flows through the indicating instrument in the same direction. This circuit, too, gives full-wave rectification.

Controlled rectifiers using diode valves can be arranged in the same circuits as those based on semiconductors. With three- and

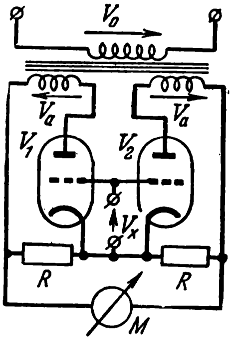

Fig. 3-15

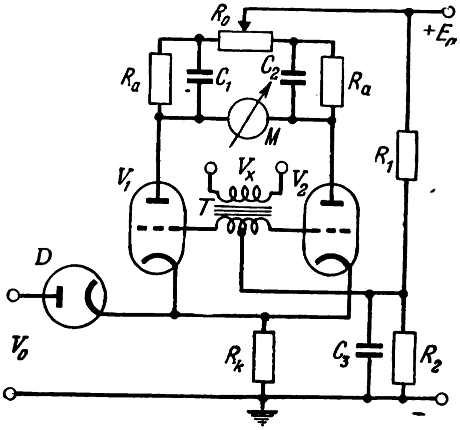

Fig. 3-16

multi-electrode valves, it is possible to construct circuits which can operate on both a.c. (synchronous feed) and d.c. Synchronous-fed circuits are exemplified in Fig. 3-15. The reference voltages applied to the anodes are in anti-phase, while the signal voltage is impressed on the grids in-phase. As a result, a vectorial sum and difference of V_0 and V_x are obtained which are fed into the anode circuits. This arrangement calls for a relatively high reference voltage, which is a certain inconvenience (especially at high frequencies). The reference voltage can be reduced by using mixer valves.

Among controlled valve circuits, use is most often made of gating circuits. An elementary circuit of this type is shown in Fig. 3-16. In the absence of the reference voltage V_0, the operating points of the valves V_1 and V_2 are in the middle of the linear portion of their dynamic characteristics, and the circuit is balanced for direct current. The reference voltage alternately blocks the valve amplifier whose grids are fed with the signal voltage V_x in anti-phase. As a result, synchronous rectification takes place. A major drawback of this circuit is that it draws power from the controlling circuit. This can, however, be eliminated by replacing the diode in the controlling circuit of

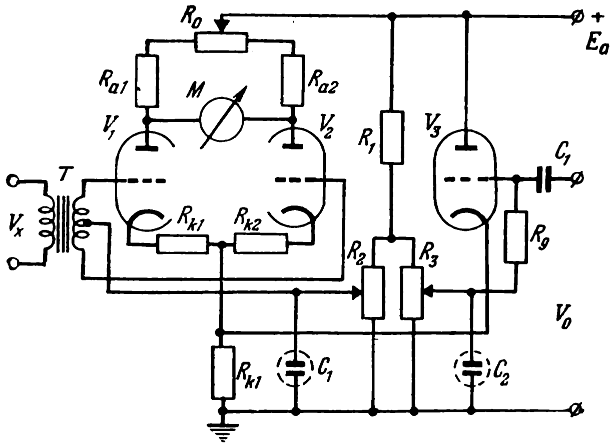

Fig. 3-17

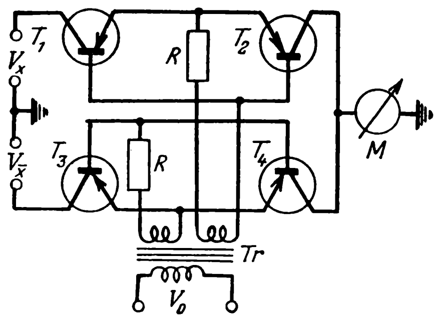

Fig. 3-18

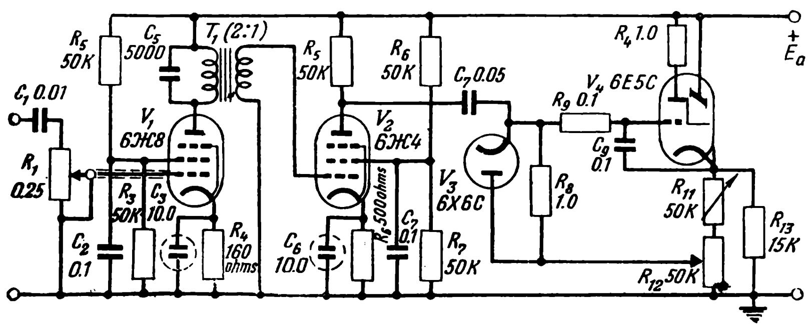

Fig. 3-19

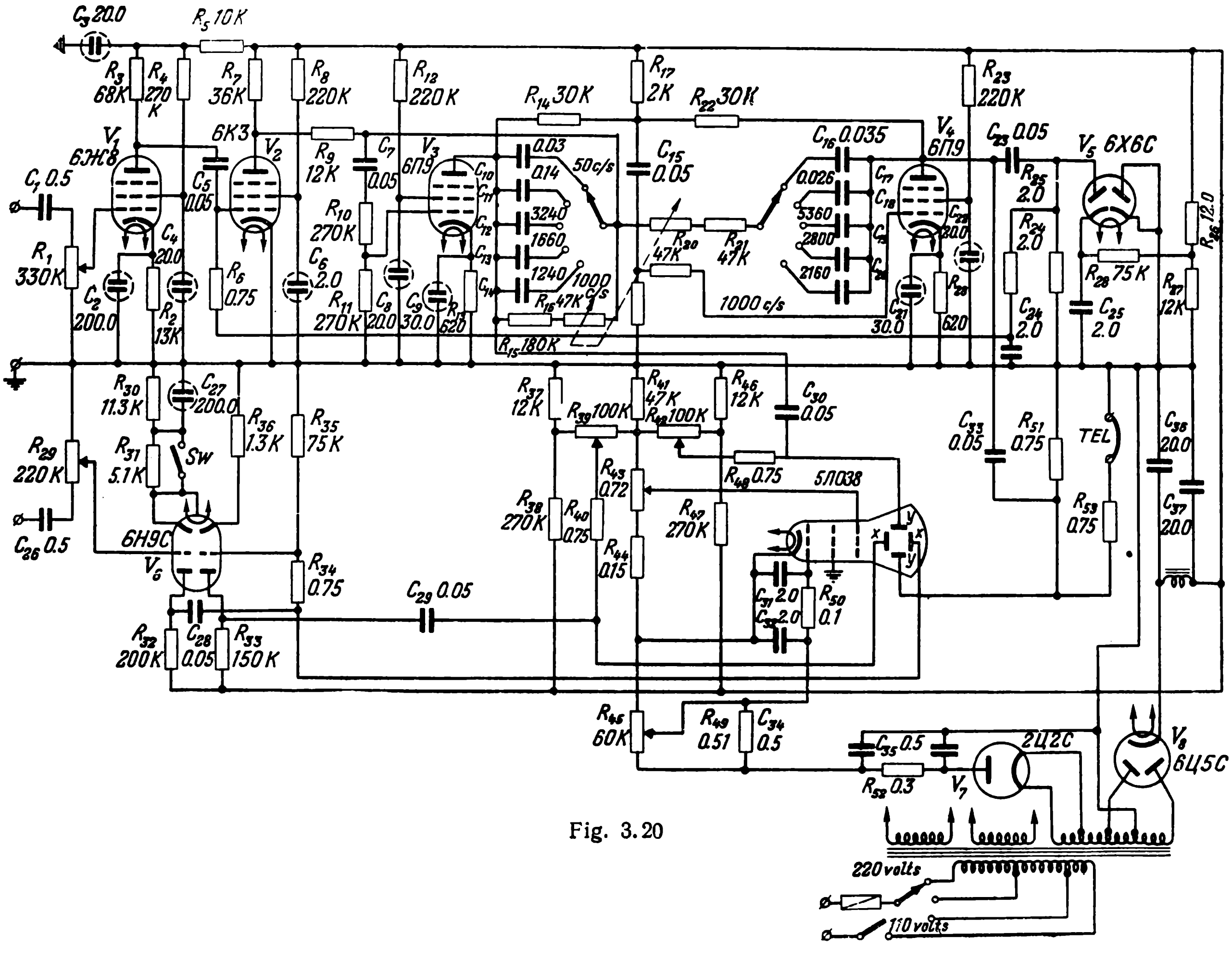
C3 20.0
R5 10K
R3 68K
R4 270 K
R7 36K
R8 220K
R12 220K
R14 30K
R17 2K
R22 30K
R23 220K
V1 6Ж8
V2 6K3
V3 6П9
V4 6П9
V5 6X6C
V6 6H9C
V7 2Ц2C
V8 6Ц5C
C1 0.5
R1 330K
C2 200.0
C4 20.0
R2 13K
C5 0.05
R6 0.75
C6 2.0
R9 12K
C7 0.05
R10 270K
R11 270K
C8 20.0
C9 30.0
620
0.03
0.14
3240
1660
1240
1000 c/s
50c/s
R16 47K
R15 180K
C15 0.05
R20 47K
R21 47K
1000 c/s
C16 0.035
0.026
5360
2800
2160
C17
C18
C22 20.0
C21 30.0
R26 620
C23 0.05
R25 2.0
R24 2.0
C24 2.0
R28 75K
C25 2.0
R26 12.0
R27 12K
R30 11.3K
C27 200.0
R36 1.3K
R35 75K
R29 220K
R31 5.1K
SW
C26 0.5
R34 0.75
C29 0.05
R32 200K
C28 0.05
R33 150K
R37 12K
R39 100K
R41 47K
R42 100K
R46 12K
C30 0.05
R43 0.72
R48 0.75
5Л038
R38 270K
R40 0.75
R44 0.15
R47 270K
C31 2.0
C32 2.0
R50 0.1
R45 60K
R49 0.51
C34 0.5
C35 0.5
R52 0.3
C33 0.05
R51 0.75
TEL
R53 0.75
C36 20.0
C37 20.0
220 volts
110 volts

Fig. 3.20

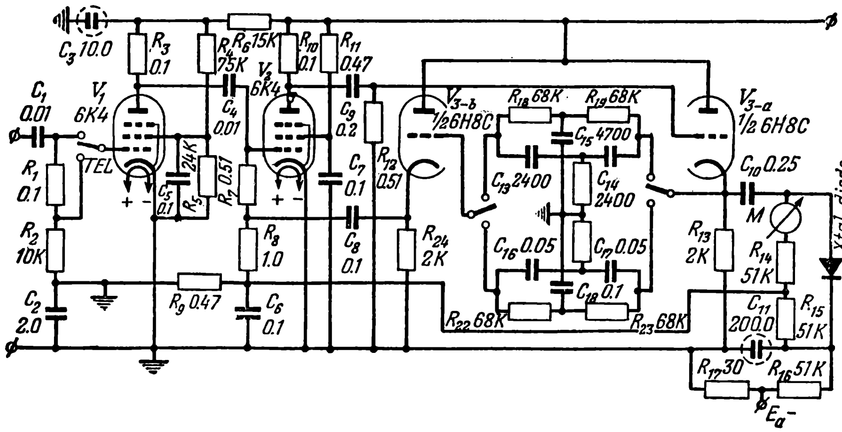

Fig. 3-21

Fig. 3-16 with a triode operating as a cathode follower (Fig. 3-17). Then the control voltage can be fed to the grid of the cathode follower (V_3). During a positive half-cycle of the control voltage, the valves V_1 and V_2 are nonconducting, and the signal cannot reach the indicating instrument. During a negative half-cycle, V_3 is nonconducting, and V_1/V_2 operate as a conventional amplifier. Since V_1 and V_2 are periodically blocked, the circuit operates as a synchronous rectifier.

Figure 3-18 shows a controlled transistor circuit coming into ever wider use now. As will be noted, the circuit consists of two identical gates. First consider operation of the transistors T_1 and T_2. Their emitters and collectors are connected in parallel. The control voltage is applied to them so that both transistors are either conducting or blocked simultaneously. When both are nonconducting, the resistance between the collectors runs into several megohms. When the polarity of the control voltage is reversed, the transistors become conducting, and the resistance of the gate drops to a few ohms. It should be stressed that such a gate is symmetrical with respect to the polarity of the gated voltage.

The above arrangement operates as a full-wave rectifier in which the gates (T_1/T_2 and T_3/T_4) are conducting in turn. It is analogous to a full-wave rectifier built around a polarized electromechanical relay. As distinct from the latter, the transistor rectifier can operate at both audio and higher frequencies.

Now let us turn to a few practical systems of null detectors.

Figure 3-19 shows a null detector whose output is coupled to a magic eye. The desired high sensitivity is obtained due to the high amplification of the signal and the high resolving power of the tuning indicator. In order to reduce

the influence of the higher harmonics and interference, the first stage is arranged into a fixed-tuned amplifier. The sensitivity can be adjusted by means of potentiometers R_1 and R_{12}. When the signal is applied to R_8, a rectified voltage is produced, bias to the grid of V_4 is reduced, and the dark sector of the magic eye opens. The threshold sensitivity of the null detector is about 20 μV.

Figure 3-20 shows the ИНО-3М cathode-ray-tube null detector manufactured in the Soviet Union. In addition to the *CRT* a pair of headphones TEL can be connected to the output of V_4. The null detector has two amplifiers: a four-stage quasi-resonant amplifier for vertical deflection and a two-stage aperiodic amplifier for horizontal deflection. The first two valves in the Y-amplifier use *RC* coupling. Bias to the grid of V_2 is taken from across the AGC detector (R_{24}) the delay for which is provided by R_{27}. Connected between the valves V_3 and V_4 of the paraphase stage is a Wien bridge made up of R_{14}/R_{22} and networks C_{10-14}/R_{15-16} and C_{16-20}/R_{20-21}. The signal applied to the grid of V_4 is taken from part of the anode resistor of the preceding stage (R_{17}). The same resistor R_{17} is connected to the upper bridge corner to provide an a.c. ground. The feedback voltage derived from across the bridge proper is fed to the grid of V_3 via the network C_7/R_{10-11}. At the frequency of quasiresonance the bridge is at balance, the negative feedback voltage is zero, and the amplifier has the highest gain factor. At frequencies other than of quasiresonance, the negative feedback voltage is other than zero, and the output voltage taken from the anodes of V_3 and V_4 to the vertical deflection plates is reduced. The frequency of quasi-resonance can be set at 50, 80, 400, 800 and 1,000 c/s by means of a capacitance selector switch. The frequency can also be trimmed to within ± 5 per cent of each setting by means of R_{16} and R_{20}.

The X-amplifier section uses the paraphase arrangement which provides for symmetrical supply to the X-plates. With the auxiliary voltage applied to the input of the X-amplifier, the measuring circuit can be balanced by watching the ellipse on the screen. Both the Y- and X-amplifiers have provisions for continuous manual gain adjustment. The gain of the X-amplifier can also be adjusted stepwise (using the switch Sw). At all working frequencies, the sensitivity of the

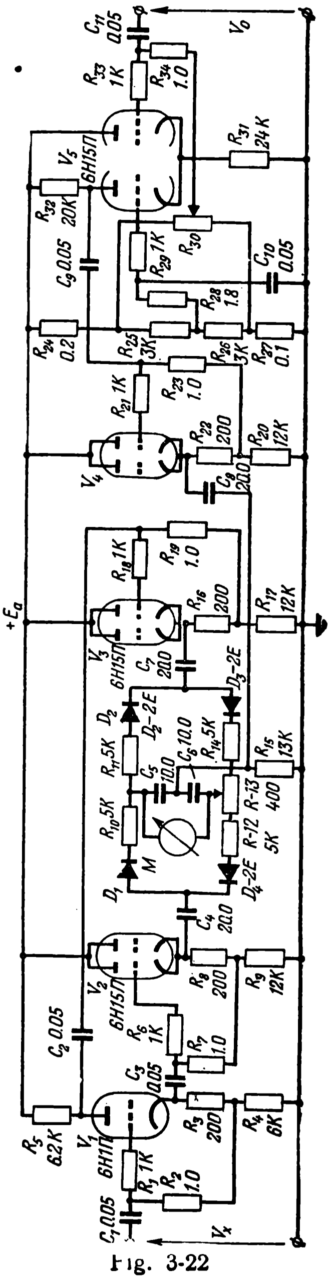

Fig. 3-22

null detector is not below 100 μV per mm of beam deflection. The attenuation of the 2nd harmonic is at least 20 db, and the input impedance is not less than 0,25 megohm.

Figure 3-21 shows the circuit diagram of a null detector employed in capacitance measurements. The null detector is in fact a quasi-resonance amplifier with a double parallel-T network in the negative feedback circuit. Feedback covers the second amplifying stage (V_2). From the anode of V_2, the voltage is fed to the output cathode follower ($V_{3\text{-}a}$) loaded by the parallel-T network. From the output of the parallel-T network, the feedback voltage is taken to the second cathode follower ($V_{3\text{-}b}$) and then to the grid of V_2. On the output side, the bridge operates at no-load. This is one of the best quasi-resonance circuits.

The presentation device is a pointer instrument connected to a shunt-fed rectifier. The rectified voltage developing across R_{15} serves to operate the AGC circuit which covers both amplifying stages. Initial bias is impressed on V_1 and V_2 from across R_{17} inserted in the common earth circuit.

The tuning frequency (50 and 1,000 c/s) can be selected by bringing the respective filter in circuit.

Figure 3-22 shows a phase-sensitive null detector. The detector is built around a ring rectifier. After phase reversal, the signal is applied to the grids of the cathode followers (V_2 and V_3) and then to the ring rectifier using Type D2-E crystal diodes. After it has been clipped on both sides by a cathode-coupled clipper-limiter based on V_5, the control voltage is fed to the cathode follower (V_4) and then to R_{15}. This voltage alternately blocks the diodes D_1/D_3 and D_2/D_4, thereby producing gating action in the circuit. Connected in series with the diodes are temperature-compensating resistors which enhance zero stability under varying temperature.

It is only too obvious that the above examples do not cover the whole field of valve-type null detectors which are described in detail in the technical literature.

Chapter Four

AUXILIARY EQUIPMENT

4-1. Types of Auxiliary Equipment. Requirements

In most cases, a measuring system consists of a measuring circuit proper, a null detector (or deflection-type instrument for nonbalanced circuits) and, finally, assorted auxiliaries which, though not connected into the measuring circuit, are essential to the normal operation of the system. The functions and character of the auxiliaries vary from system to system and also depend on the requirements that a given system is to meet: sensitivity, accuracy, etc.

The list of auxiliaries is topped by power supplies, both d.c. and a.c., since no measuring system can operate without them. Next in importance are auxiliary amplifiers, mainly a.c. and, though less frequent, d. c. Then come instruments for frequency measurements, waveform analysis, current and voltage measurements other than of the unknown signal, etc. Finally, there are devices for adjusting current and voltage (rheostats, variable transformers, etc.). We shall limit ourselves to power supplies and amplifiers, though a few general remarks will also be made about auxiliaries as a whole.

The most important requirements that auxiliaries must meet are: maximum reliability, low rate of failures, and simplicity of operation. Special emphasis is placed on the operating stability of auxiliaries.

The above requirements apply mainly to power supplies and amplifiers. As for auxiliary instruments, their accuracy should never be overspecified. One should never use a method and apparatus more accurate (and consequently, more complicated and expensive) than is actually necessary. This remark is the more so true of auxiliary instrumentation which may be of much lower accuracy than the basic equipment.

Here is an example in point. Suppose we are to measure variations in the capacitance of a capacitor as a function of frequency in the audio range (say, between 50 and 10,000 c/s). Under these conditions, we may expect that the relative change in capacitance will not exceed 1-2 per cent of the nominal value. Consequently, if this relative quantity is to be measured accurate to within 1 per cent, we should use a method measuring the absolute values of capacitance accurate to within 0.01-0.02 per cent. On the other hand, the frequency, which is also to be measured, varies by a factor of 200. It is obvious that it would be a sheer waste of time and effort to measure the frequency also accurate to within 0.01 per cent. A frequency meter with an accuracy of 1-2 per cent will do.

The situation is entirely different where frequency is the objective of measurement such as in evaluating the frequency stability of a.c. generators. Naturally, a far better measuring technique would be required so as to reduce the error many times.

4-2. D. C. Power Supplies

First and foremost, d. c. power supplies must generate an e.m.f. of high permanence. This is the reason why storage batteries are most often used to power measuring circuits. Dry cells whose e.m.f. is also fairly permanent are mainly employed in portable measuring sets, although for some time past there has been an increasing trend towards using them in laboratory work as well. Rectifiers are predominant as the power supplies of auxiliary circuits; when supplemented by stabilizers they also find use in the main circuits of measuring systems. Motor-generator sets are resorted to in special cases only.

The two types of storage cells most commonly used at present are the *lead-acid cell* and the *alkaline* (*nickel-iron or nickel-cadmium*) *cell*. The alkaline cell is more robust both mechanically and electrically; it can operate at high discharge rates, stands up well to rough handling, and to transient short-circuits. The average discharge voltage is 1.2 V per cell. One thing about the alkaline cell, however, makes the experimentor be decisively in favour of the lead-acid type. The thing is that when an alkaline cell is on discharge, its e.m.f. decreases although by small increments but continuously. There is practically no portion in the discharge curve where the e.m.f. could be regarded as constant.

Things are different with the lead-acid type: the discharge curve has a fairly sharp bend in the beginning and at the end, while its middle portion runs almost parallel to the time axis (curve *a* in Fig. 4-1 shows the discharge of a lead-acid cell, and curve *b* the discharge of an alkaline cell). This fact is of vital importance to laboratory work, and although the lead-acid type weighs more and is

not so strong mechanically, it is more suitable as a power supply for measuring circuits. When properly used and maintained, its stability is fairly high. It is important to remember, however, that the current of a lead-acid cell attains a steady-state value (to within 0.01 per cent) a long time after the load has been connected up or changed. Furthermore, this degree of permanence can only be obtained if the current drawn from the cell does not exceed 1 per cent of the 10-hr discharge rate. It is also essential that the cell be kept at a constant temperature: a change of 1°C causes a change of 0.01-0.02 per cent in current. Obviously, the bigger the cell, the

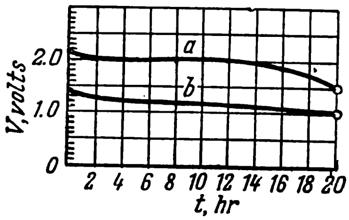

Fig. 4-1

steadier it is in operation and the greater the current that may be drawn from it.

So, with due care, lead-acid cells can deliver a fairly constant voltage. This is, however, of minor importance in some applications. In null methods, for example, the state of balance is independent of the supply voltage, and small variations in the latter, only slightly affecting the sensitivity of the measuring circuit, are quite tolerable. On the other hand, in any type of potentiometer work (or with an unbalanced bridge) all variations in the supply voltage will be present in the result as complimentary errors.

The main measuring circuits usually operate on currents in the milliampere range. Consequently, their storage cells can be discharged at low rates and at a steady voltage.

Auxiliary circuits draw various power, sometimes in considerable amounts (for example, when testing wide-range instruments). Consequently, it would be extremely difficult to maintain a high degree of stability in them. Generally, auxiliary circuits should also run on storage batteries, but the latter will have to operate under more adverse conditions (especially, as far as the discharge rate is concerned) than is the case with the main circuits.

4-3. Voltage Stabilizers

Recent advances in the design of voltage stabilizers, usually employed in conjunction with rectifiers, have stimulated their use in measurements. Intended mostly for auxiliary circuits, they have also proved advantageous in some types of main circuits, since they are convenient in service and produce good results.

The theory of voltage regulation has been given full coverage in the technical literature and will not be taken up here. We shall only examine the most commonly used types of voltage stabilizers.

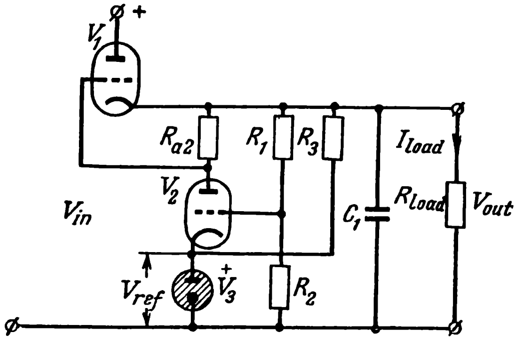

Fig. 4-2

Good use can be made of parametric regulators which employ nonlinear resistances (ballast valves, incandescent lamps, thermistors, stabilivolts, etc.), especially for auxiliary circuits. More attractive are, however, electronic regulators which act by negative feedback. The method is simply explained by the circuit of Fig. 4-2. It consists of a regulating or series valve V_1, a single-stage d.c. amplifier built around valve V_2, a stabilivolt V_3 which acts as a reference-voltage source, and a voltage divider R_1/R_2.

V_1 is connected in series with the load and acts as a variable resistance whose value changes with the input voltage and load current. When, for example, the output voltage increases due to an increase in the supply voltage or a reduction in load current, the grid voltage of V_2, which is the difference between the voltage across R_1 and the reference voltage V_{ref}, will also increase. As a result, the anode current of V_2 increases, the voltage drop across its load R_{a_2} rises, and the grid potential of the series valve drops. At the same time, the resistance of the series valve goes up, and the voltage drop across it increases by approximately as much as does the input voltage. Finally, the output voltage of the stabilizer remains practically un-

changed. When the output voltage decreases, the potentials and voltages in the stabilizer change in the opposite sense.

The load current, or output power, of a given stabilizer is solely dependent on the type of series valve employed. For a load current of several ten or hundred milliamperes, use should be made of power triodes such as the 6C4C and the 6H5C, or triode-connected tetrodes and pentodes of the Types 6П1П, 6П3С, ГУ-50, etc. Triode connection considerably reduces the internal resistance of these valves. Low internal resistance in a regulating valve is important for several reasons. Firstly, a low output impedance is obtained; secondly, the efficiency of the stabilizer is improved. As a way of increasing load current, it is customary to connect several regulating valves of the same type in parallel. It should be noted that the series valve must draw no grid current, or the operation of the stabilizer will be greatly upset.

A few words about the remaining circuit components of the stabilizer shown in Fig. 4-2. R_3 governs the current of the stabilivolt. C_1 prevents the circuit from going into self-excitation. The d.c. amplifier controls the resistance of the series valve. The greater the gain factor of the d.c. amplifier, the greater the stabilization factor. As will be recalled, the gain factor is determined by the valve characteristics and the resistance of the anode load. Therefore, the d.c. amplifier usually employs triodes or pentodes with a high gain factor.

The stabilization factor of the electronic regulator examined above is of the order of several hundreds. It can be further increased by using several (usually two) d.c. amplifier stages. Then the stabilization factor will be of the order of several thousands.

Electronic regulators provide for continuous control of output voltage, for which purpose the divider R_1/R_2 is made variable.

One advantage of such a stabilizer is that it removes ripple as well as d.c. mains variations. Since ripples may be regarded as fast changes in output voltage, they will be reduced at the output in comparison with the input by a factor of K (or the stabilization factor).

Commercially available stabilized rectifiers, such as the BC-11, BC-12 and BC-13, are mainly intended for use in the auxiliary circuits of measuring systems. In most cases, they can reduce a ± 10 per cent mains voltage variation to $\pm(0.1$-$0.2)$ per cent, with the supply frequency varying within ± 1 per cent and the load between 10 and 100 per cent of the rated value. The stabilizers are intended for laboratory work at an ambient temperature of $20° \pm 5°$C and a relative humidity of not over 80 per cent.

The output voltage of the BC-11 can be adjusted between 150 and 300 V in two ranges: from 150 to 225 V and from 225 to 300 V. Within each range, voltage adjustment is continuous. Load current can be varied between 10 and 100 mA.

The BC-12 consists of two independent units, one being an output voltage regulator similar to the BC-11 with a load current of 30 to 300 mA, and the other with an output voltage of zero to 75 V and a load current of up to 5 mA.

The BC-13 has an output voltage of 6.3 V ±5 per cent and a permissible load current of up to 5 A.

When using electronic regulators in measuring circuits, it should be remembered that the stabilivolt which supplies the reference voltage is subject to sudden changes in voltage (up to ±0.1 to 0.2 per cent). This will produce a proportional change in the stabilizer output.

The Vibrator Works in Moscow manufactures quite an unorthodox voltage regulator which is employed in the Type У1136 test set (Fig. 4-3). It operates on the principle that a photoconductive cell amplifier automatically holds a

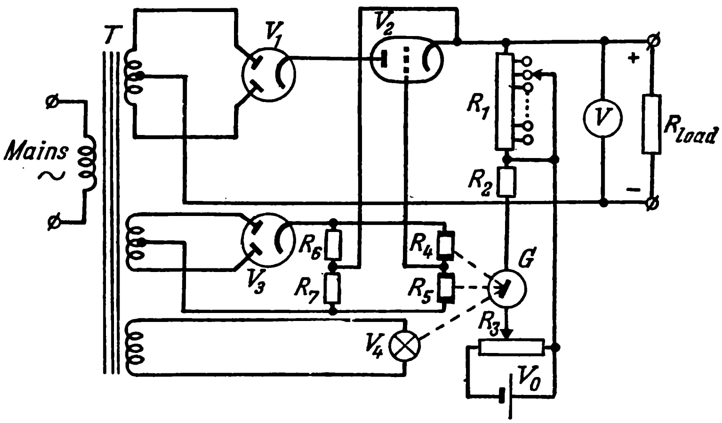

Fig. 4-3

preset voltage by comparing it with a reference voltage. The voltage drop across R_2 is compared by a taut-suspension galvanometer with the voltage across the voltage divider R_3 fed by a constant-voltage source (usually, a dry cell). Reflected from the mirror, the light beam falls onto two photoconductive cells R_4 and R_5. When the system is at balance, both photoconductive cells are illuminated equally, and there is no current flowing across the bridge made up by the two photoconductive cells and resistors R_6 and R_7. When the output voltage of the stabilizer increases or decreases, the galvanometer deflects, and different amounts of light fall on the photoconductive cells. As a result, a current begins to flow through the bridge, developing the voltage that controls the regulating valve V_2. The voltage which can be taken from across the divider R_1/R_2 can be changed in eight steps: 3-7-5-15-30-75-150-300-450 V. Within each step, the voltage can be continuously adjusted from 0.1 V up. The stabilizer reduces a ±10 per cent mains voltage variation to ±0.01 per cent. After a 10-minute warm-up, the stabilized voltage remains constant to within ±0.005 per cent per minute. Load current may be as great as 300 mA in the ranges up to 75 V, 150 mA in the 150-V and 300-V ranges, and 30 mA in the 450-V range.

There are also several types of electronic regulators for low voltages (20-30 V). They are, however, complicated, bulky and uneconomi-

cal devices which have not found any appreciable use in measurements. Yet, the need for them is great.

The increasing use of crystal diodes and transistors in recent years has given big impetus to work on voltage regulation by means of semiconductor devices. Some of the circuits devised are of definite interest to measurements.

Semiconductor voltage regulators which act by negative feedback are constructed along the same lines as valve-type stabilizers. Regulation is effected by the collector-emitter region of a junction-type transistor whose resistance is controlled by the base current. The constant-voltage source may be either low-voltage dry cells which supply a fairly constant voltage since the discharge rate is low, or avalanche (Zener) diodes, such as Types D-808 through D-813. These are silicon diodes made by a special process. When a reverse voltage is applied to them, an abrupt (or avalanche) breakdown occurs at certain values of voltage and current. This breakdown resembles the firing of gas-filled valves (such as stabilivolts). When an avalanche breakdown takes places, the voltage across the Zener diode remains constant over a fairly broad range of currents (Fig. 4-4) owing to two factors: (a) impact ionization in the solid and (b) the avalanche-like (cumulative) multiplication of mobile current-carriers due to a sharp increase in diffusion.

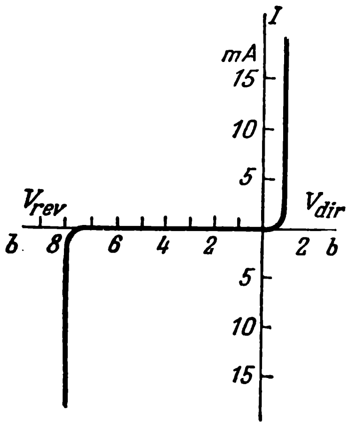

Fig. 4-4

The advantages that Zener diodes have over gas-filled valves may be summed up as follows:

(1) When manufactured by a suitable process and from appropriate source materials, Zener diodes can be made for voltages from a few volts to several hundred at working currents from a few milliamperes to several amperes.

(2) They have no firing potential, which in other diodes exceeds the stabilization voltage.

(3) The volt-ampere characteristic has no drooping portion, and so Zener diodes are not liable to self-excitation.

(4) Zener diodes are free from spontaneous abrupt changes in the stabilization voltage.

(5) Provided the supply voltage and current are sufficiently constant, the reproducibility of the stabilized voltage in consecutive operations is extremely high.

(6) There is no ageing, i.e., irrevocable changes in the stabilized voltage, after several thousand hours of operation.

(7) Zener diodes are compact in size and have a greater overload capacity than gas-filled valves.

(8) When used on the straight portion of the volt-ampere characteristic (Fig. 4-4), Zener diodes can operate as low-voltage regulators (0.6-0.8 V) with a sufficiently low dynamic resistance.

(9) The voltage-temperature coefficient is of opposite sign for the forward and reverse direction. For the reverse direction it is ± 0.05

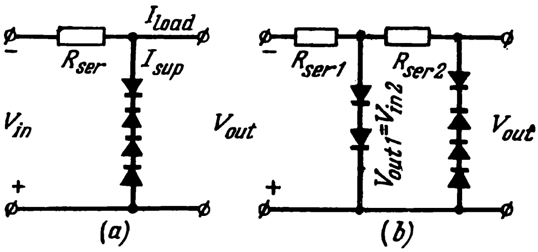

Fig. 4-5

to 0.08 per cent/°C change in temperature, and is greater for diodes with a higher stabilized voltage.

By virtue of the property in (9), several Zener diodes can be connected both aiding and in opposition, giving a voltage-temperature coefficient of ± 0.001 per cent/°C. Such a battery may well be used as a standard source in precision semiconductor stabilizers (which are discussed later), and also independently as the parametric type of regulator to power low-power measuring circuits, such as the working circuits of high-resistance potentiometers.

Figure 4-5 gives the circuits of (a) a single-stage stabilizer and (b) a two-stage stabilizer.

With Type D-808 Zener diodes arranged in a single-stage stabilizer, the voltage-temperature coefficient can be reduced to ± 0.001 per cent/°C. In a two-stage stabilizer, it is not practicable to employ temperature compensation in the first stage, since this appreciably increases the dynamic resistance of the diode battery. Instead, temperature variations should be taken care of in the second stage, the more so that they are reduced by the second stage many times. With approximately the same voltage-temperature coefficient (± 0.001 per cent/°C), two-stage stabilizers reduce ± 10 per cent mains voltage variations to ± 0.01 per cent at a practically constant load. In other words, the permanence of the output voltage of such a stabilizer is about the same as that of a standard Weston cell.

Where the load is great and varying within broad limits, resort is made to compensation transistor regulators, such as one shown schematically in Fig. 4-6. It operates as follows. When the voltage across the load increases, the current through R_1 also increases. As a result, the base current of the amplifying transistor T_2 rises, causing its collector current to increase and the base current of the regulating transistor T_1 to decrease. The resistance of the collector-emitter region of T_1 goes up, and the voltage across the load is restored to about the previous one. The regulator operates similarly when the output voltage decreases.

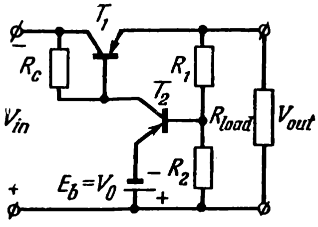

Fig. 4-6

It should be noted that, as distinct from valves, the junction-type transistor is controlled by current rather than by voltage. Therefore, for a high stabilization factor to be obtained, the d.c. amplifier must have a high current gain factor.

Since the space available does not permit a detailed examination of the semiconductor regulators described in the technical literature, we shall limit ourselves to the fairly perfect arrangement of Fig. 4-7.

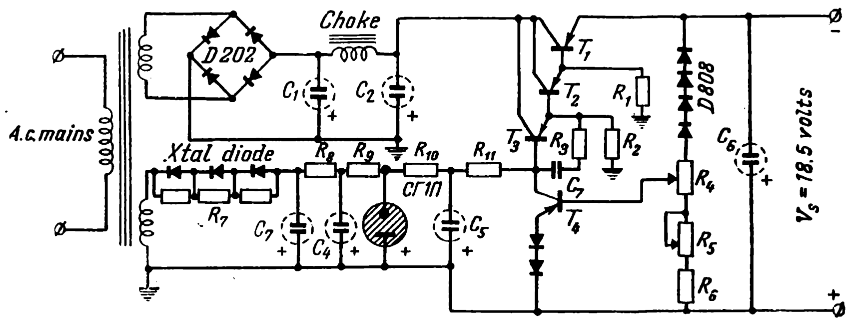

Fig. 4-7

The constant-voltage source in this circuit is a battery of Type Д-808 Zener diodes, with two of them connected for reverse and four for forward conduction.

The regulating component is a composite transistor consisting of two Type П13Б junction-type low-power transistors and a Type П4Д junction-type power transistor. A Type СГ1П stabilivolt feeds the collector circuit of the amplyfying transistor T_4.

The performance of this stabilizer has been found to be as follows:

(1) a ± 15-per cent mains voltage variation is reduced to ± 0.003 per cent at the output;

(2) reduction in the load current by half the maximum value causes the output voltage to change by not more than ± 0.006 per cent;

(3) in the temperature range from $+15°$ to $+50°C$ the output voltage changes by not more than ± 0.01 per cent;

(4) the drift of the output voltage does not exceed ± 0.01 per cent in fifteen days.

(5) the ripples in the output voltage do not exceed 1 mV, or 0.005 per cent.

That is a fairly good performance.

Like electronic stabilizers which act by negative feedback, semiconductor stabilizers are efficient smoothing filters, since they remove both ripple as well as d.c. mains variations. This feature is of particular importance where low voltages and low load resistances are involved. Ordinary *LC* and *RC* filters in these applications turn out to be very bulky and inefficient.

As more know-how is acquired in the manufacture of semiconductor diodes and triodes, especially of silicon, better designs of stabilizers will appear in the near future. Indeed, it may be expected that they will be given preference in measurement applications.

4-4. A. C. Supplies

According to frequency, the a.c. supplies used in measurements may be classed into three groups: low-frequency, audio-frequency, and radio-frequency. The latter group will not be taken up, since it belongs in radio engineering.

Measurement circuits are fed with commercial frequency mainly where a.c. instruments have to be tested. This purpose is served by suitable a.c. generators. If accuracy is not essential, an ordinary mains supply may be used. Where necessary, transformers are employed, and also filters if the higher harmonics have to be supressed.

In quite a number of applications, audio frequencies will do the job best. In addition to general requirements, audio-frequency generators should have high frequency and amplitude stability and generate pure sine waveforms (with no higher harmonics). Also, they should operate in the desired frequency range and deliver ample power at the output. As for the frequency range, existing audio-frequency generators are built for operation between 20 c/s and 200 kc/s; there are no firmly established limits.

There are also audio-frequency generators with fixed frequencies. With them the selection of the working frequency and, consequently, of the generator type depends on the frequency at which the object of measurement operates and on conditions of measurement.

Normally, generators for measuring applications have a power output of 0.1 to 1.0 W. This is quite enough for most systems (especially of the bridge type) employing audio-frequency generators. It may be noted that greater outputs can be obtained without difficulty, but any excessive stress on a generator and additional amplification, may result in failure to meet the key requirement—production of a sine waveform. It should also be remembered that the output of a generator varies with the load resistance.

Thus, the fundamental requirements are frequency stability and an undistorted voltage sine waveform. It is beyond the scope of this book to examine the ways and means of maintaining frequency stability and sine waveform. It will suffice to mention that generators (especially those with fixed frequencies) are fairly good in this respect. Ordinarily, frequency stability is one part in 100 to 1,000, reaching one part in a million or ten millions in standard signal generators. As for the waveform, its departure from the sine law is described by the distortion factor, ν, which is the ratio of the r.m.s. value of all the harmonics to the r.m.s. value of the fundamental frequency:

$$\nu = \sqrt{(V_2^2 + V_3^2 + V_4^2 + \ldots)/V_1^2} \times 100 \text{ per cent.}$$

In existing generators, the distortion factor is usually 1 to 3 per cent. Where necessary, it may be reduced to 0.1-0.2 per cent by means of suitable filters. Further reduction is difficult to obtain, the more so that what can be obtained without it is quite enough for most practical cases.

According to principle of operation, audio-frequency generators may be classed into three groups: the rotating type, the buzzer type, and the electronic type (or valve oscillators).

The rotating type and, to a considerable degree, the buzzer type have fallen out of use because of obsolescence, and we shall not take them up. It should only be noted that they cannot be used for precision measurements, since their output waveform is heavily distorted. Also, buzzer-type generators are incapable of continuous frequency adjustment. Although this is feasible in the rotating type, the frequency stability is poor for the reason that it depends on the r.p.m. of the drive motor which may vary within broad limits.

So, the main type of a.c. source for measurement applications is the valve oscillator. It is simple in operation, reliable in service, delivers sufficient power and has an almost sinusoidal waveform over a broad frequency range with a fairly high frequency stability. The impedance of the oscillator output stage can be conveniently matched with the impedance of the load (i.e., the measurement circuit) by means of a tapped transformer.

According to principle of operation, valve oscillators may be further subdivided into three types: the LC type (very seldom used at the present time), the beat-frequency type, and the RC type, of which the latter is most commonly used.

The Beat-frequency Type. As distinct from the LC oscillator, it can be easily constructed to cover any frequency range down to very low frequencies. This is the reason why it should preferably be used as a broad-band oscillator.

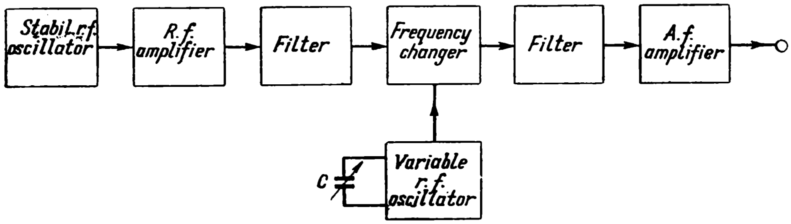

Fig. 4-8

A typical beat-frequency oscillator is a combination of two r.f. oscillators, a frequency changer, filters and amplifiers. As will be recalled, the beat frequency is one of the two additional frequencies produced when two different frequencies are combined. One beat frequency is the sum of the two original frequencies; the other is the difference between them.* Therefore, any low frequency can be obtained by combining very high frequencies, provided they are sufficiently close to each other. Normally, the r.f. oscillators making up the beat-frequency unit are built for frequencies of the order of 100 kc/s, one of them being of the fixed-frequency type and the other with a provision for varying the frequency to suit the desired audio-frequency range.

Beat-frequency oscillators may vary greatly in their circuitry. Functionally, they conform to the block diagram of Fig. 4-8.

Without going into detail, the beat-frequency principle can be conveniently embodied in a single-band audio oscillator for any frequency range from zero to f_{max}. For this, it is essential that the frequency of the first r.f. oscillator be f_1, and the frequency of the second r.f. oscillator vary from f_1 to f_1+f_{max}. Since f_1 and f_2 are radio frequencies, f_2 can be easily varied within the above limits by means of an ordinary variable air capacitor. On the other hand, it is not simple to maintain the frequency stability of the b.f. oscillator, since a relatively small change in the original frequency brings about

* Actually, a fairly complicated frequency spectrum is obtained in which the sum or difference frequency can be singled out. For a more detailed discussion the reader is referred to books on radio engineering.

a considerable change in the beat frequency for the reason that the latter is much lower than the former. This is especially noticeable at the lowest frequencies, at the beginning of the scale. As a way out b.f. oscillators often have suitable zero adjusters which are small trimmer capacitors. They make it possible to trim the frequency of the variable r.f. oscillator so that it equals the frequency of the other, with the tuning knob set at zero.

The obvious advantages of beat-frequency oscillators are, above all, a very broad frequency range from the lowest frequencies up, low nonlinear distortion, simplicity and flexibility in operation. These advantages offset the more complicated circuitry and lower frequency stability in comparison with other types of oscillators, the more so that, where necessary, the stability can be enhanced by special measures. Generally, beat-frequency oscillators are simpler, smaller in size and lighter in weight than *LC* oscillators owing to the fact that their r.f. oscillators are very simple. Such have been the reasons behind their wide use as broadband oscillators in measurements. They, however, compare unfavourably with *RC* oscillators.

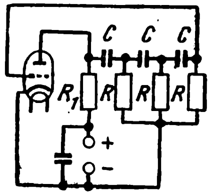

Fig. 4-9

RC oscillators first appeared in the USSR in the late thirties. Simple design, small size and low cost spurred their use. Today, most of the measurement oscillators are made in the *RC* class.

A typical *RC* oscillator, the main features of which are shown in Fig. 4-9, is in fact a resistance-coupled amplifier with feedback. As will be recalled, such circuits tend to operate as relaxation oscillators, producing an output waveform far from being sinusoidal (such as in the well-known two-stage multivibrator). This is because in normal relaxation oscillators the condition for oscillation, i.e., a definite relationship between the phases of the anode and grid voltages, is satisfied simultaneously for a whole range of frequencies. By suitable measures, the requisite phase relationship can be obtained for a single frequency; then the oscillator output will be sinusoidal at exactly that frequency.

In the circuit of Fig. 4-9, the resistance R_1 in the anode lead provides a feedback path to the grid. This feedback path incorporates a phase-shifting network consisting of three capacitors and three resistors.

The network produces the phase shift necessary for oscillation. Obviously, this shift depends on frequency, and the condition for oscillation will only be satisfied for a single frequency. As is proved in books on radio engineering, for steady oscillation in the circuit of Fig. 4-9:

$$f = 1/15.4\ RC \text{ when } R \gg R_1.$$

One advantage of *RC* oscillators is that very low frequencies are obtainable with small components. While in *LC* oscillators, the resonant frequency is inversely proportional to the square root of capacitance times inductance, in *RC* oscillators it is inversely proportional to resistance times capacitance. To put it differently, a ten-fold change in the resonant frequency in an *LC* oscillator requires a 100-fold change either in the capacitance or in the inductance. In an *RC* oscillator, a tenfold change in the capacitance or the resistance will do. As a result, very simple and small *RC* oscillators can be built. Furthermore, the use of resistors instead of inductors also simplifies the construction of the oscillator, especially where very low frequencies are to be generated.

As compared with beat-frequency oscillators, *RC* oscillators also offer certain practical advantages, mainly due to simple circuitry and frequency stability. Incidentally, *RC* oscillators can in principle be built of very high stability, especially on relatively low frequencies, since variations in frequency are caused by variations in capacitors and resistors which are among the most stable circuit components.

As for output waveform, *RC* oscillators are comparable with any other type of oscillator.

Figure 4-10 shows the schematic diagram of the Type 3Г-10 audio-frequency *RC* oscillator. It incorporates a two-stage exciter with a positive feedback path through a phase-shifting network $R_1 C_1 / R_2 C_2$ and regenerative feedback via

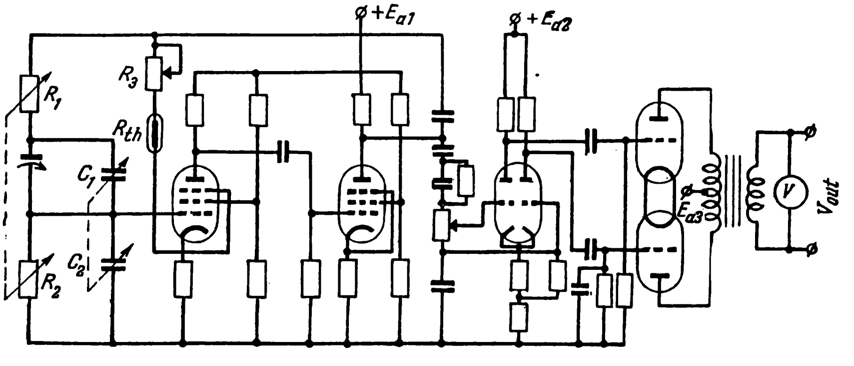

Fig. 4-10

R_3 and R_{th}. The latter is a Type TП6/2 thermistor which automatically controls the amplitude of oscillation.

The entire frequency range of the oscillator extending from 20 c/s to 20 kc/s is divided into three bands which are selected by varying the values of R_1 and R_2 in the proportion of 1 : 10 : 100. Within each band, continuous adjustment is effected by means of ganged tuning capacitors C_1 and C_2.

The basic technical data of the oscillator are:

Frequency accuracy . . . . . $\pm (0.02f_i + 1)$ c/s
Rated power output . . . . . 0.5 W
Maximum power output . . . 5 W
Nonlinear distortion:
at rated power output . . . below 0.7 per cent
at maximum power output . . below 1.5 per cent
at maximum power output
into 5 kiloohms load . . . below 2 per cent

The output impedance of the oscillator is designed for matched loads of 50, 200, 600 and 5,000 ohms.

The frequency response with respect to the 400 c/s level is:

50-10,000 c/s . . . . flat to within ± 1 db
20-20,000 c/s . . . . flat to within ± 3.5 db

The range of the output voltmeter is 60 V.

The Type 3Г-11 and the Type 3Г-12, also available commercially, do not differ in principle from the Type 3Г-10 except that they have a fourth frequency band which has extended their frequency range to include 200 kc/s.

Irrespective of the type of oscillator chosen, its operation and, consequently, its frequency stability will depend on the quality of power supplies. Most existing oscillators are designed to operate on mains supply and incorporate provisions for voltage stabilization in conditions of normal mains variations.

4-5. Auxiliary Amplifiers

The measurement of low voltages usually involves what are known as *instrument amplifiers.* They are electronic amplifiers of a sufficiently high gain and stabilized output. Their output is coupled to an indicating instrument whose scale is graduated in units of the input quantity with an allowance for the constant gain factor. In fact, these amplifiers are valve voltmeters for the millivolt range. They can be used in both a.c. and d.c. circuits.

It is not difficult to see that any variation in the gain factor will constitute an error in the final result. Therefore, the requirements for gain factor stability are very stringent. Very often, the designer has to go to the full extent of the measures known in radio engineering in order to meet this requirement. Therefore, instrument amplifiers are very complex special-purpose devices lying outside of the scope of this book.

Wide use in measurements is also made of simpler auxiliary amplifiers. Their function in most cases is to build up the signal (voltage) at the output of a measuring circuit (ordinarily, the circuit to be balanced) where the signal is too weak. In principle, both a.c. and d.c. auxiliary amplifiers can be built; practically, the former predominate.

The use of a d.c. amplifier entails additional difficulties, and the experimentor gives preference to an a.c. amplifier wherever possible.

Auxiliary amplifiers are almost invariably connected to the output of a.c. bridges. It is obvious that the requirements for them cannot be but easy. Since the objective is to enhance the sensitivity of a null method, i.e., a method by which the complete balance or zero-output condition is sensed, some variations in the gain factor will not affect the final result.

The frequency range in which auxiliary amplifiers are to operate is fairly narrow, and two approaches are possible here. Where the measuring system operates at commercial frequency, the amplifier is designed for a very narrow range of the order of 30-60 c/s. Where the working frequency is in the audio range, the amplifier should operate satisfactorily in the range 100-10,000 c/s. Most often, the working frequencies are 50 c/s and 1,000 c/s.

In special cases, use is made of tuned amplifiers with resonators such as a tuning fork or a magnetostrictive resonator.

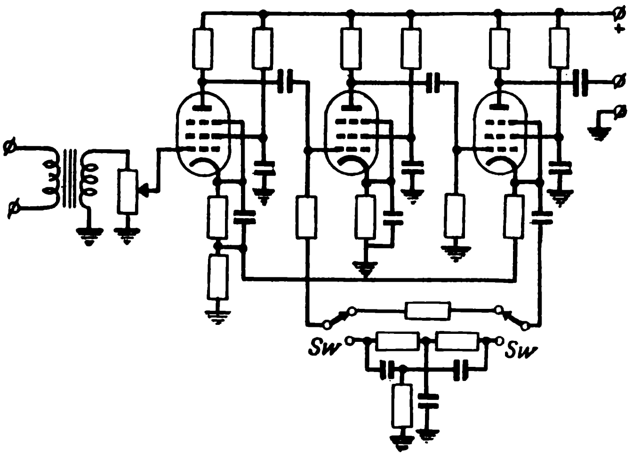

Fig. 4-11

To sum up, any a.f. amplifier employed in radio engineering meets the above requirements, which incidentally are easier than for receiving amplifiers (for example, there are no specified limits for frequency response, inherent nonlinear distortion, etc.).

As an example, Fig. 4-11 shows a simplified diagram of the Soviet-made Type 28 ИМ amplifier. Its frequency range is 200-10,000 c/s and it has three amplifying stages. There are two feedback paths, one embracing all the three stages, and the other covering the last two stages. The switch *Sw* in the other feedback path effects a change-over from broadband to selective amplification. The parallel-T network governs the frequency response of the amplifier.

Some of the measuring apparatus, including a.c. amplifiers, use transistors which are small in size, highly economical and durable. Unfortunately, their characteristics are temperature-dependent. This necessitates the use of temperature compensation and regenerative feedback.

Figure 4-12 shows the circuit of a transistor amplifier, with a resistance strain-gauge connected to its input. The amplifier is loaded by the vibrator of a loop oscillograph. The first two stages built

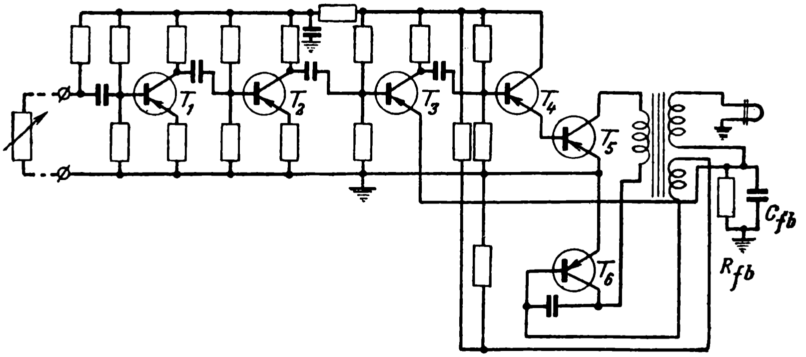

Fig. 4-12

around the transistors T_1 and T_2 make up a voltage amplifier, while the subsequent stages form a power amplifier. Each stage of the voltage amplifier is furnished with regenerative current feedback for temperature compensation and gain stabilization.

The power amplifier and the output transformer are covered by a tight frequency-dependent feedback. The feedback voltage is taken from across R_{fb} and C_{fb} and is fed to the emitter of T_3. In addition to stabilizing the characteristics of the amplifier, this feedback path (especially C_{fb}) improves the frequency response at high frequencies. As a result, the transformer can be made simpler and smaller.

Naturally, where the stability of characteristics is not essential, any type of transistor amplifier can be used to advantage as a way of improving the sensitivity of null methods.

Chapter Five

THE D. C. BRIDGE METHOD

5-1. The Bridge Circuit. Definition of the Bridge Method

The electrical bridge is a term referring to any one of a variety of electric networks used for measurement purposes in which supply voltage is applied across two opposite junctions, a null detector across two other junctions, and the unknown element is placed in one branch.

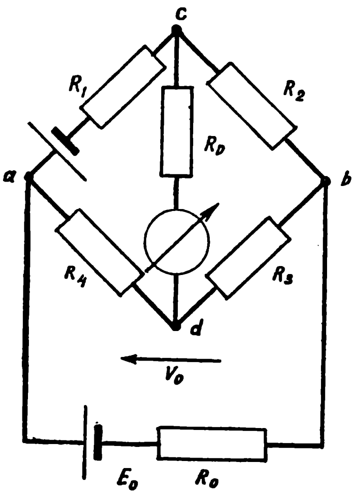

Fig. 5-1

The most common form of bridge is the four-arm bridge shown in Fig. 5-1. It was discovered by Christie in 1833, but its potentialities were not recognized until 1843, when Wheatstone called attention to them. Hence, this network is commonly known as the Wheatstone bridge.

The bridge consists of four resistors connected as shown. A d.c. source is connected across *ab* and a null detector across *cd*. The resistors R_1, R_2, R_3 and R_4 are called the *arms* of the bridge. R_1+R_2 and R_3+R_4 make up the two *branches* of the bridge. The points *a*, *b*, *c*, and *d* are the *junctions* of the bridge.

The term "bridge" originally applied only to the circuit across *ab*, which is still referred to as the bridge proper. In Soviet terminology, the circuits across *ab* and *cd* are termed the *source* or *generator diagonal* and the *detector* or *indicator diagonal*, respectively.

The circuit of Fig. 5-1 is the basic bridge network, since all other bridge forms can be obtained by simple modifications. Although the network components can be arranged in a variety of ways, the most common arrangement is such that of the four resistors one is the unknown, say, R_1; the one next to, and having a common junction with it, is the *standard arm*. The remaining two arms are the *ratio arms.*

Generally, there are two broad classes of bridges: *balanced* and *unbalanced*. In a balanced bridge, the voltage appearing across the detector circuit is brought to zero (the bridge is *balanced*) by adjustment of the standard arm (which is then referred to as the *rheostat arm*). As will be shown later, the balance condition is independent of the resistances of the voltage source R_0 and of the null detector R_D, and the same balance point is obtained when the source and detector are interchanged. The general balance equation which expresses the relationship between the resistances of the arms of a general four-arm bridge at balance is:

$$R_1/R_2 = R_3/R_4, \text{ or } R_1R_3 = R_2R_4.$$

Then, the unknown resistance and, indeed, any of the four arms can be determined in terms of the three other arms by the above general balance equation:

$$R_1 = R_2R_4/R_3.$$

In unbalanced bridges, balancing is dispensed with, and the unknown is determined from the deflection of the indicator.

The most common bridge circuits are the four-arm (Wheatstone) bridge and the double Kelvin bridge. The latter has two more arms to compensate for the effects of the lead and contact resistances, and is generally used for d.c. measurements of very low resistances. The four-arm bridge comes in for discussion first.

5-2. The Four-arm (Wheatstone) Bridge

The basic relationships describing the operation of the four-arm bridge are as follows.

The most common is the balanced four-arm bridge in which no current flows through the detector at balance. Since the resistance of the null detector is finite, zero deflection will be obtained only when the potentials at *c* and *d* are equal or, which is the same, their potential difference is nil. (Fig. 5-2). For $V_{cd}=0$, there must be equal voltage drops across the respective arms of the bridge (i.e., across the 1st and 4th, or across the 2nd and 3rd), or:

$$V_{ac} = V_{ad}; \; V_{cb} = V_{db}. \tag{5-1}$$

Since there is no current flowing through the detector, the respective voltage drops across the arms are given by:

$$V_{ac}=I_1R_1;\ V_{cb}=I_1R_2;$$
$$V_{ad}=I_2R_4;\ V_{db}=I_2R_3.$$

Substituting these expressions in Eq. (5-1) and dividing them termwise, we obtain the classical general balance equation:

$$R_1/R_2=R_4/R_3;\ R_1/R_4=R_2/R_3;\ R_1R_3=R_2R_4. \qquad (5\text{-}2)$$

From Eq. (5-2) it follows that the detector and the source can be interchanged without affecting the balance condition. If in the circuit

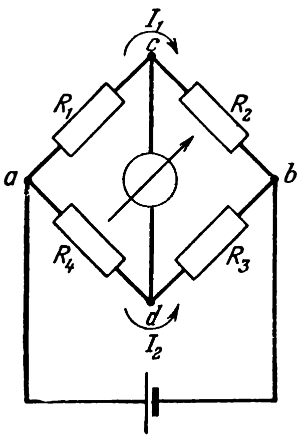

Fig. 5-2

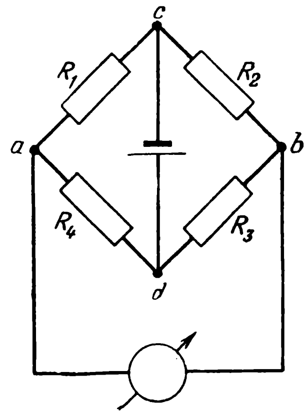

Fig. 5-3

of Fig. 5-2 the voltage source and the null detector are interchanged, this will give the circuit of Fig. 5-3 for which, as may be readily shown, the balance condition is the same. To put it another way, the position of the source and detector in four-arm bridges is of no consequence to the results. In unbalanced bridges, however, the current through the detector will depend on the relative position of all the elements, including the source and detector. Since practical bridges are all unbalanced and the current through the detector affects the sensitivity of the network, it is not the same where to connect the voltage source or the detector. Their placement in a bridge should be such as to give maximum sensitivity. This requirement will be discussed in greater detail later.

Equation (5-2)is sufficient for the unknown resistance to be found. In some cases, however (for example, in precision measurement with an inadequate source or with power dissipation in an arm, etc.),

knowledge of other constants of the bridge may be essential (input and output impedance, currents in the branches and the detector circuit, etc.).

It should be qualified that for all the simplicity of the bridge network, the determination of the above relationships and constants may prove a fairly involved procedure, depending on the method of circuit and network calculations chosen. Thus, it is possible to determine all the currents and voltages in a bridge by Kirchhoff's laws, but this involves a system of six simultaneous equations. The Maxwell circulating current theorem reduces their number to three equations. The simplest procedure, however, is to use Helmholtz's (or Thévenin's) theorem. As to the equivalent resistances of the network, use

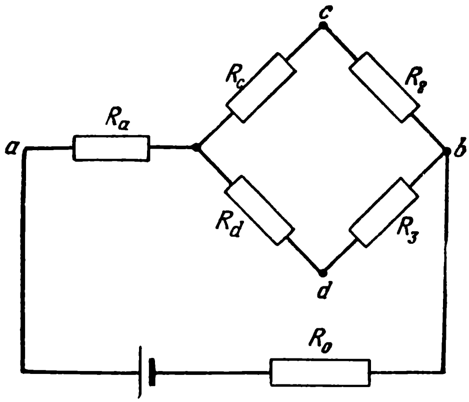

Fig. 5-4

should preferably be made of star-mesh conversions (also known as wye-delta transformations). It is these two latter methods that we shall use in the subsequent discussion.

The first to be determined is the input resistance R_{ab} of the bridge network of Fig. 5-1. The conversion of the mesh *acd* into an equivalent star gives the network of Fig. 5-4. The resistances of the star branches respectively are:

$$\left.\begin{aligned} R_a &= R_1R_4/(R_1+R_4+R_D); \\ R_c &= R_1R_D/(R_1+R_4+R_D); \\ R_d &= R_4R_D/(R_1+R_4+R_D). \end{aligned}\right\} \qquad (5\text{-}3)$$

As follows from the network of Fig. 5-4:

$$R_{ab} = R_a + (R_2+R_c)(R_3+R_d)/(R_2+R_3+R_c+R_d). \qquad (5\text{-}4)$$

Substituting Eq. (5-3) in Eq. (5-4) and rewriting, we finally obtain:

$$R_{ab}=[R_D(R_1+R_2)(R_3+R_4)+\Pi]/[R_D(R_1+ \\ +R_2+R_3+R_4)+(R_1+R_4)(R_2+R_3)], \quad (5\text{-}5)$$

where

$$\Pi=R_1R_2(R_3+R_4)+R_3R_4(R_1+R_2)= \\ =R_1R_2R_3+R_2R_3R_4+R_3R_4R_1+R_4R_1R_2. \quad (5\text{-}6)$$

The input resistance of a balanced bridge will be a particular case of Eq. (5-5). Since the current through the detector is zero, it may be assumed that $R_D=\infty$. Substituting it in Eq. (5-5) gives:

$$R_{ab}=(R_1+R_2)(R_3+R_4)/(R_1+R_2+R_3+R_4). \quad (5\text{-}7)$$

Equation (5-7) gives the resistance of the bridge with the circuit *cd* open and can be easily obtained immediately from the network of Fig. 5-1.

By the same procedure, we can determine the output resistance of the network, R_{cd}, which has, naturally, a form close to that of Eq. (5-5):

$$R_{cd}=[R_0(R_1+R_4)(R_2+R_3)+\Pi]/[R_0(R_1+R_2+R_3+R_4)+ \\ +(R_1+R_2)(R_3+R_4). \quad (5\text{-}8)$$

Here, Π is the same as in Eq. (5-6).

In many cases the resistance of the source (especially of d.c. ones) may be taken equal to zero, i.e., $R_0=0$. Then Eq. (5-8) will be greatly simplified:

$$R_{cd}=[R_1R_2/(R_1+R_2)]+[R_3R_4/(R_3+R_4)]. \quad (5\text{-}9)$$

The physical implication of this equation is clear. Equation (5-9) can be obtained directly, if in the network of Fig. 5-1 the terminals *a* and *b* were short-circuited and the resistance with respect to the terminals of the detector determined.

In the case of a balanced bridge, for which the balance equation is $R_1R_3=R_2R_4$, Eq. (5-9) may be rewritten in a variety of ways, giving it a more convenient form. For this, any of the four arms is found in terms of the remaining three from the balance equation, the result is substituted in Eq. (5-9), the terms are reduced to a common denominator, and the common factors are cancelled to give:

$$R_{cd}=R_1(R_2+R_3/R_1+R_2)=R_2(R_1+R_4/R_1+R_2)= \\ =R_3(R_1+R_4/R_3+R_4)=R_4(R_2+R_3/R_3+R_4),$$

and also

$$R_{cd}=(R_2+R_3)(R_1+R_4)/(R_2+R_3)+(R_1+R_4). \quad (5\text{-}10)$$

From Eq. (5-10) it follows that R_{cd} is the resistance of two parallel branches, (R_2+R_3) and (R_1+R_4). Referring to the network of Fig. 5-1, such an output resistance will be obtained if the source circuit is open, i.e., $R_0=\infty$, while for the original equation (5-9) we assumed $R_0=0$. Thus, for a balanced bridge we have obtained the same value of R_{cd}, irrespective of whether the resistance of the source circuit is assumed to be equal to infinity or zero. In a more general form, we may state that for balanced bridges the output resistance is independent of the resistance of the source circuit.

This feature is a very important property of bridge networks, which will come into the picture more than once in the subsequent discussion.

Now the detector current I_D is to be determined (or the voltage V_{cd} across the terminals of the detector) when the bridge is out of balance. From Helmholtz's theorem it follows that

$$I_D=V_{cd}/(R_{cd}+R_D), \tag{5-11}$$

where V_{cd} is the open-circuit voltage across terminals c and d, and R_{cd} is the output resistance of the bridge network.

From the network of Fig. 5-2:

$$\begin{aligned} V_{cd}&=I_1R_1-I_2R_4=V_0\,[R_1/(R_1+R_2)-R_4/(R_3+R_4)]=\\ &=V_0\,[(R_1R_3-R_2R_4)/(R_1+R_2)(R_3+R_4)]. \qquad (5\text{-}12)\end{aligned}$$

The output resistance of the bridge is given by Eq. (5-8) and Eq. (5-9). In many cases, especially for d.c. bridges, it may be assumed that $R_0=0$ and the simpler Eq. (5-9) used. Then, substituting Eqs. (5-9) and (5-12) in Eq. (5-11) gives:

$$\begin{aligned} I_D&=V_0\left[\frac{R_1R_3-R_2R_4}{(R_1+R_2)\;(R_3+R_4)}\right]\left\{1\Big/\left[\frac{R_1R_2\,(R_3+R_4)+R_3R_4\,(R_1+R_2)}{(R_1+R_2)\;(R_3+R_4)}+R_D\right]\right\}=\\ &=V_0\,(R_1R_3-R_2R_4)/[R_D\,(R_1+R_2)\,(R_3+R_4)+\\ &+R_1R_2\,(R_3+R_4)+R_3R_4\,(R_1+R_2)]=\\ &=V_0\,(R_1R_3-R_2R_4)/R_D\,(R_1+R_2)\,(R_3+R_4)\,\Pi, \qquad (5\text{-}13)\end{aligned}$$

where Π is the same as in Eq. (5-6).

A more complete expression for the unbalance current when $R_0\neq 0$ (Fig. 5-1), as derived from Eq. (5-8) will be:

$$\begin{aligned} I_D&=E_0\,(R_1R_3-R_2R_4)/[R_0R_D\,(R_1+R_2+R_3+R_4)+\\ &+R_D(R_1+R_2)\,(R_3+R_4)+R_0\,(R_1+R_4)\,(R_2+R_3)+\Pi]. \qquad (5\text{-}14)\end{aligned}$$

Finally, it may sometimes prove more convenient to express the current through the detector not as a function of its voltage but as a function of the total current I_0 supplied to the bridge network (which may be the case with high-resistance multipliers and low input resist-

ance). Again for R_0=infinity, Helmholtz's theorem gives:

$$V_{cd}=I_1R_1-I_2R_4=[I_0(R_3+R_4)/(R_1+R_2+R_3+R_4)]R_1-$$
$$-[I_0(R_1+R_2)/(R_1+R_2+R_3+R_4)]R_4=$$
$$=I_0(R_1R_3-R_2R_4)/(R_1+R_2+R_3+R_4);$$
$$I_D=V_{cd}/(R_{cd}+R_D)=V_{cd}\Big/\left[\frac{(R_1+R_4)(R_2+R_3)}{R_1+R_2+R_3+R_4}+R_D\right]=$$
$$=I_0(R_1R_3-R_2R_4)/[R_D(R_1+R_2+R_3+R_4)+$$
$$+(R_1+R_4)(R_2+R_3)].$$

Naturally, assuming $I_D=0$ in any of the expressions for the detector current we arrive at the original balance equation, $R_1R_3=R_2R_4$.

In some cases (when, say, measuring the load on separate resistances), it is essential to know the currents in the arms of an unbalanced bridge. For reference, the following equations of the arm currents as functions of the total supply current, I_0, in the bridge network are given:

$$\left.\begin{aligned}I_1&=I_0[R_4(R_2+R_3)+R_D(R_3+R_4)]/N;\\I_2&=I_0[R_3(R_1+R_4)+R_D(R_3+R_4)]/N;\\I_3&=I_0[R_2(R_1+R_4)+R_D(R_1+R_2)]/N;\\I_4&=I_0[R_1(R_2+R_3)+R_D(R_1+R_2)]/N,\end{aligned}\right\}\quad(5\text{-}15)$$

where $N=R_D(R_1+R_2+R_3+R_4)+(R_1+R_4)(R_2+R_3)$.

Equations (5-15) can be easily resolved to functions of the voltage applied to the bridge. Indeed, the total current

$$I_0=V_0/R_{ab}. \quad (5\text{-}16)$$

Substituting the input resistance R_{ab} from Eq. (5-5) in Eq. (5-16) and then in Eq. (5-15) we obtain another series of the arm currents:

$$\left.\begin{aligned}I_1&=V_0[R_4(R_2+R_3)+R_D(R_3+R_4)]/M;\\I_2&=V_0[R_3(R_1+R_4)+R_D(R_3+R_4)]/M;\\I_3&=V_0[R_2(R_1+R_4)+R_D(R_1+R_2)]/M;\\I_4&=V_0[R_1(R_2+R_3)+R_D(R_1+R_2)]/M,\end{aligned}\right\}\quad(5\text{-}17)$$

where $M=R_D(R_1+R_2)(R_3+R_4)+R_1R_2(R_3+R_4)+R_3R_4(R_1+R_2)$.

The two series of equations, (5-15) and (5-17), for the arm currents of an unbalanced bridge are rather unwieldy. Of course, they become simpler in the particular case of a balanced bridge, because R_D is assumed to be infinity.

It is a fairly tedious task to find currents even in an ordinary four-arm bridge. As far as multiple-arm bridges are concerned, the usual methods based on Kirchhoff's laws are frustrating. A more elegant solution is provided by such sophisticated methods of circuit and net-

work calculations as the four-terminal theorem, the Maxwell circulating-current theorem and the parallel-generator (nodal voltage) theorem.

The classical balance condition, $I_D=0$ when $R_1R_3=R_2R_4$, has been derived for bridge networks using a single source of e.m.f. connected across the terminals *ab*. Naturally, the balance condition is valid only for this particular case. Meanwhile, cases are common in measuring practice where several sources of e.m.f. may be contained in a bridge, either connected on purpose (for example, in measuring some parameters under load) or originating fortuitously (such as contact potential differences, thermo-e.m.f.s). In the latter case, e.m.f.s may turn up anywhere in the network: in the arms, the detector circuit, etc. It seems interesting, therefore, to examine a more general case of a bridge containing several e.m.f.s.

On the basis of bridge properties, it may be shown that if the product of resistances in one pair of opposite arms is equal to that in the other pair of opposite arms, the detector circuit is independent of the source circuit, and vice versa.

This condition underlies many of the electrical measurements. Among other things, it implies that the opening and closing of the source circuit at balance will not affect the current flowing through the detector. Use of this condition is made in measuring the internal resistance of primary and secondary cells and batteries. The cell or battery under test is made an arm of a bridge, the source circuit is completed and interrupted repeatedly by means of a key, and the remaining arms are adjusted until the galvanometer gives the same deflection in any position of the key. It also serves as the basis for the widely used "false-zero" method by which parasitic e.m.f.s in bridge measurements can be compensated. By this method, the deflection of the indicator is first found with the external source disconnected (i. e., only due to the parasitic e.m.f.s in the arms). Then, with the source switched on, the network is so adjusted as to obtain the same deflection on the indicator. If the bridge is properly adjusted, the switching of the source on or off will not affect the deflection of the indicator.

Now we have come to the selection of a null detector for a balanced four-arm bridge. The sensitivity of the device may be taken as satisfactory if changes in the adjustable arm, equal to the specified fractional deviation (or accuracy limit of measurement), $\delta_x=\Delta R_1/R_1$, produce one-division deflection on the detector, as a minimum.

The output resistance, R_{cd}, of a balanced bridge slightly out of balance changes very little and can be determined either by Eq. (5-9) or Eq. (5-10):

$$R_{cd}=R_1R_2/(R_1+R_2)+R_3R_4/(R_3+R_4).$$

As follows from Eq. (5-12), the open-circuit output voltage when $\delta_x \ll 1$ and $R_1R_3 = R_2R_4$, will be:

$$V_{cd} = V_0\left[(R_1+\Delta R_1)/(R_1+\Delta R_1+R_2) - R_4/(R_3+R_4)\right] =$$
$$= V_0 \frac{(\Delta R_1/R_1)\,R_1R_3}{(R_1+\Delta R_1+R_2)\,(R_3+R_4)} \cong V_0\delta_x\,(R_1R_3/R_1+R_2)\,(R_3+R_4).$$

From the arm ratios at balance:

$$R_3/(R_3+R_4) = R_2/(R_1+R_2),$$

we have

$$V_{cd} \cong V_0\delta_xR_1R_2/(R_1+R_2)^2 \cong V_0\delta_xR_3R_4/(R_3+R_4)^2. \quad (5\text{-}18)$$

The detector current will be

$$I_{cd} = V_{cd}/(R_{cd}+R_D).$$

If this current is to produce a deflection of at least one scale division, the current constant of the galvanometer must not exceed I_{cd}. Also, the galvanometer must have an external critical damping resistance roughly equal to R_{cd} so that its moving system may come to equilibrium in the shortest time after deflection.

It is obvious that the above criteria hold only in the measurement of any one value of resistance. Most often, however, the detector must permit its use over the entire range of resistances for which the bridge is designed. As the unknown, ratio arms and supply voltage vary, R_{cd} and V_{cd} also vary. The range of changes in the former is especially important, since it determines the conditions of damping of the galvanometer. Preference should be given to a galvanometer with an external critical damping resistance equal to the arithmetic mean of the lowest and highest values of R_{cd}. Finally, the instrument should be checked for compliance with Eq. (5-18) in order to see whether or not it has sufficient sensitivity at the extremes of the resistance range.

For a galvanometer with a considerable margin of sensitivity, it will be a good plan to use shunts and multiplier resistors. Then, by reducing the sensitivity to the requisite limits, critical damping can be obtained conveniently. The shunts and multiplier resistors can be so chosen that even considerable variations in R_{cd} will not affect the condition of damping of the detector.

5-3. The Kelvin Double Bridge

Four-arm Wheatstone bridges are best suited for measuring medium resistances from about 5-10 ohms up. Accuracies of the order of 0.1 to 0.01 per cent are obtainable in most cases, while first-class bridges can be accurate to within 0.001 per cent.

In measuring lower resistances (from 1 to 5-10 ohms), the effect of variable contact resistances becomes noticeable, and the errors due to them may be as great as 0.1 per cent or more. This is because, for reasons intrinsic to the nature of bridges, the contact resistances, being in series with the arm resistors, are included in the measuring network and falsify the results, being added to them.

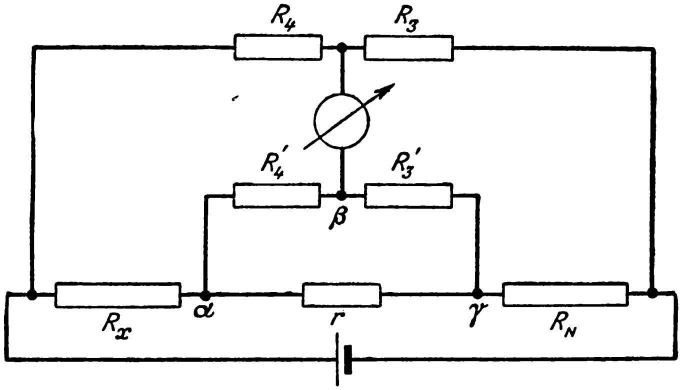

Fig. 5-5

Where very low resistances (from 1 ohm down to a few hundredths or even thousandths) are involved, contact resistances become predominant. Indeed, they minimize the advantages of the bridge method as such and handicap the use of the four-arm bridge. This necessitates the employment of a modified bridge method specifically designed for low-resistance measurements and free from the major drawback—the influence of contact resistances. A variety of modifications has been suggested to date, the most commonly used among them being the Kelvin double bridge, the term "double" being used due to the presence of a double set of ratio arms.

The general arrangement of the Kelvin double bridge is shown in Fig. 5-5, where R_N and R_x are a standard resistance and the unknown resistance, respectively.

As will be recalled, low resistances (down from 1 ohm) employed in measurements are always constructed with four terminals so that the current circuit and the potential circuit can be connected separately. Referring to Fig. 5-5, it will be seen that the Kelvin-bridge arrangement is ideally suited for connection of four-terminal resistances R_N and R_x. Most of the contact resistances will then be placed either before the outer (main) arms R_x, R_4 and R_N, R_3, or in series with the inner (auxiliary) arms R_3, R_3', R_4 and R_4', normally having values less than 10 ohms. With such an arrangement, the effect of

contact resistances will be negligible. The only weak point in the Kelvin double bridge is the contact resistances connected in series with the low-resistance link (or yoke) r. By choice of the circuit parameters (see below), however, their effect, too, can be made negligible. For precise work, use may be made of a special technique which would exclude or take care of all extraneous effects.

The balance condition for the Kelvin double bridge may be most conveniently found by applying the mesh-star conversion to the mesh

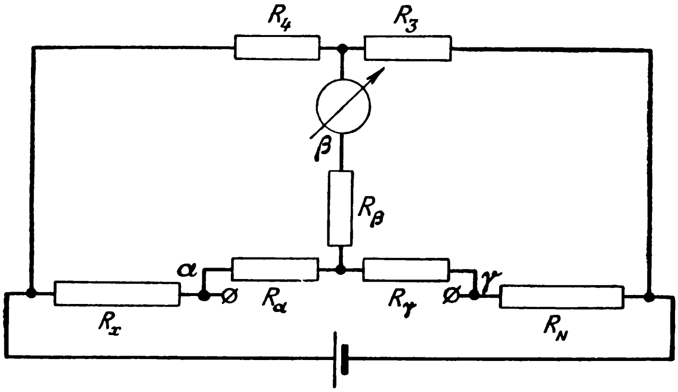

Fig. 5-6.

$\alpha\beta\gamma$. This will give the arrangement of Fig. 5-6. It is obvious that the converted network satisfies the balance condition for the Wheatstone four-arm bridge. Modified to take into account the special features of the double bridge, this condition will be:

$$R_4(R_N+R_\gamma)-R_3(R_x+R_\alpha)=0. \tag{5-19}$$

Recalling that

$$R_\alpha=rR'_4/(r+R'_3+R'_4); \quad R_\gamma=rR'_3/(r+R'_3+R'_4),$$

and after substituting in Eq. (5-19), we obtain:

$$R_4\left(R_N+\frac{rR'_3}{r+R'_3+R'_4}\right)-R_3\left(R_x+\frac{rR'_4}{r+R'_3+R'_4}\right)=0.$$

Rationalizing, we have

$$(R_3R_x-R_4R_N)(r+R'_3+R'_4)-r(R_4R'_3-R_3R'_4)=0 \tag{5-20}$$

Having solved Eq. (5-20) for R_x, we finally obtain

$$R_x=R_N(R_4/R_3)+\frac{r(R_4R'_3-R_3R'_4)}{R_3(r+R'_3+R'_4)}=R_N(R_4/R_3)+d, \tag{5-21}$$

where

$$d=(r/R_3)\frac{R_4R_3'-R_3R_4'}{r+R_3'+R_4'}=\frac{rR_3'}{r+R_3'+R_4'}(R_4/R_3-R_4'/R_3'). \quad (5\text{-}22)$$

As follows from Eq. (5-21), the balance condition for the Kelvin double bridge differs from that for the Wheatstone four-arm bridge in that there is an additional term d which, according to Eq. (5-22) vanishes when

$$R_4/R_3=R_4'/R_3'. \quad (5\text{-}23)$$

Ordinarily, the standard resistance R_N in existing forms of the Kelvin double bridge is a fixed resistance coil. Therefore, the bridge is balanced to satisfy Eq. (5-21) by adjusting the ratio R_4/R_3, so that Eq. (5-23) is also satisfied.

To this end, it is customary to make $R_4=R_4'$ and $R_3=R_3'$.* One pair (R_4 and R_4' or R_3 and R_3') is arranged as a twin set of dial resistances, with the decades coupled mechanically. The other pair is made up of a twin set of plug-type decades. Both the dial and the plug resistances are arranged so that movement of the dial or repositioning of the plug increases or decreases both the outer and inner ratio arms together, thus keeping

$$R_4=R_4', \quad R_3=R_3', \quad \text{and} \quad R_4/R_3=R_4'/R_3'.$$

The condition of Eq. (5-23) can be satisfied and the term d in Eq. (5-22) can be made to disappear with a high degree of accuracy (solely dependent on deviation of the arm resistances from their nominal values). Though high, this accuracy is not ideal. Therefore, it appears expedient additionally to reduce, as far as possible, the second factor of the term d in Eq. (5-22). This can be done by making the resistance of the link r very small, using a short length of a heavy copper wire or strip for the link.

As a result of all the above measures, the term d is made so negligibly small (the error due to it is not over one-hundredth of one per cent) that it may be disregarded. Therefore, in most cases it is assumed that $d=0$. Consequently,

$$R_x=R_N(R_4/R_3).$$

Yet, it should be remembered that in some cases one cannot drop the term d. This is true of testing shunts under heavy currents (and generally of the measurement of very low resistances), where r may be comparable to or even exceed R_x many times.

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* This equality is not essential, since Eq. (5-23) can be satisfied anyhow. Yet, this is convenient from a practical point of view.

For these reasons, the effect of d must be taken into account. It would be interesting, therefore, to analyze the error introduced by neglecting d and to determine its limits encountered in practical work. For this, it would be convenient to rewrite Eq. (5-22) thus:

$$d = r\sigma, \tag{5-24}$$

where

$$\sigma = [R_3'/(r + R_3' + R_4')](R_4/R_3 - R_4'/R_3').$$

Then, introducing errors, we have:

$$\left.\begin{aligned} R_3 &= R_{3N}(1-\delta_3); \\ R_3' &= R_{3N}(1-\delta_3'); \\ R_4 &= R_{4N}(1-\delta_4) = kR_{3N}(1-\delta_4); \\ R_4' &= R_{4N}(1-\delta_4') = kR_{3N}(1-\delta_4'), \end{aligned}\right\} \tag{5-25}$$

where

R_{3N} and R_{4N} = nominal arm resistances;
δ_3, δ_3', δ_4, δ_4' = fractional errors in the arm resistances;
k = arm ratio equal to

$$k = R_{4N}/R_{3N} \cong R_4/R_3 \cong R_x/R_N.$$

Substituting system (5-25) in Eq. (5-24), dropping r in comparison with $(R_4' + R_3')$, and neglecting the second-order terms we have:

$$\sigma = [k/(k+1)](\delta_3 - \delta_4 + \delta_4' - \delta_3'). \tag{5-26}$$

Since we are interested in the limiting error, σ_{lim}, then, assuming $\delta_3 = -\delta_3' = -\delta_4 = \delta_4' = \delta_{\mathrm{lim}}$ we substitute it in Eq. (5-26) to obtain:

$$\sigma_{\mathrm{lim}} = [k/(k+1)]\, 4\delta_{\mathrm{lim}}. \tag{5-27}$$

Using Eq. (5-27), we finally have

$$R_x = R_N \frac{R_4}{R_3} + r\frac{k}{k+1} + \delta_{\mathrm{lim}} \cong R_N \frac{R_4}{R_3}\left[1 + \frac{r}{R_x} \times \frac{k}{k+1} \times 4\delta_{\mathrm{lim}}\right]. \tag{5-28}$$

As follows from Eq. (5-28), the error introduced by neglecting d depends on three factors: the ratio r/R_x, the arm ratio k, and the limiting error, δ_{lim}, in the adjustment of the arm resistances, and decreases as they decrease. For most of the practical cases we may assume that

$$0.001 < r/R_x < 100 \text{ and } 0.01 < k < 1{,}000.$$

As to δ_{lim}, it cannot exceed 0.0002, 0.0005 or 0.001, depending on the accuracy class of the bridge in use (the figures are quoted for bridges of accuracy classes 0.02, 0.05 and 0.1). Substitution of the limiting values in Eq. (5-28) will show that the error due to neglecting

the term d can generally vary within broad limits. This necessitates the use of experimental techniques which would make it possible to determine the term d or to isolate it in some simple way.

The term d can be calculated from the data in the test certificate of a given bridge by Eq. (5-24) and Eq. (5-26), if r is known. The latter can be determined by a variety of methods, the simplest of them being the ammeter-voltmeter method. It is obvious, however, that this procedure would involve a good deal of computation.

Computation can be simplified considerably, if there is some possibility for rearranging the network during measurement. The term σ, for example, can be easily found as follows: take two measurements, one with a negligibly small r and the other with r^* which is at least 100 times as great as r. Then for the first measurement

$$R_x = R_N(R_4/R_3) + r\sigma;$$

for the second measurement

$$R_x = R_N(R_4^*/R_3) + r^*\sigma.$$

From the above two equations we have

$$\delta = (R_N/R_3)\,[(R_4 - R_4^*)/(r^* - r)].$$

Since $r^* \gg r$, we can write that

$$\sigma \approx (R_N/R_3)\,[(R_4 - R_4^*)/r^*],$$

where R_4 and R_4^* are the values for the first and the second measurement, respectively.

With σ thus found and r known, the value of d can be determined.

A more attractive proposition, however, would be to exclude the effect of d experimentally, during measurement. Here is one such technique which involves double balance.

It boils down to the following: R_x is measured for the first time, the leads connected to the potential terminals of each of the coils R_x and R_N are interchanged, and R_x is measured for the second time. With the leads of R_x and R_N thus interchanged, the arms R_3, R_3' and R_4, R_4' will also change places in the network.

At a second balance, we obtain for the new values of R_4^* and $R_4'^*$:

$$R_x = R_N(R_4'^*/R_3') + [rR_3/(r + R_3 + R_4^*)]\,(R_4'^*/R_3' - R_4^*/R_3)$$

where

$$R_4^* = R_4 + \Delta R_4 \text{ and } R_4'^* = R_4' + \Delta R_4,$$

or

$$R_x = R_N(R_4'^*/R_3') - [rR_3/(r + R_3 + R_4 + \Delta R_4)] \times$$
$$\times (R_4/R_3 - R_4'/R_3' + \Delta R_4/R_3 - \Delta R_4/R_3'). \quad (5\text{-}29)$$

Since ΔR_4 is negligibly small (being equal to the two last decimal places in the value of the rheostat arm) and the value d may be taken approximately, we have

$$R_x = R_N(R_4'^{*}/R_3') - [rR_3/(r+R_3+R_4)]\ (R_4/R_3 - R_4'/R_3'). \quad (5\text{-}30)$$

A comparison of Eq. (5-30) and Eq. (5-21) shows that its second term on the right-hand side is equal to d, because nominally and with an accuracy sufficient for its determination we may assume that $R_3 = R_3'$ and $R_4 = R_4'$.

Adding Eq. (5-21) and Eq. (5-30) together gives:

$$R_x = (R_N/_2)\,(R_4/R_3 + R_4'^{*}/R_3'). \quad (5\text{-}31)$$

The above expression is still inconvenient for use, since the actual values of the resistances must be substituted in it. For the most common cases, however, where the results are only required to within the limits of accuracy of the bridge, this expression may be simplified by assuming that $R_3 = R_3'$. Then with the same accuracy we may write

$$R_x = (R_N/R_3)\,[R_4^{*} + R_4)\,2]. \quad (5\text{-}32)$$

The term d is present neither in Eq. (5-21) nor in Eq. (5-32), i.e., its value does no affect the result.

Also, the double-balance method makes it possible to determine d directly, without measuring r or σ. Indeed, subtracting Eq. (5-32) from Eq. (5-21), we obtain

$$d = (R_N/R_3)\,[(R_4^{*} - R_4)\,/2].$$

From Eq. (5-31), one can conclude that with the double-balance method no measures have to be taken in order to reduce the resistance of the lead r, as is usually the case, because it does not affect the result.

Here we come to the selection of a detector for the double bridge. As has been noted, the conversion of the mesh $\alpha\beta\gamma$ into a star reduces the Kelvin double bridge of Fig. 5-5 to a simple Wheatstone bridge (Fig. 5-6). Therefore, the reasoning for the choice of a detector for the Wheatstone bridge holds for the converted Kelvin bridge. Account must, however, be taken of some special features of the Kelvin bridge and of the operation of galvanometers in it.

Firstly, there is a marked asymmetry in the relative magnitude of the resistances in the Kelvin bridge. Indeed, the resistances R_x, R_N and r may be neglected in determining the output resistance, since they are very small in comparison with $R_3 = R_3'$ and $R_4 = R_4'$. Noting that, we have

$$R_{out} \cong R_3R_4/(R_3+R_4) + R_3'R_4'/(R_3'+R_4') \cong 2R_3R_4/(R_3+R_4).$$

Now determine the change in the converted arm, $R_x + rR'_4/(r+R'_3+R'_4)$ for a given change $\delta_x = \Delta R_x/R_x$ in the unknown R_x. Dropping r in comparison with $R'_3+R'_4$, we get:

$$\delta'_x = \frac{\Delta R_x}{R_x + rR'_4/(R'_3+R'_4)} = \delta_x \frac{1}{1+(r/R_x)\,[R'_4/(R'_3+R'_4)]}\ . \quad (5\text{-}33)$$

Noting that at balance

$$R_3/(R_3+R_4) = R_x/(R_x+R_N) = R'_4/(R'_3+R'_4), \quad (5\text{-}34)$$

and substituting these expressions in Eq. (5-33), we finally have

$$\delta'_x = \delta_x \frac{1}{1+r/(R_x+R_N)}\ . \quad (5\text{-}35)$$

The open-circuit voltage at the bridge output can be expressed as a function of the supply current I, assuming that practically no current branches off into R_3R_4.

Assuming $V_0 \simeq I(r+R_x+R_N)$ and substituting Eq. (5-34) and Eq. (5-35) in Eq. (5-18), we get

$$V_{cd} = I\,(r+R_x+R_N)\,\delta_x \frac{1}{1+r/(R_x+R_N)} \times$$
$$\times\ [R_3R_4/(R_3+R_4^2)] = I\delta_x R_x\,[R_x+R_N)/R_x] \times$$
$$\times\ [R_4/(R_3+R_4)]\,[R_3/(R_3+R_4)] = IR_x\delta_x R_3/(R_3+R_4).$$

As before, the change δ_x in the unknown R_x is taken to be within the accuracy limits of the bridge.

Thus, we have obtained all the data necessary for the selection of a null detector for the Kelvin bridge. The choice is the same as in the case of the Wheatstone bridge. It should be remembered, however, that the Kelvin bridge is less sensitive (see Sec. 5-4) and would require a more sensitive null detector. This is especially true of the measurement of very low resistances (of the order of 10^{-4} to 10^{-6} ohm). Because of this, galvanometers should be selected for the measurement of very low resistances, since they constitute the most unfavourable cases.

The final point to be mentioned in connection with the selection and operation of galvanometers in bridge networks is the one related to the balancing of the Wheatstone and the Kelvin bridge. Every bridge invariably has a regulating resistor (a rheostat), usually connected in series with the galvanometer and protecting it when the bridge is far out of balance. It is more advantageous, however, to have this regulating resistor in series with the source rather than with the galvanometer. This disposition of the rheostat ensures the constancy of damping, prevents the generation of thermal e.m.f.s, facilitates the use of the false-zero method, and speeds up measurement.

5-4. The Sensitivity of D. C. Bridges

If a bridge is to be accurately balanced, it is essential that the out-of-balance voltage across the detector circuit produce sufficient deflection in the detector. This goal can be achieved by: (1) selecting a galvanometer (or any other detector) of the requisite sensitivity; (2) selecting an ample source (of voltage or current); and (3) selecting the optimum bridge parameters.

In most cases, preference is given to galvanometers of a relatively low sensitivity. This is because they are more reliable in service (they have a shorter periodic time, require no complicated hook-ups, etc.). The maximum power that may be applied to a bridge is governed by the load capacity of the various circuit components, most often by that of the resistance being measured. Therefore, in the majority of cases, the requisite sensitivity is attained through the proper selection of bridge constants.

Sensitivity is of particular importance to unbalanced bridges (see Sec. 5-5), for the reason that the unbalance signal is the measurand, and the instrument must therefore be able to measure it with the requisite accuracy.

The sensitivity of a bridge may be considered in terms of three output parameters: current, voltage, or power. The optimum relationships between them may vary within broad limits, depending on the particular case on hand, viz., the supply voltage (or current), the maximum load of the arms, the sensitivity of the detector, etc. In a general case, design equations are too unwieldy. Therefore, our discussion will be limited to a few cases encountered most often in practical work.

1. The supply voltage is limited by the power that may be dissipated in the unknown resistance; the conductance of the detector is negligibly small.

This situation often arises in measuring nonelectrical quantities by electrical methods, when the detector circuit contains an electronic amplifier. Then, the power dissipated in the other three arms is not limited and, consequently, $R_2 \gg R_1$, and $R_4 \ll R_1$. It is obvious that only voltage sensitivity would make any sense. The voltage across the detector circuit is given by

$$V_{cd} = V_0 [(R_1 + \Delta R_1) / (R_1 + \Delta R_1 + R_2) - R_4 / (R_3 + R_4)], \quad (5\text{-}36)$$

where ΔR_1 is the change in the unknown resistance.

Noting that $\Delta R_1 \ll R_1 + R_2$ and that the bridge is at balance when $\Delta R_1 = 0$, we get from Eq. (5-36):

$$V_{cd} = V_0 [\Delta R_1 / (R_1 + R_2)]. \quad (5\text{-}37)$$

The supply voltage can be easily found in terms of the unknown resistance R_1 and the permissible power dissipation P_1 in it:

$$V_0 = \sqrt{P_1/R_1} \times (R_1 + R_2).$$

Substituting Eq. (5-38) into Eq. (5-37) we get

$$V_{cd} = \sqrt{P_1 R_1}\,(\Delta R_1/R_1) = \sqrt{P_1 R_1}\,\varepsilon, \qquad (5\text{-}39)$$

where $\varepsilon = \Delta R_1/R_1$ is the relative change in the arm R_1.

As follows from Eq. (5-39), for the same permissible power dissipation, maximum sensitivity will be obtained with sensing elements having maximum resistance.

2. As in (1), the supply voltage is limited by the permissible power dissipation in the unknown resistance; it is required to obtain maximum output power in the detector circuit.

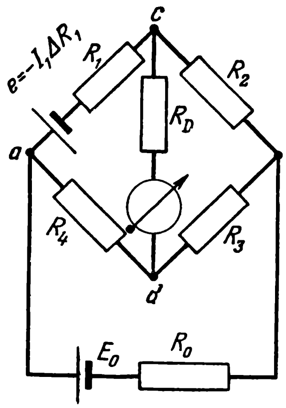


Fig. 5-7

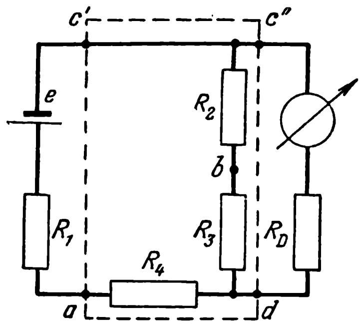


Fig. 5-8

Likewise, this case often occurs in measuring nonelectrical quantities by electrical methods, when the detector is directly connected to the output of a bridge.

The problem can best be solved by using the compensation theorem, or rather an extension to it, which reads: *If the resistance of a branch of a network, carrying a current, be changed by some value, the effect on all other branch circuits is that which would be produced by inserting (in series with the changed branch) a source with an e.m.f. equal to the original current times the change in the branch, taken with the opposite sign.* By this theorem, the small current in the detector circuit in a balanced bridge for a change ΔR_1 in the unknown resistance R_1 may be regarded as being due to the insertion into the bridge, in series with R_1, of some equivalent e.m.f. of I_1, where I_1 is

the original current in R_1, assumed to be constant. * This leaves us with the equivalent circuit of Fig. 5-7, containing two sources of e.m.f., E_0 and e. In this circuit $R_1R_3=R_2R_4$, i.e., the bridge remains at balance. For this reason, the voltage across the supply circuit does not affect the current in the detector circuit. Indeed, we may well dispense with the supply circuit altogether, and the equivalent circuit will be as shown in Fig. 5-8. Referring to the diagram, it can be seen that maximum power sensitivity will be obtained when the source $e(R_1)$ delivers maximum power into the load R_D through the network $ac'c''d$. As will be recalled, for better energy transfer the input resistance of the network and the internal resistance of the source must be matched, i.e., made equal. Then the maximum power delivered by the source will be

$$\Delta P_D = e^2_{cd(xx)}/4R_{cd}. \tag{5-40}$$

Here $e_{cd\,(xx)}$ is the output voltage across the unloaded network which, according to the diagram of Fig. 5-8, is equal to

$$\begin{aligned} e_{cd\,(xx)} &= e(R_2+R_3)/(R_1+R_2+R_3+R_4) = \\ &= I_1\Delta R_1\,[(R_2+R_3)/[R_1+R_2+R_3+R_4)] = \\ &= \sqrt{(P_1/R_1)}\,[(R_2+R_3)/(R_1+R_2+R_3+R_4)]\,\Delta R_1. \end{aligned} \tag{5-41}$$

Substituting Eq. (5-41) in Eq. (5-40) and noting that

$$R_{cd}=(R_1+R_4)(R_2+R_3)/(R_1+R_2+R_3+R_4),$$

we get

$$\begin{aligned} \Delta P_D &= {}^1/_4P_1\,[R_1/(R_1+R_4)]\,[(R_2+R_3)/(R_1+R_2+R_3+ \\ &+R_4)]\,(\Delta R_1/R_1)^2 = {}^1/_4P_1\,[1/(1+R_4/R_1)]\,\{1/[1+ \\ &+(R_1+R_4)/(R_2+R_3)]\}\,\varepsilon^2. \end{aligned} \tag{5-42}$$

Letting $R_4/R_1=a$ and $(R_1+R_4)/(R_2+R_3)=R_1/R_2=R_4/R_3=b$ we may rewrite Eq. (5-42) thus:

$$\Delta P_D = {}^1/_4P_1\,[1/(1+a)\,1+b)]\,\varepsilon^2. \tag{5-43}$$

From Eq. (5-43) the power in the detector will be greatest, $\Delta P_{Dmax}=\Delta P_0$, when

$$\left.\begin{aligned} a&=0\,(R_4=0); \\ b&=0\,(R_2+R_3=\infty). \end{aligned}\right\} \tag{5-44}$$

On the other hand,

$$\Delta P_0 = {}^1/_4P_1\varepsilon^2. \tag{5-45}$$

However, if the constants of the bridge were made to satisfy the condition of Eq. (5-44) as much as possible, which would be optimal

* Naturally, this assumption holds only for small ΔR, i. e., near the balance condition of a bridge, which is quite legitimate in an analysis of sensitivity.

from the view-point of sensitivity, there would be a sharp increase in the power drawn by the bridge as a whole. Indeed, near balance, when the current through the detector may be neglected, the powers in the arms R_2, R_3 and R_4 are given by:

$$P_2 = I_1^2 R_2 = P_1 (R_2/R_1);$$
$$P_4 = V_1^2 R_4 = P_1 (R_1/R_4);$$
$$P_3 = I_4^2 R_3 = P_1 (R_1/R_4)(R_3/R_4).$$

Hence the power taken by the bridge network as a whole at balance is

$$P = P_1 + P_2 + P_3 + P_4 = P_1 (1 + R_2/R_1 + R_1/R_4 + $$
$$+ R_1/R_4 \times R_3/R_4) = P_1 (1 + R_1/R_4)(1 + R_3/R_4),$$

or, substituting a and b,

$$P = P_1 (1 + 1/a)(1 + 1/b) = P_1 [(1 + a)(1 + b) /ab]. \qquad (5\text{-}46)$$

From Eq. (5-46) it follows that with decreasing a and b (i.e., closer to the optimum conditions), the total power P sharply increases, tending to infinity. However, comparing Eq. (5-45) and Eq. (5-43) and taking into account Eq. (5-41), we find that already at $a=0.1$ and $b=0.1$ the difference between P_D and P_0 does not exceed 20 per cent. The power dissipated in the four arms of a bridge is a hundred times the power taken by the unknown resistance. In many bridge designs, the ratio arms will stand up to a far greater thermal load, and so our choice seems to be a safe one.

Equation (5-46) makes it possible to determine the power taken by the detector circuit for any ratios of the bridge arms. For the equal ratio-arm bridge, with $a=1$ and $b=1$, the loss is four times as great as in the optimal case given by Eq. (5-45). This must be always kept in mind, since many an experimentor erroneously thinks that the equal ratio-arm bridge is best.

3. All the arms of the bridge, including the unknown resistance, have approximately the same permissible power dissipation; it is required to obtain maximum power in the detector circuit.

This requirement often arises in precise work, such as the testing of resistance standards and the like. It is clear that the supply power will be limited by the capacity of the most loaded arm.

The power dissipated in the four arms (P_1, P_2, P_3, P_4) can be related to the supply voltage thus:

$$\left.\begin{aligned} P_1 &= \frac{V_0^2}{(R_1+R_2)^2} R_1; \quad P_2 = \frac{V_0^2}{(R_1+R_2)^2} R^2; \\ P_3 &= \frac{V_0^2}{(R_3+R_4)^2} R_3; \quad P_4 = \frac{V_0^2}{(R_3+R_4)^2} R_4. \end{aligned}\right\} \qquad (5\text{-}47)$$

Solving the above expressions for V_0, it is possible to determine the maximum safe voltage for a given load on the arms. Assume, for example, that

$$R_1+R_2<R_3+R_4 \text{ and } R_1<R_2.$$

Then the current in the branch *acb* will be greater than in the branch *adb*. The most loaded arm will be R_2. If the maximum permissible power dissipation for this arm be P_2, the best supply voltage, from Eq. (5-47), will be

$$V_0=(R_1+R_2)\sqrt{P_2/R_2}.$$

As was noted earlier, the optimum condition is $R_D \longrightarrow R_{cd}$. Since this condition is satisfied as often as not, we shall introduce a matching coefficient s, so that

$$R_D=sR_{cd}.$$

Then, noting that the bridge is out of balance due to a change ΔR_1 in the unknown resistance R_1 and recalling that

$$R_{cd}=R_4(R_3+R_2/R_3+R_4),$$

we obtain from Eq. (5-13):

$$\Delta I_D=(R_1+R_2)\sqrt{\frac{P_2}{R_2}}\times$$

$$\times\frac{\Delta R_1R_3}{sR_4\left(\frac{R_3+R_2}{R_3+R_4}\right)(R_1+R_2)(R_3+R_4)+R_1R_2(R_3+R_4)+R_3R_4(R_1+R_2)}=$$

$$=(R_1+R_2)\sqrt{\frac{P_2}{R_2}}\times$$

$$\times\frac{\Delta R_1R_3}{s\frac{R_1R_3}{R_2}(R_2+R_3)(R_1+R_2)+R_1R_2\left(R_3+\frac{R_1R_3}{R_2}\right)+R_3\frac{R_1R_3}{R_2}(R_1+R_2)}=$$

$$=(R_1+R_2)\sqrt{\frac{P_2}{R_2}}\times$$

$$\times\frac{\Delta R_1R_2}{R_1\left[s(R_2+R_3)(R_1+R_2)+(R_2+R_3)(R_1+R_2)\right]}=\frac{\Delta R_1\sqrt{P_2R_2}}{R_1(R_2+R_3)(1+s)}. \tag{5-48}$$

The power applied to the detector will be

$$\Delta P_D=\Delta I_D^2R_D.$$

Substituting the expression for ΔI_D from Eq. (5-48) and assuming $R_D=sR_{cd}$, we get:

$$\Delta P_D=\frac{\Delta R_1^2P_2R_2}{R_1^2(R_2+R_3)^2(1+s)^2}\,sR_1\frac{(R_2+R_3)}{(R_2+R_1)}=\left(\frac{\Delta R_1}{R_1}\right)^2P_2\times$$

$$\times\frac{R_1R_2}{(R_2+R_3)(R_2+R_1)}\cdot\frac{s}{(1+s)^2}.$$

When $R_1R_3=R_2R_4$, we have

$$(R_2+R_3)(R_2+R_1)=R_2(R_1+R_2+R_3+R_4).$$

Finally,

$$\Delta P_D=\varepsilon^2 P_2\frac{R_1}{R_1+R_2+R_3+R_4}\times\frac{s}{(1+s)^2}. \tag{5-49}$$

Equation (5-49) has been derived on the assumption that R_1 has the lowest resistance of the four arms; accordingly, the most loaded arm is R_2. However, this is not the only arrangement possible. An examination of all other dispositions will produce identical expressions. From their comparison, a general form for Eq. (5-49) will be

$$\Delta P_D=\varepsilon^2 P\,(R_{min}/\sum R)\times s/(1+s)^2, \tag{5-50}$$

where

P = power dissipation in the most loaded arm;

R_{min} = arm of lowest resistance;

$\Sigma R=R_1+R_2+R_3+R_4$ = sum of all arm resistances.

Notation for the other terms has been given earlier.

As follows directly from Eq. (5-50), in the case of an equal-arm bridge, where all the arms have the same permissible load, the power delivered into the detector will be 1/16th of the power P dissipated in each arm (or, in a more general case, in the most loaded arm) times the square of the relative change in the unknown resistance, $\varepsilon=\Delta R_1/R_1$. This, naturally, agrees well with the previous case when $a=1$ and $b=1$.

4. All the bridge arms and the supply voltage are given; it is required to determine the best disposition of the source and detector.

This problem often arises in connection with permanent bridges using an external galvanometer and an external battery.

As has been noted, the balance condition is independent of the resistances of the voltage source and null detector, and the same balance point is obtained when the source and detector are interchanged. In the case of an unbalanced bridge, however, the current and, consequently, the sensitivity of the detector will remain unchanged only if all the arms are equal, changing in any other case. For a better understanding, let us examine the two dispositions of Fig. 5-9.

The detector current is given by:

$$\Delta I_D=V_0\frac{R_1R_3-R_2R_4}{R_D(R_1+R_2)(R_3+R_4)+\Pi}\ \text{(for the arrangement of Fig.5-9}a\text{);}$$

$$\Delta I_D=V_0\frac{R_1R_3-R_2R_4}{R_D(R_1+R_4)(R_2+R_3)+\Pi}\ \text{(for the arrangement of Fig. 5-9}b\text{).}$$

where $\Pi=R_1R_2R_3+R_2R_3R_4+R_3R_4R_1+R_4R_1R_2$.

As follows from these expressions, both the balance condition (the numerator) and the function Π, as a fairly symmetrical one, remain unchanged with any disposition of the source and detector. The only term that changes is the factor at R_D. Denote it as $\varphi(R)$.

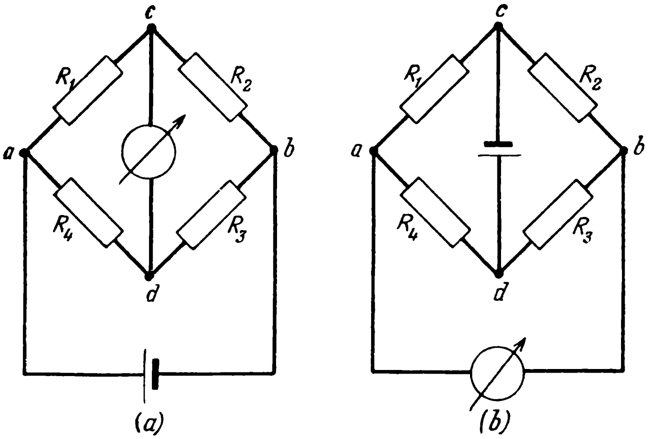


Fig. 5-9

It is obvious, the smaller φ (R), the greater the current and sensitivity of the network. This function for the two arrangements of Fig. 5-9 can be written thus:

$$\varphi_a(R)=(R_1+R_2)(R_3+R_4);$$
$$\varphi_b(R)=(R_1+R_4)(R_2+R_3).$$

Let us determine the conditions under which

$$\varphi_b(R)>\varphi_a(R),$$

i. e.,

$$\varphi_b(R)-\varphi_a(R)>0.$$

This difference can be presented thus:

$$\varphi_b(R)-\varphi_a(R)=(R_1+R_4)(R_2+R_3)-(R_1+R_2)(R_3+R_4).$$

Rationalizing, we get:

$$\varphi_b(R)-\varphi_a(R)=(R_2-R_4)(R_1-R_3).$$

Apparently, this expression will be positive and, consequently, the arrangement of Fig. 5-9*a* will have a greater sensitivity, if each of the inequalities

$$R_2>R_4;\ R_1>R_3,$$

is satisfied separately.

Comparing these inequalities with the arrangements of Fig. 5-9, we can state that for the given R_0 and R_D, the highest sensitivity will be obtained from a given bridge with the two smaller resistances connected in one branch and the two greater resistances in the other. This rule determines why more power is dissipated in one arrangement than in the other. The rule holds if $R_0 < R_D$; otherwise, the reverse is the case.

5. Low resistances are to be measured with a Kelvin double bridge; it is required to obtain maximum power in the detector circuit.

It should be noted above all that since R_x and R_N (Fig. 5-5) are usually very small, the voltage supply that may be used is governed by the permissible power dissipated either by R_x or by R_N. This leaves us with a situation similar to case (3) above. It can be shown that Eq. (5-50) is valid for both a Wheatstone and a Kelvin bridge.

Assuming $R_4 = R_4'$ and $R_3 = R_3'$ (Fig. 5-5), we convert the mesh $R_4' \, r R_3'$ into a star, i.e., to the network of Fig. 5-6. Then, dropping ΔR_4 in the denominator and using the balance condition $R_x R_3 = R_N R_4$, we determine by any convenient method the change in the detector current for a change in the resistance R_4:

$$\Delta I_D = I_0 \frac{\Delta R_4 R_N}{R_D (R_3 + R_4) + R_4 (R_N + 2R_3)} \,. \tag{5-51}$$

Denoting the resistance which the bridge presents to the detector terminals as R_{out}, we re-introduce the matching coefficient:

$$s = R_D / R_{out}.$$

The resistance r, being very small, may be safely dropped in determining the output resistance of the bridge. Then

$$R_{out} = \frac{(2R_4 + R_x)(2R_3 + R_N)}{2R_3 + 2R_4 + R_x + R_N} = R_3 \frac{2R_4 + R_x}{R_3 + R_4} \,. \tag{5-52}$$

Expressing R_D in terms of s and R_{out} by Eq. (5-52) and Eq. (5-51) and proceeding as in the case of Eq. (5-48) and Eq. (5-49), we get:

$$\Delta I_D = I_0 (\Delta R_4 / R_4) [R_x / (R_x + 2R_4)] [1/(1+s)];$$

$$\Delta P_D = (\Delta R_4 / R_4)^2 [I_0^2 R_x R_N / (2R_4 + 2R_3 + R_x + R_N)] [s/(1+s)^2].$$

In a double bridge, R_x and R_N are known to be small in value and loaded more than the other arms. Therefore, it is safe to assume that

$$I_0^2 R_x R_N = P R_{min},$$

for the greater of them is more loaded, while the smaller is the lowest of all the resistances in the bridge. Denoting

$$\sum R = 2R_4 + 2R_3 + R_x + R_N,$$

we again have Eq. (5-50) which is thus proved to be common to both Wheatstone and Kelvin bridges.

A comparison of the design formulas for both types of bridges will show that the output power of Kelvin bridges is considerably lower, because the ratio $R_{min}/\Sigma R$ in Eq. (5-50) is much smaller for them. This is the reason why in measuring very low resistances with a Kelvin bridge, use has to be made of more sensitive galvanometers than for the measurement of medium resistances with a Wheatstone bridge, and of the maximum possible current.

6. Other cases.

Both in designing and using bridge networks, one may run into situations differing from those just discussed. For reference, there is Table 5-1 summarizing optimum bridge parameters for the four-arm bridge of Fig. 5-1, for the most common cases. It uses the same notation and also includes the cases discussed.

5-5. Unbalanced Bridges. Comparison Sets

Some of the commercial measurements do not call for the high accuracy given by balanced bridges. Also, the necessity for their balancing and the time they take for a measurement may prove major and, perhaps, decisive drawbacks.

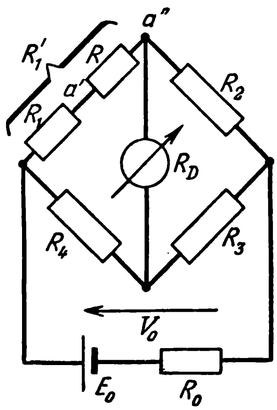


Fig. 5-10

Thus, while the use of balanced bridges is warranted for precise measurements (accurate to within a few hundredths of one per cent), less important (and especially large-scale) commercial measurements accurate to within 0.1 to 1.0 per cent may well be taken with simpler devices and with readings taken directly from the detector. Such devices have for some time past been finding ever wider use, since the measurement of nonelectrical quantities by electrical methods are in most cases based on exactly this principle.

The desire to simplify and speed up measurements has resulted in several special types of bridges among which are unbalanced bridges and comparison sets.

Unbalanced bridges operate on the principle that when a balanced bridge is thrown out of balance by a change in the resistance of one of its arms, the current through the detector is a function of the change in the resistance of that arm. Moreover, when the change is small and the supply voltage is constant, the detector current may be assumed proportional to the change.

In an analysis of an unbalanced bridge, one must determine the law of the scale of the moving-coil detector connected across the bridge and the sensitivity of the bridge. For this, the detector current should be related to changes in the resistance of an arm. This can be done, say, by Eq. (5-14). Assume that the arm R_1 of an initially balanced bridge, i.e., when $R_1R_3=R_2R_4$, has changed by R (Fig. 5-10). As a result, there will be a current I_D flowing through the detector, which, on the basis of Eq. (5-14), may be written thus:

$$I_D=E_0\frac{R_1'R_3-R_2R_4}{R_0R_D(R_1'+R_2+R_3+R_4)+R_D(R_1'+R_2)(R_3+R_4)+R_0(R_1'+R_4)(R_2+R_3)+\Pi}; \tag{5-53}$$

where

$$\Pi=R_1'R_2(R_3+R_4)+R_3R_4(R_1'+R_2)$$

and

$$R_1'=R_1+R.$$

Equation (5-53) may be rewritten thus:

$$I_D=R_3E_0\frac{R}{aR+b}, \tag{5-54}$$

where

$$\left.\begin{aligned} a&=(R_D+R_2)(R_3+R_4)+R_3R_4+R_0(R_D+R_2+R_3);\\ b&=R_D(R_1+R_2)(R_3+R_4)+R_1R_2(R_3+R_4)+\\ &\quad+R_3R_4(R_1+R_2)+R_0[R_D(R_1+R_2+R_3+R_4)+\\ &\quad+(R_1+R_4)(R_2+R_3)]. \end{aligned}\right\} \tag{5-55}$$

From Eq. (5-54) it follows that all the quantities included in a and b are constant for a given bridge. Consequently, the detector current is solely a function of R. This relationship, $I_D=f(R)$, is shown in Fig. 5-11. With only one arm being variable, this relationship will be the same for any bridge. The actual magnitude of the curve depends on the value of E and of all the bridge parameters (R_1, R_2, R_3, R_4, R_0, R_D).

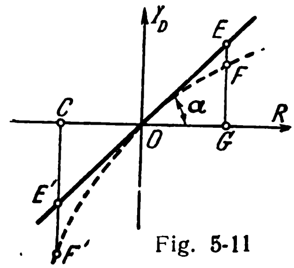


Fig. 5-11

For maximum sensitivity, it is essential that the angle α between the tangent to the curve and the X-axis be greatest and that the deviation of the curve from a straight line, i.e., the distance EF

Table 5-1

Calculation of D. C. Wheatstone Bridge for Maximum Sensitivity

Nos	Given \ Find	R_1	R_2	R_4	R_D	E
1	R_0, E	$R_1=R_0$	$R_2=R_0$	$R_4=R_0$	$R_D=R_0$	—
2	R_0, R_D, E	$R_1= =\sqrt{R_0R_D}$	$R_2=R_1$	$R_4=R_1$	—	—
3	R_1, R_0^*; E	—	$R_2=R_1\sqrt{1+\frac{R_0}{R_1}}\times \times\cos\left[\frac{1}{3}\arccos\times \times\left(-\sqrt{\frac{R_1}{R_0+R_1}}\right)\right]$	$R_4=\frac{R_1\sqrt{\frac{R_0}{R_0+R_1}}}{\cos\left[\frac{1}{3}\arccos\times \times\left(-\sqrt{\frac{R_0}{R_0+R_1}}\right)\right]}$	$R_D=R_2\times \times\frac{R_1+R_4}{R_1+R_2}$	—
4	R_1, R_0^*; P_1; —	—	$R_2\to\infty$	$R_4\to\infty$	Ditto	$E\leqslant\sqrt{\frac{P_1}{R_1}}\left(R_0+ +R_1+R_2+\frac{R_0R_1}{R_4}\right)$
5	R_1, R_0^*; P_1; E	—	$R_2=\frac{R_1(E-\sqrt{P_1R_1})}{\sqrt{P_1R_1}(R_1-R_0)}\times \times\left[R_1- -\sqrt{R_0\frac{ER_1-R_0\sqrt{P_1R_1}}{E-\sqrt{P_1R_1}}}\right]$	$R_4= =\frac{R_0R_1}{E\sqrt{\frac{R_1}{P_1}}-(R_0+R_1+R_2)}$	Ditto	—

6			P	—	$R_2 = R_1 \dfrac{\sqrt{P} - \sqrt{P_1}}{\sqrt{P_1}}$	$R_4 = R_1 \dfrac{\sqrt{P_1}}{\sqrt{P} - \sqrt{P_1}}$	Ditto	$E \leqslant \sqrt{\dfrac{P}{R_1}(R_1+R_0)}$
7		P_n		—	$R_2 = R_1$	$R_4 = R_1$	Ditto	$E \leqslant \sqrt{\dfrac{P_n}{R_1} \times 2\,(R_0+R_1)}$
8		P		—	$R_2 = R_1$	$R_4 = R_1$	Ditto	$E \leqslant \sqrt{PR_1}\left(1+\dfrac{R_0}{R_1}\right)$
9		E		—	$R_2 = \sqrt{R_1 R_D \dfrac{R_1+R_0}{R_1+R_D}}$	$R_4 = \sqrt{R_0 R_1 \dfrac{R_1+R_D}{R_1+R_0}}$	—	—
10		P_1	—	—	$R_2 \to \infty$	$R_4 \to 0$	—	$E \leqslant \sqrt{\dfrac{P_1}{R_1}\left(R_0 + R_1 + R_2 \dfrac{R_0 R_1}{R_4}\right)}$
11	R_1, R_0^*, R_D	P_1	E	—	$R_2 = \dfrac{\sqrt{R_D}}{\sqrt{R_D}+\sqrt{R_0}} \times \left(E\sqrt{\dfrac{R_1}{P_1}} - R_1 - R_0\right)$	$R_4 = \dfrac{R_0 R_1}{E\sqrt{\dfrac{R_1}{P_1}} - (R_0+R_1+R_2)}$	—	—
12			P	—	$R_2 = \dfrac{R_1(P-P_1)}{P_1 + \sqrt{PP_1 \dfrac{R_1}{R_D}}}$	$R_4 = R_1 \dfrac{P_1 + \sqrt{PP_1 + \dfrac{R_D}{R_1}}}{P-P_1}$	—	$E \leqslant \sqrt{\dfrac{P}{R_m}(R_A+R_0)}$

(Continued)

Nos	Given	Find	R_1	R_2	R_4	R_D	E
13	R_1, R_0^*, R_D	P_n	—	$R_2=R_1$	$R_4=R_1$	—	$E \leqslant 2\sqrt{\frac{P_n}{R_1}} \times (R_0+R_1)$
14		P	—	$R_2=\frac{R_1\sqrt{\frac{R_D}{R_1+R_D}}}{\cos\left[\frac{1}{3}\arccos\times\left(-\sqrt{\frac{R_D}{R_1+R_D}}\right)\right]}$	$R_4=R_1\sqrt{1+\frac{R_D}{R_1}\cos\times\left[\frac{1}{3}\arccos\times\left(-\sqrt{\frac{R_1}{R_1+R_D}}\right)\right]}$	—	$E\leqslant \leqslant\sqrt{\frac{P}{R_m}(R_0+R_m)}$
15		E	—	$R_2=\frac{R_1}{4}\times\left[\sqrt{9+8\left(\frac{R_0}{R_1}+\frac{R_0}{R_4}\right)}- -1\right]$	$R_4=\frac{R_1R_0}{2(R_1+R_2+R_0)}\times\left[1+ +\sqrt{9+8\left(\frac{R_1}{R_0}+\frac{R_2}{R_0}\right)}\right]$	$R_D=R_2\frac{R_1+R_4}{R_1+R_2}$	—
16	$R_1, R_0^*, R_4\times$ (or R_2)	P_1	—	$R_2\to\infty$	$R_4\to R_0$	$R_D=R_2\frac{R_1+R_4}{R_1+R_2}$	$E\leqslant\sqrt{\frac{P_1}{R_1}}\left(R_0+ +R_1+R_2+\frac{R_0R_1}{R_4}\right)$
17		P_n	—	$R_2=R_1$	$R_4=R_1$	$R_D=R_2\frac{R_1+R_4}{R_1+R_2}$	—

4*

18		P	—	$R_2=R_1$	$R_4=R_1$	$R_D=R_2\dfrac{R_1+R_4}{R_1+R_2}$	$E\leqslant\sqrt{\dfrac{P}{R_m}}\times(R_0+R_m)$
19		E	—	$R_2=\sqrt{R_1R_D\dfrac{R_1+R_0\left(1+\dfrac{R_1}{R_4}\right)}{R_1+R_4+R_D}}$	$R_4=\sqrt{R_1R_0\dfrac{R_1+R_D\left(1+\dfrac{R_1}{R_2}\right)}{R_0+R_1+R_2}}$	—	—
20	R_1, R_0^*, R_D, $R_4\times$ $\times$ (or R_2)	P_1	—	$R_2\to\infty$	$R_4\to 0$	—	$E\leqslant\sqrt{\dfrac{P_1}{R_1}}\left(R_0+R_1+R_2+\dfrac{R_0R_1}{R_4}\right)$
21		P_n	—	$R_2=R_1$	$R_4=R_1$	—	—
22		P	—	$R_2=\dfrac{R_1R_D}{2(R_1+R_4+R_D)}\times\left[1+\sqrt{9+8\left(\dfrac{R_1}{R_D}+\dfrac{R_4}{R_D}\right)}\right]$	$R_4=\dfrac{R_1}{4}\left[\sqrt{9+8\times\left(\dfrac{R_D}{R_1}+\dfrac{R_D}{R_2}\right)}-1\right]$	—	$E\leqslant\sqrt{\dfrac{P}{R_m}}\times(R_0+R_m)$
23	R_1, R_2, R_4, R_0^*	E	—	—	—	$R_D=R_2\dfrac{R_1+R_4}{R_1+R_2}$	—

* If R_1 is not given, take $R_0=0$.

or $E'F'$, be smallest. Let us examine the two requirements in turn.

As follows from Eq. (5-54),

$$S=\partial I_D/\partial R=E_0R_3\frac{b}{(aR+b)^2}\,, \tag{5-56}$$

where S is the sensitivity of a bridge.

Actually, the angle gives the sensitivity of the bridge near balance, or the balance sensitivity, S_0.

Assuming $R=0$ in Eq. (5-56) gives:

$$S_0=ER_3/b. \tag{5-57}$$

Now we can relate both forms of sensitivity:

$$S=(ER_3/b)\times 1/(1+aR/b)^2=S_0\frac{1}{(1+aR/b)^2}\,. \tag{5-58}$$

In order to understand the physical implication of the ratio a/b, let us determine the input resistance for the circuit of Fig. 5-10 with respect to the terminals $a'a''$. Let this resistance be R_a. The equivalent circuit is shown in Fig. 5-12.

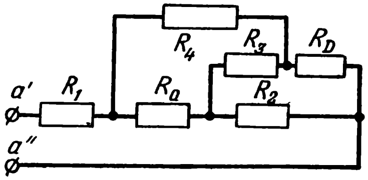


Fig. 5-12

By converting the star $R_3/R_4/R_D$ into an equivalent mesh we get

$$R_a=R_1+$$

$$+\frac{\left[\dfrac{R_0(R_3+R_4+R_3R_4/R_D)}{R_0+R_3+R_4+R_3R_4/R_D}+\dfrac{R_2(R_D+R_3+R_3R_D/R_4)}{R_2+R_D+R_3+R_DR_3/R_4}\right](R_4+R_D+R_4R_D/R_3)}{\dfrac{R_0(R_3+R_4+R_3R_4/R_D)}{R_0+R_3+R_4+R_3R_4/R_D}+\dfrac{R_2(R_D+R_3+R_DR_3/R_4)}{R_2+R_D+R_3+R_DR_3/R_4}+R_4+R_D+R_4R_D/R_3},$$

or, rationalizing:

$$R_a=\frac{R_D(R_1+R_2)(R_3+R_4)+R_1R_2(R_3+R_4)+R_3R_4(R_1+R_2)+R_0[R_D(R_1+R_2+R_3+R_4)+(R_1+R_4)(R_2+R_3)]}{(R_D+R_2)(R_3+R_4)+R_3R_4+R_0(R_D+R_2+R_3)}\,. \tag{5-59}$$

Comparing Eq. (5-59) and Eq. (5-55), we see that

$$R_a=b/a. \tag{5-60}$$

Substituting Eq. (5-60) in Eqs. (5-54) and (5-58) and taking into account Eq. (5-57), we finally get

$$I_D = S_0 R/(1+R/R_a);$$
$$S = S_0 1/(1+R/R_a)^2. \tag{5-61}$$

Substituting for b in Eq. (5-57) and noting that $R_1R_3 = R_2R_4$ we have

$$S_0 = \frac{E}{R_1[R_1+R_2+R_3+R_4+R_D(2+R_3/R_4+R_4/R_3)+R_0(2+R_3/R_2+R_2/R_3)+ +R_0R_D(1/R_1+1/R_2+1/R_3+1/R_4)]}. \tag{5-62}$$

Equation (5-62) may serve as a point of departure for a complete analysis and selection of optimum parameters for bridges with maximum current sensitivity. We shall limit ourselves to a particular case involving symmetrical bridge networks most commonly used in practice.

Assume that

$$\left.\begin{aligned} R_1 &= R_2; \\ R_3 &= R_4. \end{aligned}\right\} \tag{5-63}$$

Let the resistance of the galvanometer be equal to the output resistance R_{cd} of the bridge. Then, noting Eq. (5-63), we get

$$R_D = \frac{R_0(R_1+R_4)(R_2+R_3)+R_1R_2(R_3+R_4)+R_3R_4(R_1+R_2)}{R_0(R_1+R_2+R_3+R_4)+(R_1+R_2)(R_3+R_4)} = \frac{R_1+R_3}{2}. \tag{5-64}$$

Thus, from the seven quantities included in Eq. (5-62) only two, R_1 and R_3, are independent variables. Denoting

$$R_0 = nR_1 \tag{5-65}$$

and substituting Eqs. (5-63), (5-64) and (5-65) in Eq. (5-62) we finally have

$$S_0 = \frac{E_0}{4R_1\left[R_1+R_3+\frac{R_0}{2}(2+R_3/R_1+R_1/R_3)\right]} = \frac{E_0}{4R_1^2(1+n)\left[1+\left(\frac{1+n}{2n}\right)\frac{R_0}{R_1}\right]}. \tag{5-66}$$

For the other type of symmetry

$$\left.\begin{aligned} R_1 &= R_4; \\ R_2 &= R_3, \end{aligned}\right\} \tag{5-67}$$

the output resistance R_{cd} of the bridge and, consequently, the resistance of the galvanometer R_D will be

$$R_D = 2R_1R_3/(R_1+R_3), \tag{5-68}$$

for which reason a different expression is obtained for balance sensitivity:

$$S_0 = \frac{E_0}{4R_1(R_1+R_3+2R_0)} = \frac{E_0}{4R_1^2\left(1+n+2\frac{R_0}{R_1}\right)}. \tag{5-69}$$

As should be expected, for a fully equal-arm bridge, i. e., for $n=1$, any of Eqs. (5-66) and (5-69) gives the same expression for S_0, namely

$$S_0 = \frac{E_0}{8R_1^2(1+R_0/R_1)}. \tag{5-70}$$

On the basis of Eqs. (5-63), (5-65) and (5-67) and substituting in Eq. (5-59), we get respectively:

(a) for the symmetry $R_1=R_2$, $R_3=R_4$:

$$R_a = 4\frac{(n+1)\left[1+\left(\frac{n+1}{2n}\right)\frac{R_0}{R_1}\right]}{2n+3+3\left(\frac{n+1}{2n}\right)\frac{R_0}{R_1}}R_1; \tag{5-71}$$

(b) for the symmetry $R_1=R_4$, $R_2=R_3$:

$$R_a = 4\frac{n+1+2\frac{R_0}{R_1}}{n+4+2\left(\frac{n+2}{n+1}\right)\frac{R_0}{R_1}}R_1. \tag{5-72}$$

Substituting $n=1$ into any of Eqs. (5-71) and (5-72), we obtain for an equal-arm bridge:

$$R_a = \frac{8(1+R_0/R_1)}{5+3R_0/R_1}R_1. \tag{5-73}$$

To sum up, Eqs. (5-66), (5-69), (5-70), (5-71), (5-72) and (5-73) give all that is necessary for determining by Eq. (5-61) the detector current in unbalanced symmetrical and equal-arm bridges.

Obviously, in the most common case, when $E_0=V_0$ and $R_0=0$, the expressions for S_0 and R_a become much simpler. Thus, when $R_1R_3=R_2R_4$, Eqs. (5-62) and (5-69) for an asymmetrical bridge give:

$$S_0 = \frac{V_0}{R_1[R_1+R_2+R_3+R_4+R_D(2+R_3/R_4+R_4/R_3)]};$$

$$R_a = \frac{R_D(R_1+R_2)(R_3+R_4)+R_1R_2(R_3+R_4)+R_3R_4(R_1+R_2)}{(R_D+R_2)(R_3+R_4)+R_3R_4} =$$

$$= \frac{R_D(R_1+R_2)+R_1(R_2+R_3)}{R_D+R_2+R_1R_3/(R_1+R_2)}.$$

When $R_0=0$, for symmetrical bridges we get:
(a) from Eqs. (5-66) or (5-69) for any of the two types of symmetry:

$$S_0=\frac{V_0}{4R_1(R_1+R_3)}=\frac{V_0}{4R_1^2(1+n)}; \qquad (5\text{-}74)$$

(b) from Eq. (5-71) for the symmetry $R_1=R_2$

$R_3=R_4$:

$$R_a=\frac{4(n+1)}{2n+3}R_1; \qquad (5\text{-}75)$$

(c) from Eq. (5-72) for the symmetry $R_1=R_4$, $R_2=R_3$:

$$R_a=\frac{4(n+1)}{n+4}R_1. \qquad (5\text{-}76)$$

Finally, for an equal-arm bridge we find by substituting $R_0=0$ in Eqs. (5-70) and (5-73) or $n=1$ in Eqs. (5-74) and (5-75) or (5-76) that

$$S_0=V_0/8R_1^2; \quad R_a=8/5R_1.$$

Now consider the nonlinearity of the scale. A measure of its nonlinearity is given by the deviation of the curve $I_D=f(R)$ from a straight line. Presenting the tangent $E'E$ (Fig. 5-11) in equation form as

$$I_D^0=S_0R, \qquad (5\text{-}77)$$

the distance $EF=I_D^0-I_D$ gives us the absolute nonlinearity of the scale at point G.

Denoting the relative nonlinearity of the scale as D, we get

$$D=(I_D^0-I_D)/I_D. \qquad (5\text{-}78)$$

Substituting the expressions for currents from Eqs. (5-61) and (5-77) in Eq. (5-78) we get

$$D=R/R_a.$$

Then Eq. (5-61) for the current I_D can be given the form:

$$I_D=S_0R/(1+D).$$

This is the final equation giving the detector current. As can be seen, the current depends on the balance sensitivity, scale nonlinearity, and the amount of bridge out-of-balance.

Now we have obtained all the formulas necessary for the calculation of a bridge and its scale.

The results can be conveniently presented in graph form, as a nomogram, using the so-called projection construction.

Projection construction boils down to the following: from a point C (called the pole) in a rectangular coordinate system (Fig. 5-13) * a straight line is drawn, giving two similar triangles AOD and ABC.

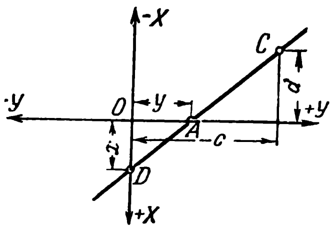


Fig. 5-13

From examination of these triangles we may write

$$d/x=(s-y)/y,$$

whence $$y=cx/(d+x). \qquad (5\text{-}79)$$

Comparing Eqs. (5-61) and (5-79), we note that these equations are identical when

$$I_D=y; \quad S_0R_a=c;$$
$$R=x; \quad R_a=d.$$

Consequently, in order to be able to determine the law of the scale of an unbalanced bridge (presuming that the scale of the galvanometer is uniform) it is necessary to find the values of R_a (ohms) and S_0R_a(mA); these values will locate the pole C ** (Fig. 5-14). Rotation of the line l about the pole in accordance with the given values of R will produce the corresponding values of I_D. The points of intersection of the abscissas of the resistance and of the ordinates of the current will be the points on the curve $I_D=f(R)$, giving the shape of the scale of an unbalanced bridge.

The angle α between the line l and the X-axis is solely dependent on the balance sensitivity:

$$\tan\alpha=S_0.$$

Putting it another way, the sensitivity of a bridge increases with increasing angle α. It is obvious that this angle is equal to the angle α in Fig. 5-11.

This same nomogram can be used for the solution of a reverse problem: that of calculating a bridge network from a given curve $I_D=$ $=f(R)$. It will be sufficient to take two points on the curve, say G and H, and to construct rectangles $AGBO$ and $DHEO$. Then the point of intersection of their diagonals will give the position of the pole C from which both R_a and S_0 can be easily found.

We have so far ignored that the balance sensitivity and the detector current are directly proportional to the e. m. f. E_0 (or, respectively, V_0) of the source. Because of this, the scale of an unbalanced

* In a general case, projection construction also holds for a curvilinear coordinate system.

** Since the pole C may be located in any quadrant, let it be in the second. Then, the curve $I_D=f(R)$ will be located exactly as it is in Fig. 5-11, which answers the physical nature of the process.

bridge will be linear only for constant E_0 (or V_0). This can be ensured by using moving-coil ratiometers.

The sensitivity of bridges can sometimes be enhanced by placing several sensing elements in different arms of the bridge. This may

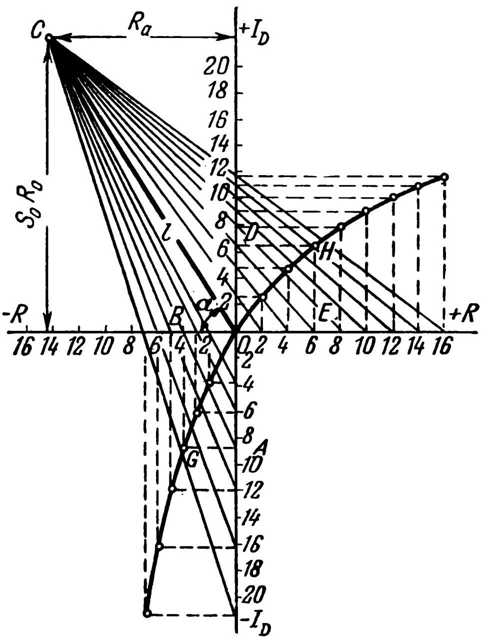


Fig. 5-14

also appreciably reduce the nonlinearity of the scale of the presentation instrument. Such bridges can be calculated by the techniques described above. The results of the calculations are summarized in Table 5-2 for reference.

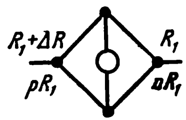
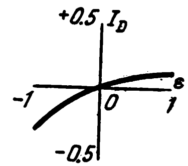
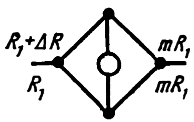
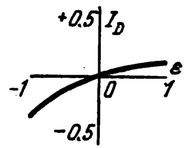
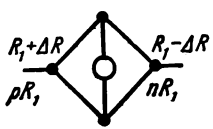
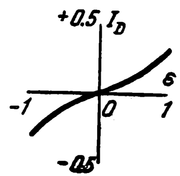
In some applications, it may prove practicable to use unbalanced Kelvin double bridges. Their calculation is done by a technique similar to the one discussed above.

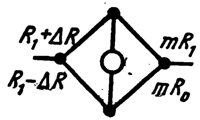
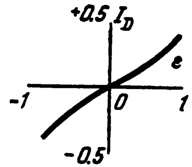
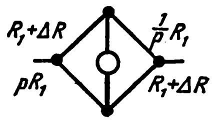
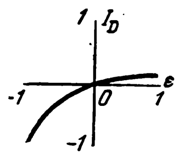
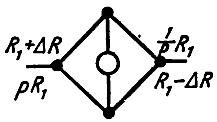
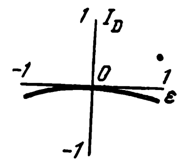
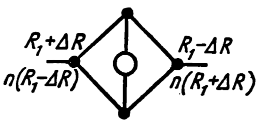
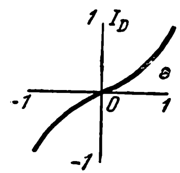
There is another variety of simplified bridge networks intended for rapid measurements. Termed comparison sets or *percentage bridges*, they are mainly used for the routine checking of mass-produced components of a single rating. In such cases, it is important to know the percentage deviation of a given quantity from the true value, rather than the true value as such. Hence the name "percentage bridge".

Percentage bridges can be both balanced and unbalanced. The former variety, encountered more often, is an ordinary four-arm bridge

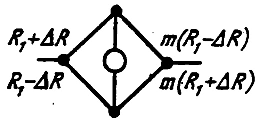
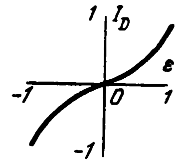
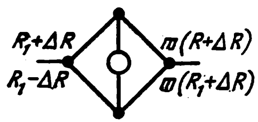
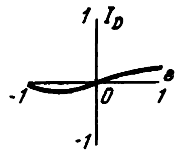
Table 5-2

Characteristics of Unbalanced Bridge Scales

Diagram	Detector current, I_D	Balance sensitivity, S_0	Sensitivity index	Optimum detector resistance, R_D	Scale nonlinearity $D = \frac{\overset{\circ}{I}_D - I_D}{I_D}$	Scale law when $n=m=p=1$, $V_0=1$, $R_1=1$
	$I_D = \frac{V_0}{R_1} \times \frac{\varepsilon}{4(1+n)+\varepsilon(3+2n)}$	$S_0 = \frac{V_0}{R_1} \times \frac{1}{4R_1(1+n)}$	1	$R_D = \frac{1+n}{2} R_1$	$D = \frac{3+2n}{4(1+n)}\varepsilon$	
	$I_D = \frac{V_0}{R_1} \times \frac{\varepsilon}{4(1+m)+\varepsilon(4+m)}$	$S_0 = \frac{V_0}{R_1} \times \frac{1}{4R_1(1+m)}$	1 $(m=n)$	$R_D = \frac{2m}{1+m} R_1$	$D = \frac{4+m}{4(1+m)}\varepsilon$	
	$I_D = \frac{V_0}{R_1} \frac{\varepsilon}{2(1+n)-\varepsilon^2}$	$S_0 = \frac{V_0}{R_1} \times \frac{1}{2R_1(1+n)}$	2	$R_D = \frac{1+n}{2} R_1$	$D = -\frac{1}{2(1+n)}\varepsilon$	

	$I_D=\frac{V_0}{R_1}\times \frac{\varepsilon}{2(1+m)-\varepsilon^2\frac{3+m}{2+m}}$	$S_0=\frac{V_0}{R_1}\times\frac{1}{2R_1(1+m)}$	2 $(m=n)$	$R_D=\frac{2m}{1+m}R_1$	$D=-\frac{2+m}{2(1+m)^2}\varepsilon^2$	
	$I_D=\frac{V_0}{R_1}\frac{2\varepsilon+\varepsilon^2}{2\left(2+\frac{1}{p}+p\right)+\varepsilon\left(4+3p+\frac{3}{p}\right)+\varepsilon^2\times\left(1+p+\frac{1}{p}\right)}$	$S_0=\frac{V_0}{R_1}\times\frac{1}{R_1\left(2+\frac{1}{p}+p\right)}$	2 $(p=n)$	$R_D=R_1$	$D\approx\frac{3p^2+4p+3}{2p^2+4p+2}\varepsilon$	
	$I_D=-\frac{V_0}{R_1}\times\frac{\varepsilon^2}{2\left(2+\frac{1}{p}+p\right)+\varepsilon\left(p-\frac{1}{p}\right)-\varepsilon^2\left(1+p+\frac{1}{p}\right)}$	$S_0=0$	0	$R_D=R_1$	$D\approx\frac{p-1}{2(p+1)}\times\varepsilon-\frac{p^2+p+1}{2(p^2+2p+1)}\varepsilon^2$	
	$I_D=\frac{V_0}{R_1}\times\frac{\varepsilon}{(1+n)-\varepsilon^2\frac{1+n}{2}}$	$S_0=\frac{V_0}{R_1}\times\frac{1}{R_1(1+n)}$	4	$R_D=\frac{1+n}{2}R_1$	$D=-\frac{1}{2}\varepsilon^2$	

(Continued)

Diagram	Detector current, I_D	Balance sensitivity, S_0	Sensitivity index	Optimum detector resistance, R_D	Scale nonlinearity $D=\dfrac{\overset{0}{I}_D-I_D}{I_D}$	Scale law when $n=m=p=1$, $V_0=1$, $R_1=1$
	$I_D=\dfrac{V_0}{R_1}\dfrac{\varepsilon}{(1+m)+\varepsilon(1-m)-\varepsilon^2\dfrac{2m}{1+m}}$	$S_0=\dfrac{V_0}{R_1}\times\dfrac{1}{R_1(1+m)}$	4 $(m=n)$	$R_D=\dfrac{2m}{1+m}R_1$	$D=\dfrac{1-m}{1+m}\varepsilon-\dfrac{2m}{(1+m)^2}\varepsilon^2$	
	$I_D=\dfrac{V_0}{R^1}\dfrac{\varepsilon+\varepsilon^2}{2(1+m)+\varepsilon(1+4m)+2\varepsilon^2(m-1)}$	$S_0=\dfrac{V_0}{R_1}\times\dfrac{1}{2R_1(1+m)}$	2 $(m=n)$	$R_D=\dfrac{2m}{1+m}R_1$	$D=\dfrac{1+4m}{2(1+m)}\times\varepsilon+\dfrac{m-1}{m+1}\varepsilon^2$	

which becomes balanced when the actual value of the unknown resistance is equal to its nominal value. If the actual value of the specimen differs from its rating, the deflection of the galvanometer will obviously give a measure and sign of this difference. Naturally, for a certain definite rating of the variable in question the galvanometer can be calibrated to read per cent deviation directly.

A serious drawback of unbalanced percentage bridges is that, if the supply voltage is left unchanged when testing several nominal sizes of specimens, the associated change in the input resistance of the bridge will inevitably affect its sensitivity (i.e., the detector current which produces a one-percent deflection in the galvanometer). As a way out and, consequently, in order to maintain the percentage calibration of the galvanometer which, theoretically, must be the same for any magnitude of the unknown (the current through the galvanometer being proportional to the ratio $\Delta R/R$), it is customary to use shunts or to regulate supply voltage. The latter can be accomplished automatically, if the bridge is fed from a suitably selected voltage divider instead of a battery. Then any change in the sensitivity due to variations in the input resistance of the bridge will be compensated by changes in supply voltage due to redistribution of voltage drops across the divider. It is obvious that, as with any unbalanced bridge, the e.m.f. of the source must be maintained constant.

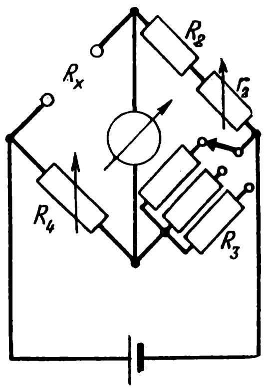


Fig. 5-15

Of course, balanced percentage bridges are somewhat more complicated in use and take more time and effort to adjust. On the other hand, they are more precise and are less affected by external effects. The simplest balanced percentage bridge is shown diagrammatically in Fig. 5-15. Initially, the bridge is adjusted by means of the resistor R_4 which is then left undisturbed. In the adjusted bridge, the resistance r_2 should be set to zero, i.e., to its mid-position. Subsequent balancing is accomplished by means of r_2. Since the main resistance R_2 is fixed, r_2 may be calibrated directly in per cent deviation. Whenever necessary, a change-over to another nominal size or nominal setting can be accomplished by selecting the appropriate resistance R_3.

In some quality-control applications, even per cent deviation may be unnecessary, it being sufficient to ascertain that the controlled object is within specified tolerance limits. This purpose can be served by a still simpler bridge network, the limit bridge, in which one of the arms (preferably, R_4) consists of two resistors which can be brought

in curcuit in turn by a switch or commutator. These are variable resistors adjusted so that the bridge will be at balance when the controlled specimen is either greater than the nominal value by the amount of tolerance (in which case one of the two variable resistors is brought in circuit and the switch is in one position) or smaller by the same amount (in which case the other resistor is in circuit and the switch is in the other position). Any further adjustments are unnecessary, and the checking procedure consists in that a specimen is inserted in the bridge, and the sign of galvanometer deflection is noted for the two positions of the switch. If the galvanometer deflections are in opposite directions, the specimen lies within the tolerance limits. If the galvanometer deflects in the same direction both times, the unknown is outside the limits. A limit bridge functions like a go-no-go gauge widely used in engineering industries and greatly simplifies the quality-control procedure.

In conclusion, it may be added that percentage bridges can be made to a high degree of accuracy, sufficient for large-scale precise work.

5-6. Construction of D. C. Bridges

In some cases, where a bridge network is to meet especially stringent requirements, it may be temporarily assembled of suitably selected components (for example, high-precision resistance coils). But the most commonly used type is a permanent or a built-up bridge, factory-assembled and built into a case as a single whole.

According to the relevant USSR State Standard, all factory-made bridges are classed into seven accuracy classes: 0.02, 0.05, 0.1, 0.2, 0.5, 1.5 and 5.

Bridges can be balanced either (1) by adjusting one of the arms while the ratio of the remaining two arms is kept constant; or (2) by adjusting the ratio of two arms while keeping the third arm unchanged. Those employing the former method of balancing are termed box (or constant ratio) bridges, and those employing the latter method are referred to as slide-wire (or variable ratio) bridges.

As the name implies, slide-wire bridges use a slide-wire of uniform cross section whose resistance per unit length is therefore constant (hence the name' calibrated wire'), made from a resistance alloy, with a sliding contact moving on it. Usually, the slide-wire is wound into a spiral and comprises two ratio arms of a four-arm bridge, while the third arm is a plug-type resistance box.

One such bridge is the Type MMB miniature multirange bridge shown in Fig. 5-16. The galvanometer and battery are built into the bridge. The bridge has five ranges extending from 0.05 to 50,000 ohms. The accuracy (in the range from 2 tó 2,000 ohms) is within 5 per cent. Over the broader range, the accuracy is 15 per cent.

Box bridges use a high-grade resistance box as a standard and two fixed resistors as the ratio arms whose ratio is precisely known and remains unchanged during measurement. The bridge is balanced by adjusting the standard. The range of a bridge can be extended by supplying it with several ratio-arm resistors so that any desired arm ratio can be obtained. The arm ratio can be changed in decimal steps, and the arm ratios themselves are proportional to 10^n and serve as decimal multipliers of the standard. The standard is usually a dial-type box, while the ratio-arm resistances can be selected by a plug switch in high-grade bridges or by a dial switch in less precise designs.

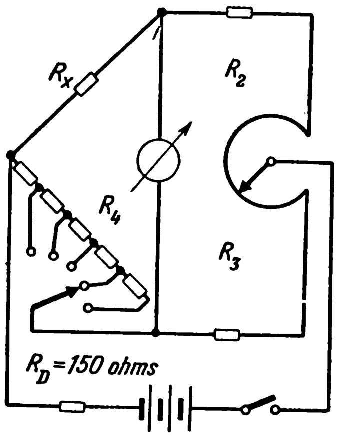


Fig. 5-16

Figure 5-17 shows diagrammatically the Type MBУ-49 box bridge. The decade box of the standard arm has four dials containing ten coils each of 1, 10 and 100 ohms and nine coils each of 1,000 ohms. The ratio-arm resistances are selected also by a dial switch. The bridge has a built-in galvanometer and a 4.5-V battery. It can measure resistances from 0.001 to 1,000,000 ohms. Its accuracy is 0.2 per cent within the range 10-100,000 ohms. It can also be used for the location of faults in overhead and cable power lines.

In modern Kelvin double bridges, the arms R_4 and R_4' (Fig. 5-5) are decade boxes with ganged dial switches (rotation of the dials simultaneously adjusts both arms. The arms R_3 and R_3' are adjustable in steps by means of plugs. The standard resistance R_N is made external to the bridge and is usually supplied as an optional item.

In most cases, Kelvin double bridges are fitted with commutators with which they can be reduced to a simple Wheatstone bridge, whenever necessary. In the latter case, one half of the twin resistance R_4/R_4' (Fig. 5-5) works as a standard and the other half is inoperative, while R_3 and R_3' are used as the ratio arms.

This "combination" principle is embodied in the Type MTB bridge diagrammatically shown in Fig. 5-18. It uses two five-dial decade boxes as the arms R_4 and R_4', the dials containing nine decades each of the 1,000, 100, 10, 1 and 0.1 ohm resistance, and two plug-type decade boxes as the arms R_3 and R'_3, consisting of decimal multiples 10, 100, 1,000 and 10,000 ohms. It also uses an external galvanometer (usually, of the reflecting type), a source of e.m.f. and a standard resistance coil for double-bridge work.

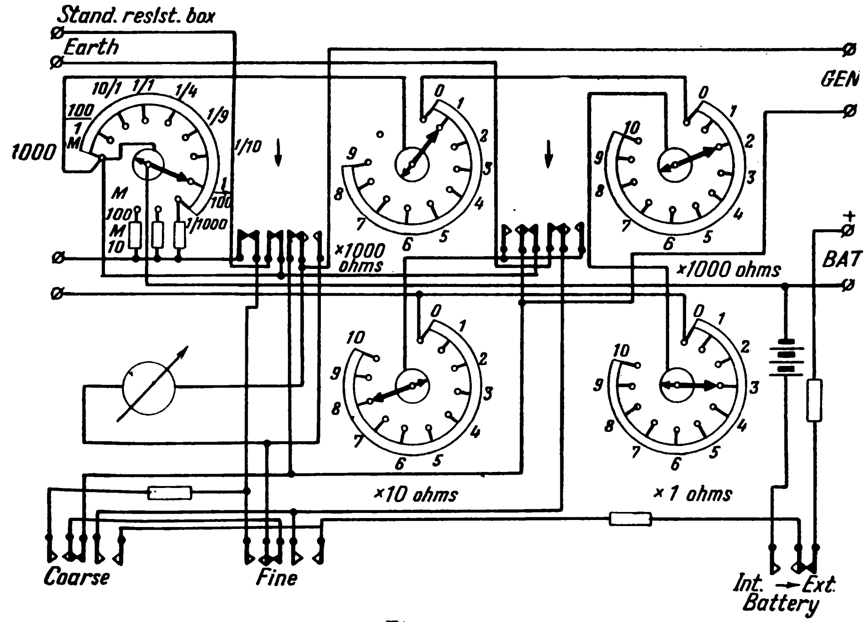


Fig. 5-17

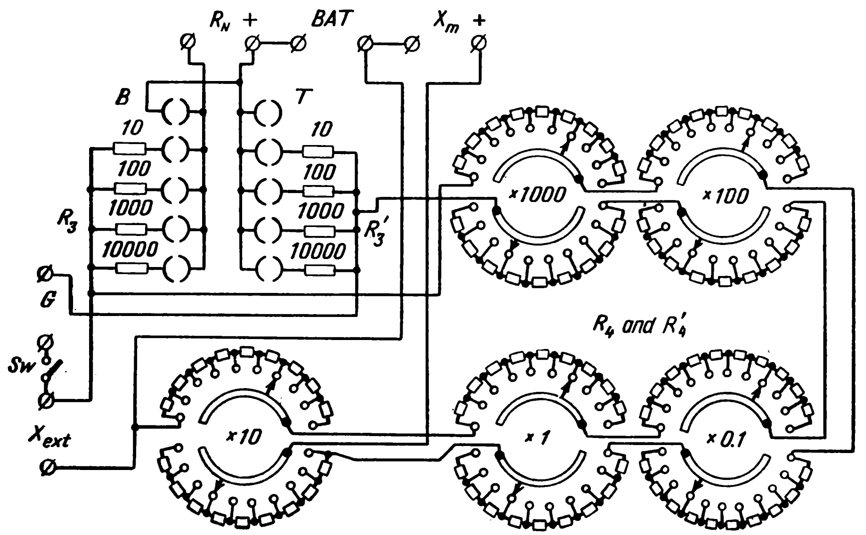


Fig. 5-18

A change-over from the Wheatstone to the Kelvin arrangement is effected by repositioning the plug from the jack B into the jack T. The factory-guaranteed range is from 10^{-5} to 10^5 ohms. The bridge is of accuracy class 0.05. For greater sensitivity, the maximum possible current should be passed through the unknown and the standard, and a check upon it maintained by means of an auxiliary ammeter.

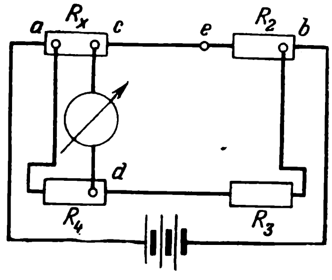


Fig. 5-19

In fact, a four-arm bridge can also measure very low resistances, provided it has four terminals for connection of the unknown resistance. Diagrammatically, such a bridge is shown in Fig. 5-19. The terminals a, b, c and d are the junctions of the bridge; a and c are the potential terminals of the unknown resistance. As follows from the circuit of Fig. 5-19, the resistances of the leads, contacts and branches between the potential and current terminals are included in the source circuit (to the left of the point a) and in the arm R_2 (to the right of the point c). If the resistance between c and e is made very low in comparison with R_2, which is easy to make when R_2 is not less than 1 ohm, the error due to the resistances of the leads and contacts will be negligibly small. Therefore, the bridge of Fig. 5-19 can in principle measure very low resistances, nearly of the same order of magnitude as measured with the Kelvin double bridge. However, R_2 cannot be very low (less than 1 ohm), and the current through the branch R_xR_2 has to be limited in order to avoid overloading the arm R_2. As a result, the bridge delivers very little power into the detector, and, other things being equal, the sensitivity of the bridge proves considerably lower in comparison with a Kelvin double bridge, a factor which necessitates the use of a detector with a greater voltage sensitivity.

The arrangement of Fig. 5-19 is embodied in the Type P-316 bridge intended for the measurement of resistances from 10^{-5} to 20 ohms (and from 20 to 10^6 ohms when arranged as a conventional four-arm bridge), accurate to within ± 0.2 per cent in the range 0.01-10,000 ohms and to within 0.5 per cent in the ranges 10^{-5}-10^{-2} and 10^4-10^6 ohms. The detector in the P-316 bridge is a highly sensitive thermo-e.m.f. galvanometer of special design.

Chapter Six

ALTERNATING-CURRENT BRIDGE MEASUREMENTS

6-1. Properties and Classification of A. C. Bridges

The bridge networks discussed in Chapter 5 were assumed to contain only resistances, be powered by d.c. sources and use a moving-coil galvanometer as the null detector. The overall picture will not change, however, if a bridge is made up of complex impedances, is powered by alternating current and uses a suitable a.c. detector (vibration galvanometer, telephones or a valve circuit) instead of a galvanometer. What we then have is commonly known as the *a.c. bridge*.

The general relationships derived for d.c. bridges remain on the whole valid for a.c. bridges as well. But there are also some specific differences, three of which have to be examined in greater detail.

In a.c. measurements, use is made not only of the conventional four-arm resistive bridge, but also of the more complicated seven- and six-arm (double) bridges and bridges containing inductances. Yet, all of these arrangements are in fact modifications of the four-arm bridge and formally can be reduced to a simple Wheatstone bridge through mesh-star and star-mesh conversions. In short, the four-arm bridge is the basic arrangement. From examination of the simplest Wheatstone bridge consisting of four resistances, one can see that this arrangement cannot be changed qualitatively. The resistances of the arms can only vary in magnitude, while they remain unchanged in character. An entirely different situation arises in the case of an a.c. bridge consisting of four complex impedances. Obviously, these impedances can be arranged into a variety of networks in which the resistances, capacitances and inductances can be connected in series or parallel or both. With four arms in a bridge and with each arm permitting connections differing from those of

the remaining three, an extremely great number of arrangements can be obtained from the basic a.c. bridge. To date, several dozen forms of a.c. bridges have been described, some of them being very complicated.

This multiplicity of network arrangements is the first distinctive feature of a.c. bridges and makes their classification and systematization quite a problem.

Another special feature of a.c. bridges is that they have to be balanced both for the magnitude and phase of the voltage drops across the arms. This calls for the inclusion of at least two adjustable elements in a network.

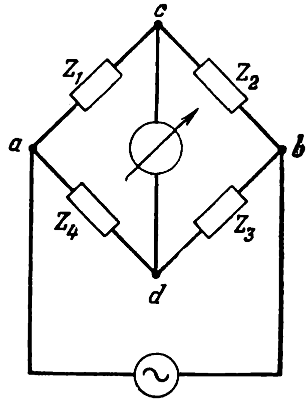


Fig. 6-1

The general form of four-arm a.c. bridge is shown in Fig. 6-1. Being structurally identical with the d.c. bridge, the fundamental constants (input and output impedances, detector current, etc.) can obviously be obtained by the same methods and given the same form (with R being replaced by Z). Therefore, the general balance equation of a general four-arm a.c. bridge will be:

$$Z_1Z_3 = Z_2Z_4. \tag{6-1}$$

In a general case, the arms of an a.c. bridge will contain capacitances, inductances and resistances.

In the complex notation:

$$Z_1 = R_1 + jX_1;\ Z_2 = R_2 + jX_2;\ Z_3 = R_3 + iX_3;\ Z_4 = R_4 + jX_4.$$

Substituting the above expressions in Eq. (6-1),

$$(R_1 + jX_1)(R_3 + jX_3) = (R_2 + jX_2)(R_4 + iX_4).$$

Rewriting, we have

$$(R_1R_3 - X_1X_3) + j(R_1X_3 + R_3X_1) = (R_2R_4 - X_2X_4) + \\ + j(R_2X_4 + R_4X_2).$$

Equating the real and imaginary terms separately, we finally obtain:

$$R_1R_3 - X_1X_3 = R_2R_4 - X_2X_4;$$
$$X_1R_3 + X_3R_1 = X_2R_4 + X_4R_2.$$

Thus, the balance condition is now given by a system of two equations which must be satisfied simultaneously. Naturally, this calls for adjustment of at least two parameters of the network. This

is a very important feature of a.c. bridges. While in a d.c. bridge the balance condition is given by a single equation and only one unknown can be determined by adjusting also one quantity, the balance condition for a.c. bridges is given by a system of two equations and two unknown quantities can be determined.

This seemingly fundamental difference is rather formal. In effect, the two unknown quantities are the resistive and reactive components of one and the same complex impedance being measured. Yet, this difference is sufficiently material to necessitate two simultaneous adjustments in the network (usually, resistive and reactive).

The physical basis of the two balance equations will become clearer, if we change the general balance equation by conversion of impedances to the polar form. In the polar form:

$$Z = |Z| e^{j\varphi},$$

where $|Z|$ represents the magnitude of a complex impedance, and φ its angle.

Since the magnitudes are multiplied and angles added in multiplication of complex numbers, then Eq. (6-1) becomes:

$$|Z_1 Z_3| e^{j(\varphi_1 + \varphi_3)} = |Z_2 Z_4| e^{j(\varphi_2 + \varphi_4)}.$$

To satisfy this relationship, it is essential that

$$\left.\begin{aligned} |Z_1 Z_3| &= |Z_2 Z_4|; \\ \varphi_1 + \varphi_3 &= \varphi_2 + \varphi_4. \end{aligned}\right\} \qquad (6\text{-}2)$$

As can be seen, Eq. (6-2) mathematically expresses the two conditions for the balance (and adjustment) of an a.c. bridge for both magnitude and phase.

As a mnemonic device, it must be remembered that the product of the magnitudes and the sum of the phase angles of one pair of opposite arms must be respectively equal to those of the other opposite pair.

Finally, the third feature. It stems directly from the equality of the sums of the phase angles. In a d.c. bridge, the balance solely depends on the quantitative relationship between the arms and can always be obtained, unless there are some technical limitations to adjustment of the ratio arms. In an a.c. bridge, on the other hand, no quantitative relationship between the parameters will result in balance, unless the sums of the phase angles are equal. Hence, for an a.c. bridge it is important which arm contains what: a resistance, inductance, or capacitance. For example, when a capacitance is compared with an inductance, they should be placed in opposite arms; when a capacitance is compared with a capacitance, they should be placed in adjacent arms.

Now let us turn to the classification of a.c. bridges. It is important both in their study and practical selection because of the great multitude of types available. Unfortunately, there is no generally accepted system, and we shall only discuss one of the possible approaches.

The objective has been the simplicity of both principle and presentation. For this reason, of all the bridges, only the fundamental four-arm types have been chosen, leaving out resonance bridges, networks containing mutual inductances, and multiple-arm bridges in explicit form.

Taking into account some additional restrictions arising from the nature of bridge networks and experimental technique, we may reduce all fundamental four-arm bridges to a single system easy to present in tabular form.

To begin with, let us determine what restrictions should and may be imposed and what features can be used for classification purposes. For this purpose, we shall examine:

(1) general principles of a.c. bridges and types of arms;

(2) disposition of the arms in a bridge;

(3) types of the standard arm (types of equivalent circuits).

In most cases, the fundamental four-arm a.c. bridge serves to measure the resistive and reactive components of an unknown complex impedance placed in one of the arms. Since the balance of the bridge depends on the phase relationships, the comparison arm should of necessity be also a complex impedance. Thus, two of the four arms, namely, the unknown and the standard, are complex impedances. The remaining two ratio arms serve an auxiliary purpose of providing the necessary difference currents and voltages. In principle, they may be resistances, inductances, capacitances, or complex impedances.

For proper guidance in selecting the actual form of ratio arms, it seems worthwhile to discuss, even though in brief, what may be called as "independent measurement" *.

By "independent measurement" we mean the following. Suppose a bridge is so constructed that each of the two variable quantities affects each of the two balance conditions separately, i. e., none of them enters both equations simultaneously. Then the resistive and reactive components of the unknown impedance must also enter these equations separately. Obviously, solution of these balance equations, one for the resistive and the other for the reactive component, will give the unknown quantities, each in terms of only one respective adjustable quantity.

* For a more detailed discussion, see Sec. 6-5.

Then, we shall know the effect of each of the adjustable quantities in advance and estimate, to a certain degree of accuracy, the amount of adjustment it requires to secure a balance. What is more important, however, is that we can then calibrate the scales of the respective elements in their units directly, and the results can be taken from the two scales, one giving the resistive and the other the reactive component of the unknown impedance. This is independent measurement. The practical advantages of the technique are obvious.

For independent measurement to be possible, however, the bridge must meet certain requirements. They can be easily determined from examination of the general balance equation. Numbering the arms in a bridge clockwise and assuming that the unknown impedance is in the first arm, we obtain:

$$Z_1 = R_1 + jX_1 = Z_2Z_4/Z_3 = (R_2 + jX_2)(R_4 + jX_4)/(R_3 + jX_3). \quad (6\text{-}3)$$

In a general form, Eq. (6-3) may be rewritten thus:

$$R_1 + jX_1 = A + jB. \quad (6\text{-}4)$$

Then for independent measurement, one of the component quantities of the standard must be included with A and the other with B. Assume that the standard is adjustable and placed in one arm, Z_4, as is often the case in practice.* Then Z_2 and Z_3 are fixed ratio arms. In a general case, since Z_2 and Z_3 are complex impedances, their ratio will also be a complex number:

$$Z_2/Z_3 = \alpha + j\beta. \quad (6\text{-}5)$$

Substituting Eq. (6-5) in Eq. (6-3), we get:

$$R_1 + jX_1 = (\alpha + j\beta)(R_4 + jX_4) = (\alpha R_4 - \beta X_4) + j(\alpha X_4 + \beta R_4). \quad (6\text{-}6)$$

Comparing Equations (6-4) and (6-6), we note

$$\left.\begin{aligned} A &= \alpha R_4 + \beta X_4; \\ B &= \alpha X_4 + \beta R_4. \end{aligned}\right\} \quad (6\text{-}7)$$

Apparently, in this case A and B simultaneously depend on both adjustments, R_4 and X_4. Then a change, say, in R_4 (or X_4) will produce a simultaneous change in A and B, and independent measurement will be unfeasible.

As follows from Eq. (6-7), however, we can make R_4 and X_4 independent of each other by making $\alpha = 0$ or $\beta = 0$. In the former case, a change in R_4 will only affect B and a change in X_4 will only affect A, and vice versa. Then independent measurement will be possible.

* It should be noted that the placement of both adjustments in one arm, although advantageous in certain respects, is not the only alternative possible (see Sec. 6-5).

The condition $\alpha=0$ gives an imaginary ratio Z_2/Z_3, and the condition $\beta=0$, a real ratio. Thus, for independent measurement to be possible, it is essential that the fixed ratio arms be such that their ratio is either purely real ($\varphi_2 - \varphi_3=0$), or purely imaginary ($\varphi_2 - \varphi_3=\pm 90°$), but not complex.

Naturally, similar results will be obtained, if we place the adjustable standard in the arm Z_2 instead of Z_4. Indeed, the result will be the same, if we place the standard in the arm Z_3 which is in the denominator. But then, the product Z_2Z_4, and not the arm ratio, must be real or imaginary (i.e., $\varphi_2+\varphi_4=0$ or $\varphi_2+\varphi_4=\pm 90°$).

To sum up, for independent measurement it is essential that the sum or difference of the phase angles of the ratio arms be either equal to zero or to $\pm 90°$. The simplest way to obtain this is either to make the ratio arms purely resistive or reactive (which will give real arm ratios and products), or to make one arm resistive and the other reactive (which will give imaginary arm ratios and products). That is how it is done in practice.

Reactances are usually made in the form of capacitors with a very small loss angle. Inductive reactances having, like good capacitors, a negligibly small resistive component or free from it altogether, are practically unfeasible. If, for some reason, inductive ratio arms ought to be used, the resultant bridge will have complex impedances in all of the four arms, all the same. This is the only exception met with in practice. Ordinarily, inductive arms are placed together, say in Z_2 and Z_3, and are made equal both in magnitude and phase. Then their ratio will be a real one ($\varphi_2 - \varphi_3=0$), and the condition for independent measurement will be thus satisfied.

In the subsequent discussion, the bridge networks will be assumed to have two main arms containing complex impedances, and two ratio (or auxiliary) arms, with either resistances in both, or capacitances in both, or a capacitance in one and a resistance in the other, or inductances (impedances) in both.

The next point to be taken up is the disposition of.the main and auxiliary arms in a bridge. Since the arms are divided into pairs (two main and two auxiliary), the main arms can only be connected in two ways: in adjacent arms with a common point at a bridge junction, or in opposite arms. For a balanced bridge, where the balance condition is symmetrical, it is theoretically immaterial which arms contain what.

Both arrangements are equally feasible in principle, the adjacent arrangement being more common. The disposition of the main arms imposes certain restrictions upon their phase relationships (i. e., on the phase angle and, as a result, the type of the standard arm). These restrictions are especially characteristic when the ratio arms are pure resistances with phase shifts equal to zero. From the

balance equation for an a. c. bridge it follows that the sums of the phase angles of opposite arms (or the differences of the phase angles of adjacent arms) should be equal. If the sum or difference of the phase angles of the ratio arms is equal to zero, the same must be true of the balance-determining arms. For example, when two capacitances are compared, they should be placed in adjacent arms; when a capacitance is compared with an inductance, they should be placed in opposite arms.

As to the disposition of the ratio arms, independent measurement requires that they be either purely resistive or purely reactive. This implies that for them the sum or difference of the phase angles may be equal either to zero or to $\pm 90°$. This has led us to the classification of bridge into product-arm bridges with opposite ratio arms (Fig. 6-2*a*) and into ratio-arm bridges with adjacent ratio arms (Fig. 6-2*b*).

To sum up, the main arms may be connected either as adjacent or as opposite arms. For the ratio arms, this results in the six possible arrangements shown in Fig. 6-2.

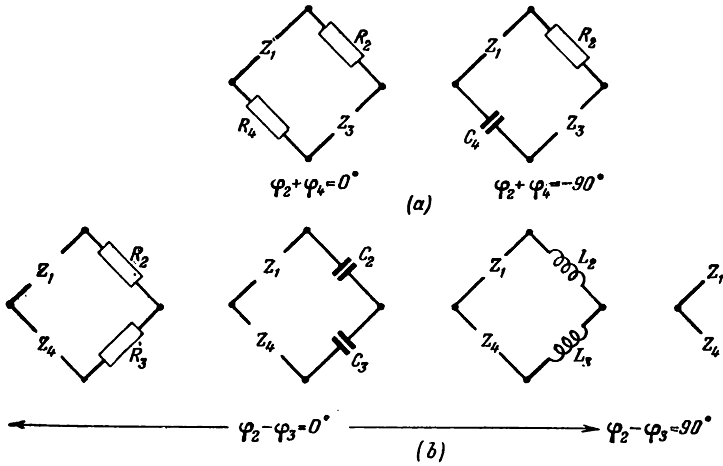


Fig. 6-2

Finally, there is the type and construction of the standard arm to be examined. In effect, the standard arm is an adjustable equivalent circuit reproducing the absolute value and phase of the unknown to a certain scale. Usually, the standard arm uses standard resistors, capacitors or inductors. Naturally, quite a number of combinations is possible here, and certain restrictions of fundamental and practical nature have to be imposed.

For one thing, inductances should be excluded, since, as noted already, a pure inductance is impossible to construct. Next to be excluded are the rarely used parallel combinations of a capacitance and an inductance, of an inductance and a resistance, and their series-parallel combinations. This leaves us with the simplest parallel and series combinations which can be reduced to five equivalent circuits most commonly used in practice. Noting that one and the

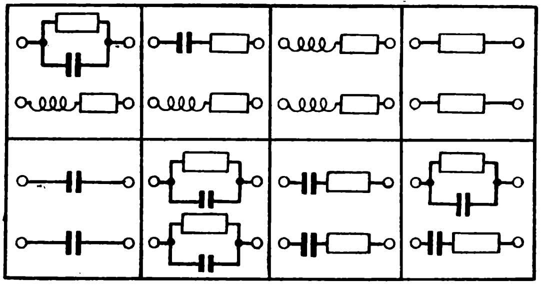


Fig. 6-3

same equivalent circuit can be employed for the comparison of different unknown quantities, we obtain the eight most common combinations of the unknown and a standard shown in Fig. 6-3. All of these combinations have been chosen from considerations of the restrictions imposed by the phase relationship. We have omitted the arrangements which do not permit balancing at all, with any combination of the ratio arms (Fig. 6-2).

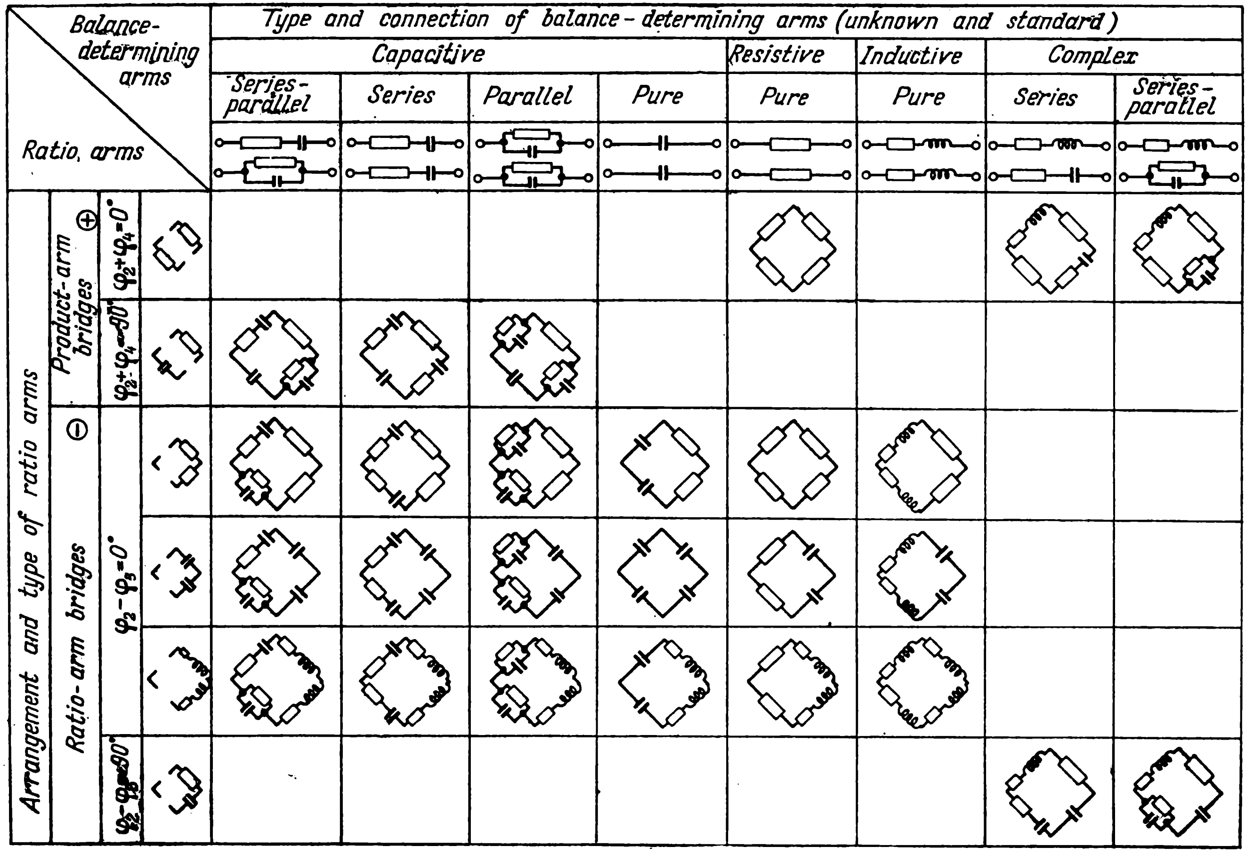
So, from the view-point of the standard arm, a. c. bridges can be divided into eight groups (Fig. 6-3).

From the foregoing, we may use the following features of bridges for the purpose of classification:

(1) type of the standard arm;
(2) connection of the standard arm;
(3) disposition of the arms in a bridge;
(4) type of the ratio arms.

It is obvious that all the four features are interrelated and affect one another. As soon as we have chosen the standard arm, for example, there remain few alternatives for the disposition of the arms in the bridge. The disposition of the arms, in turn, determines the phase angle that the equivalent circuit of the standard arm may have. Although the four features may follow in any sequence, the one given above seems substantiated by the usual procedure of bridge network calculation: with the unknown quantity given, the experimentor selects a suitable standard, and the latter governs the other constants of the network.

Table 6-1


Balance-determining arms
Ratio, arms
Type and connection of balance-determining arms (unknown and standard)
Capacitive
Resistive
Inductive
Complex
Series-parallel
Series
Parallel
Pure
Pure
Pure
Series
Series-parallel
Arrangement and type of ratio arms
Product-arm bridges ⊕
Ratio-arm bridges ⊖
$\varphi_2+\varphi_4=0°$
$\varphi_2+\varphi_4=-90°$
$\varphi_2-\varphi_3=0°$
$\varphi_2-\varphi_3=90°$

On the basis of the above features, a complete classification, like the one given in Table 6-1, can be conveniently worked out.

The eight possible combinations of the balance-determining arms (Fig. 6-3) are arranged vertically, and the six combinations of the ratio arms (Fig. 6-2) horizontally. The intersection of a vertical and a horizontal row gives the respective bridge network. Theoretically, $8\times6=48$ circuits can be synthesized; however, only 26 of them can be balanced in principle. They are shown in Table 6-1. The remaining 22 vacant places should have been occupied by the circuits which cannot be balanced altogether because of mismatch in phase relationships. Such circuits are often employed as phase-shifting or phase-inverting networks. There also exist intermediate arrangements. They can all of them be also presented in tabular form.

For all the restrictions, however, Table 6-1 contains all of the most commonly used bridge circuits, even a d. c. Wheatstone bridge.

In conclusion, a few words should be added about the symbolic notation for bridges. With a classification chart on hand, it would seem practicable to use the chess type of notation (letters and numerals). Although simple, this method would, however, require a chart to be always had. Instead, several formal methods are used for the purpose. The simplest of them presents the elements of a bridge written in the form of a determinant:

$$\begin{vmatrix} Z_1; & Z_2 \\ Z_4; & Z_3 \end{vmatrix}$$

The actual values of the bridge components are substituted in accordance with their disposition in the circuit. Such a "determinant" equated to zero gives the balance condition for the bridge.

For pure arms, the notation is fairly simple. Where complex circuits are involved, it will be a good plan to use the symbols of the respective circuit components instead of the actual expressions. For a series combination of two components (say, a resistor and a capacitor), one may write RC or R-C; for a parallel combination, $R : C$ or $R||C$. This technique may prove useful where combinations of two or three components are involved.

6-2. The Sensitivity of A. C. Bridges

Generally, what has been said about the sensitivity of d. c. bridges, including its calculation, might be extended to cover a. c. bridges as well. However, the resultant formulas would be too unwieldy for practical use.

Fortunately enough, a.c. bridges permit of a simpler approach to the problem for the reasons intrinsic in their nature. As a rule, existing a. c. bridges use amplifiers connected between the bridge output and the presentation instrument. The input impedance of

valve amplifiers can be made sufficiently high for it to be taken equal to infinity and for the bridge to be regarded as operating at no-load.

Yet, sometimes it may prove necessary to determine the conditions under which maximum current can be obtained in the detector. The evaluation of sensitivity in terms of power for a. c. bridges has not yet found any appreciable use, since power is not so easy to determine in an a. c. circuit; nor are the impedances in a bridge easy to match. All in all, the trouble of doing such evaluation may well offset its value.

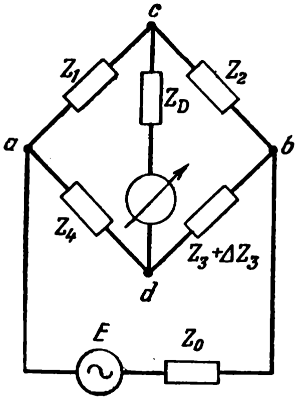


Fig. 6-4

For the above reasons, we shall first give a brief outline of the analysis of current sensitivity in a slightly modified form characteristic of a.c. bridges and then discuss in detail the technique for the calculation of the maximum output voltage of an unloaded bridge. It should be qualified from the outset that the discussion will generally apply only to four-arm bridges containing linear impedances not coupled either inductively or capacitively.

Two forms of sensitivity will be distinguished in a.c. bridges: absolute and relative. By absolute current sensitivity $\dot{S}_I$ and absolute voltage sensitivity $\dot{S}_V$ shall be meant the limits of the ratios:

$$\dot{S}_I = \lim_{|\Delta Z| \to 0} \Delta \dot{I}_{cd}/\Delta Z; \quad \dot{S}_V = \lim_{|\Delta Z| \to 0} \Delta \dot{V}_{cd}/\Delta Z,$$

where $\Delta \dot{I}_{cd}$=vector of change in detector current;

$\Delta \dot{V}_{cd}$=vector of change in voltage across detector circuit *.

Relative sensitivities are given by

$$\dot{S}_I^0 = \lim_{|\Delta Z| \to 0} \Delta I_{cd}/(\Delta \dot{Z}/Z) = \dot{S}_I Z;$$

$$\dot{S}_V^0 = \lim_{|\Delta Z| \to 0} \Delta \dot{V}_{cd}/(\Delta Z/Z) = \dot{S}_V Z.$$

Now we shall determine these sensitivities.

Consider an a. c. bridge (Fig. 6-4) thrown out of balance by a change in the impedance of an arm. Any arm may be made adjustable; this will not affect the final conclusions in the least. In order to retain the notation adopted in the classification of bridges, the

* Since for an ordinary bridge network $\dot{I}_{cd}$ and $\dot{V}_{cd}$ are analytical functions, the above ratios tend to their only limits $\dot{S}_I$ and $\dot{S}_V$, respectively.

unknown will be regarded as placed in the first arm. Then, for a product-arm bridge, the standard will be in the third arm; for a ratio-arm bridge it will in the fourth arm.

With the detector circuit open, i.e., with Z_D=infinity, determine the output voltage of the bridge for a change, say, ΔZ_3 in the arm Z_3:

$$\Delta \dot{V}_{cd} = \dot{V}_{ac} - \dot{V}_{ad} = \dot{V}_{ab} \left[\frac{Z_1}{Z_1+Z_2} - \frac{Z_4}{(Z_3+\Delta Z_3)+Z_4}\right] =$$
$$= \dot{V}_{ab} \frac{Z_1(Z_3+\Delta Z_3) - Z_2 Z_4}{(Z_1+Z_2)[(Z_3+\Delta Z_3)+Z_4]}.$$

Since the balance of the bridge has been upset only by ΔZ_3, the remaining impedances must satisfy the balance condition

$$Z_1 Z_3 = Z_2 Z_4,$$

whence

$$\Delta \dot{V}_{cd} = \dot{V}_{ab} \frac{Z_1 \Delta Z_3}{(Z_1+Z_2)[(Z_3+\Delta Z_3)+Z_4]}.$$

Since $\Delta Z_3 \ll Z_3$, the change ΔZ_3 in the denominator may be neglected as a term included in the sum. Then we get:

$$\Delta \dot{V}_{cd} = \dot{V}_{ab} \frac{Z_1 \Delta Z_3}{(Z_1+Z_2)(Z_3+Z_4)}. \tag{6-8}$$

From Helmholtz's theorem, the change in the detector current is given by

$$\Delta \dot{I}_{cd} = \frac{\Delta \dot{V}_{cd}}{Z_D + Z_{cd}} = \frac{\dot{V}_{ab} Z_1 \Delta Z_3}{(Z_1+Z_2)(Z_3+Z_4)(1+Z_D/Z_{cd})Z_{cd}}, \tag{6-9}$$

where Z_{cd} is the output impedance of the bridge, i.e., the impedance at the detector terminals. It can be determined, assuming that the source is shunted, i.e., $Z_0=0$. Although the output impedance of any generator is fairly high, this assumption holds for a. c. bridges for the reason that near balance the current in, say, the generator circuit has but a negligible effect on the current in the detector circuit, and vice versa (see Chapter 5), and any value may be given to this impedance. Then, when $Z_1 Z_3 = Z_2 Z_4$, the output impedance of an a.c. bridge is given by

$$Z_{cd} = \frac{Z_1 Z_2}{(Z_1+Z_2)} + \frac{Z_3 Z_4}{(Z_3+Z_4)} = \frac{Z_1 Z_3 (Z_1+Z_2+Z_3+Z_4)}{(Z_1+Z_2)(Z_3+Z_4)}.$$

Substituting the above expression in Eq. (6-9) and eliminating the redundant terms, we obtain:

$$\Delta \dot{I}_{cd} = \dot{V}_{ab} \delta Z_3 \frac{1}{1+Z_D/Z_{cd}} \times \frac{1}{Z_1+Z_2+Z_3+Z_4}, \tag{6-10}$$

where $\delta Z_3 = \Delta Z_3 / Z_3$ is the fractional change in the adjustable impedance.

In Eq. (6-10), the input voltage $\dot{V}_{ab}$ of the bridge depends on the internal impedance Z_0 of the source and is given by

$$\dot{V}_{ab}=\dot{E}\frac{1}{1+Z_0/Z_{ab}},$$

where Z_{ab} is the input impedance of the bridge.

Equation (6-10) relates the detector current to the constants of the bridge and makes it possible to determine its sensitivity in terms of each. This equation, however, does not suggest how the various constants of the bridge should be varied in order to obtain maximum sensitivity. On the other hand a general analysis of the equation is out of the question because it involves expressing each impedance by R and X $(Z=R+jX)$. Therefore, using Eq. (6-10) in case of need, we shall now examine the particular case of bridge operation when $Z_D=$infinity. Noting that at balance

$$Z_1/Z_2=Z_4/Z_3,$$

we divide the numerator and the denominator of Eq. (6-8) by Z_2 and Z_3 and obtain:

$$\Delta\dot{V}_{cd}=\dot{V}_{ab}\frac{(\Delta Z_3/Z_3)(Z_1/Z_2)}{(Z_1/Z_2+1)(1+Z_4/Z_3)}=\dot{V}_{ab}\delta Z_3\frac{A}{(1+A)^2}, \qquad (6\text{-}11)$$

where $A=Z_1/Z_2=Z_4/Z_3$ is the arm ratio.

From Eq. (6-10), the current sensitivities will be

$$\dot{S}_I=\dot{V}_{ab}\frac{1}{Z_3(1+Z_D/Z_{cd})(Z_1+Z_2+Z_3+Z_4)};$$

$$\dot{S}_I^0=\dot{V}_{ab}\frac{1}{(1+Z_D/Z_{cd})(Z_1+Z_2+Z_3+Z_4)}.$$

From Eq. (6-11), absolute and relative voltage sensitivity, when $Z_D=$infinity, is

$$\dot{S}_V=\dot{V}_{ab}\frac{A}{Z_3(1+A)^2};$$

$$\dot{S}_V^0=\dot{V}_{ab}\frac{A}{(1+A)^2}.$$

From the way the current sensitivities are derived and from their symmetry it follows that relative sensitivities are independent of which of the arms is adjustable. Therefore, any arm, other than Z_3 may be considered such.

Because of the wide use of valve amplifiers in conjunction with a. c. bridges, cases are most common where $Z_D=\infty$ very nearly. Such cases deserve a more detailed examination. The most interesting is relative voltage sensitivity, S_V^0, since it is a very convenient index with which to express the fractional error. Since

$$\dot{S}_V^0=\dot{V}_{ab}\frac{A}{(1+A)^2},$$

the unbalance voltage $\Delta\dot{V}_{cd}$ will be

$$\Delta V_{cd} = \dot{S}_V^0 \delta Z = \delta Z k \dot{V}_{ab}, \qquad (6\text{-}12)$$

where δZ is the relative change in the resistance of an arm; $\delta Z = \Delta Z/Z$; and $k = A/(1+A)^2$.

Thus, the unbalance voltage appearing across the terminals of the open detector branch ($Z_D = \infty$) is equal to the product of three independent variables. The first variable is the relative change in the standard and may be termed the *unbalance factor.* The second variable is a function of the arm ratio and is termed the *ratio* (or *network*) *factor.* The third variable is the supply voltage of the bridge.

Consider the effect of each of the three variables on the sensitivity of bridges.

The Unbalance Factor. In a general case, any arm may be thought of as being adjustable, including the unknown impedance. The unbalance factor of the arm containing the unknown has special importance in determining the error introduced by the threshold sensitivity of the null detector.

In determining the unbalance factor, impedance can be presented in different forms, depending on the nature of the unknown and the purpose of the measurement. The most commonly used forms are:

$$Z = R + jX = X\ (\tan\ \delta + j) = X(1/Q + j) = |Z|\ e^{j\varphi}$$

where δ is the loss angle ($\delta = \pi/2 - \varphi$);

$$\varphi = \tan^{-1} X/R = \tan^{-1} Q;$$

$$Q = X/R \text{ (quality factor).}$$

Determine the unbalance factor δZ for each of the above notations. Let $Z = R + jX$ (the form used in the measurement of the resistive and reactive components). For a change ΔR in the resistive component of an impedance, the unbalance factor δZ_R will be

$$\delta Z_R = \Delta Z_R/Z = \Delta R/Z = \frac{\Delta R}{|Z|} e^{-j\varphi}, \qquad (6\text{-}13)$$

and its magnitude

$$|\delta Z_R| = \Delta R/|Z|. \qquad (6\text{-}14)$$

For a change ΔX in the reactive component we have:

$$\delta Z_x = \Delta Z_x/Z = j(\Delta X/Z) = j\frac{\Delta X}{|Z|} e^{-j\varphi} \qquad (6\text{-}15)$$

and its magnitude

$$|\delta Z_x| = \Delta X/|Z|. \qquad (6\text{-}16)$$

Substituting the unbalance factor as given by Equations (6-13) through (6-16), in Eq. (6-12) gives the change $\Delta\dot{V}_{cd}$ in the output

voltage of the bridge for changes in the various components of the impedance. Thus, the magnitude of $\Delta\dot{V}_{cd}$ can be obtained from Equations (6-14) and (6-16):

$$\left.\begin{aligned} \Delta V_{cd} &= \frac{\Delta R}{|Z|}|k|V_{ab} \\ &\text{and} \\ \Delta V_{cd} &= \frac{\Delta X}{|Z|}|k|V_{ab}. \end{aligned}\right\} \tag{6-17}$$

From Eq. (6-17) it follows that the unbalance voltage is directly proportional to the change in the resistive (or reactive) component of the impedance and inversely proportional to the magnitude of the impedance Z.

When $Z=X$ (tan $\delta+j$) (this form is mostly used to represent impedance in capacitance measurements), the unbalance factor will be given by:

$$\delta Z_{\tan\delta}=\frac{\Delta Z_{\tan\delta}}{Z}=\frac{X\Delta\tan\delta}{Z}=\frac{\Delta\tan\delta}{\sqrt{1+\tan^2\delta}}e^{-j\varphi};$$

$$|\delta Z_{\tan\delta}|=\frac{\Delta\tan\delta}{\sqrt{1+\tan^2\delta}};$$

$$\delta Z_x=\frac{\Delta Z_x}{Z}=\frac{\Delta X}{X}. \tag{6-18}$$

When tan $\delta\leqslant 1$, we have

$$\delta Z_{\tan\delta}\cong\Delta\tan\delta e^{-j\varphi};$$

$$|\delta Z_{\tan\delta}|\cong\Delta\tan\delta. \tag{6-19}$$

Substituting Equations (6-18) and (6-19) in Eq. (6-12) we find that the change in the output voltage for a change in the reactive component is directly proportional to the fractional change $\Delta X/X$:

$$\Delta V_{cd}=\frac{\Delta X}{X}|k|V_{ab}, \tag{6-20}$$

while for a change in tan δ, it is directly proportional to the absolute change Δ tan δ:

$$\Delta V_{cd}=\Delta\tan\delta|k|V_{ab}. \tag{6-21}$$

When $Z=X(1/Q+j)$ (this form may be used in inductance measurements), we have:

$$\delta Z_Q=\frac{\Delta Z_Q}{Z}=\frac{X(-\Delta Q/Q^2)}{Z}=\frac{-\Delta Q/Q^2}{\sqrt{1+1/Q^2}}e^{-j\varphi};$$

$$|\delta Z_Q|=-\frac{\Delta Q}{Q\sqrt{Q^2+1}}; \tag{6-22}$$

$$\delta Z_x=\Delta Z_x|Z=\Delta X|X.$$

When $Q \gg 1$, we have:

$$\delta Z_Q = -\frac{\Delta Q}{Q^2} e^{-j\varphi};$$

$$|\delta Z_Q| = \Delta Q/Q^2. \tag{6-23}$$

Substituting Equations (6-22) and (6-23) in Eq. (6-12), we find that the change in the output voltage of a bridge for a fractional change $\Delta X/X$ in the reactive component of an impedance $Z=X(1/Q+j)$ is directly proportional to

$\Delta X/X$:

$$\Delta V_{cd} = \frac{\Delta X}{X} |k| V_{ab}, \tag{6-24}$$

and for a change in Q it is directly proportional to the ratio $\Delta Q/Q^2$:

$$\Delta V_{cd} = \frac{\Delta Q}{Q^2} |k| V_{ab}. \tag{6-25}$$

We have found expressions for the various forms of representation of the unbalance factor for a change in only one component of the unknown impedance. With two components of an impedance changing simultaneously, the resultant unbalance factor will be the vectorial sum of the individual components of the unbalance factor. For example, when $Z=R+jX$, the unbalance factor δZ for changes ΔR and ΔX will, on the strength of Equations (6-13) and (6-15), be:

$$\delta Z = \delta Z_R + \delta Z_x = \left(\frac{\Delta R}{|Z|} + j\frac{\Delta X}{|Z|}\right) e^{-j\varphi}. \tag{6-26}$$

The Ratio (Network) Factor. It follows from Eq. (6-12) that the ratio factor k is a complex number which can be given the following form:

$$k = \frac{A}{(1+A)^2} = \frac{d+jq}{(1+d+jq)^2} = |k| e^{j\gamma},$$

where

$$A = \frac{|Z_1| e^{j\varphi_1}}{|Z_2| e^{j\varphi_2}} = d + jq = ae^{j\theta}.$$

Also,

$$\theta = \varphi_1 - \varphi_2; \quad a = |Z_1|/|Z_2|.$$

The above relationships suggest that the output voltage and, consequently, the sensitivity of a bridge will be the greater, the greater the *magnitude of the ratio factor.* The magnitude of the ratio factor is given by

$$k = \left|\frac{d+jq}{(1+d+jq)^2}\right| = \frac{|d+jq|}{|(1+d+jq)^2|} = \frac{a}{|(1+d+jq)^2|}.$$

Since the modulus of the square of a complex number is equal to the square of the modulus of the same number, we obtain:

$$|(1+d+jq)^2| = (1+d)^2 + q^2 = 1 + 2d + d^2 + q^2 =$$
$$= 1 + 2a\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta = 1 + 2a\cos\theta + a^2,$$

whence we finally get:

$$k = a/(1 + 2a\cos\theta + a^2). \tag{6-27}$$

It is evident, in the above expression, that the sensitivity of a bridge is a phase-dependent quantity. In the literature on a.c. bridges the discussion of this matter is often limited to the optimum ratio of the arm magnitudes. There are, however, at least two reasons why special attention must be paid to the dependence of the sensitivity on the phase relationship between the arms: first, it distinguishes a.c. bridges from d.c. bridges as far as sensitivity is concerned; second, in practice the choice of phase relationships between the arms is less of a limiting factor than their magnitudes: the sensitivity of a bridge can, for example, be nearly doubled by interchanging the detector and the source for the reason that this changes the phase relationship between the arms.

Now we shall consider the choice of the magnitude and phase for the arm ratio $A = Z_1/Z_2$ giving maximum sensitivity. As follows from Eq. (6-27):

$$k = f(a, \theta).$$

Inspection of this function reveals that:

(1) k attains a maximum (i.e., k becomes infinite) when $a = 1$ or $\theta = \pm\pi$;

(2) k attains a minimum (i.e., k becomes zero) when $a = 0$ or $a = \infty$ [the function $k = f(a)$ is symmetrical with respect to unity], and $\theta = 0$.

Thus, theoretically the magnitude of the ratio factor of a bridge may take any value from zero to infinity. The character of the examined function is well illustrated by Fig. 6-5.

The magnitude of the arm ratio is unity when the arms in each pair have impedances of the same magnitude or:

$$a = \frac{|Z_1|}{|Z_2|} = \frac{|Z_4|}{|Z_3|} = 1 \text{ or } |Z_1| = |Z_2| \text{ and } |Z_3| = |Z_4|,$$

i.e., when the bridge is symmetrical.

A phase angle of 180° for the arm ratio is obtained when the arms of a bridge are made up of capacitances and inductances placed alternately, i.e.:

$$\varphi_1 = \pm 90°;\ \varphi_2 = \mp 90°;\ \varphi_3 = \mp 90°;\ \varphi_4 = \pm 90°;$$
$$\theta = \varphi_1 - \varphi_2 = \varphi_4 - \varphi_3 = \pm 180°.$$

It may thus be summed up that in order to obtain maximum sensitivity, the impedances of the arms making up the branches on either side of the detector must be as nearly equal in magnitude as possible and have the greatest possible phase displacement.

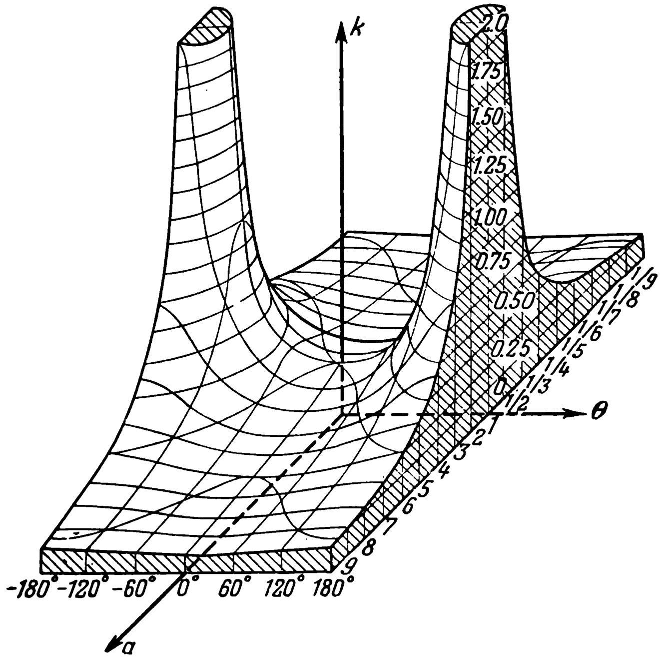


Fig. 6-5

A bridge branch (Fig. 6-6) may consist of two resistances or of two reactances, making an angle equal to zero (at a and b in Fig. 6-6). Or it may consist of a resistance and a reactance, making an angle of $\pm 90°$ (at c in Fig. 6-6). Finally, it may consist of two reactances in anti-phase, making an angle of $\pm 180°$ (at d in Fig. 6-6).

Intermediate combinations are also possible, identifiable with any of the listed above, depending on the phase angle between the arm impedances.

For a bridge symmetrical with respect to the detector circuit ($a=1$), the magnitude of the ratio factor from Eq. (6-27) is:

when $\theta = 0$

$$k = \frac{1}{1 + 2\cos\theta + 1} = 0.25;$$

when $\theta = \pm 90°$,

$$k = 0.5;$$

when $\theta = \pm 180°$;

$$k = \infty.$$

To sum up, by sensitivity, all a.c. bridges can be classed into three types:

Type of bridge	Arm ratio angle
In-phase	0
Quadrature	$\pm\pi/2$
Anti-phase	$\pm\pi$

The first type has the lowest sensitivity and applies best to d.c. bridge networks. The second type is characteristic of ordinary a.c. bridges. Finally, the third type applies to bridges displaying voltage resonance in the branches and having no theoretical limit to sensitivity. Practically, the sensitivity of these bridges is limited by the quality of the reactive components, the allowable load on the bridge elements, and the internal impedance of the source.

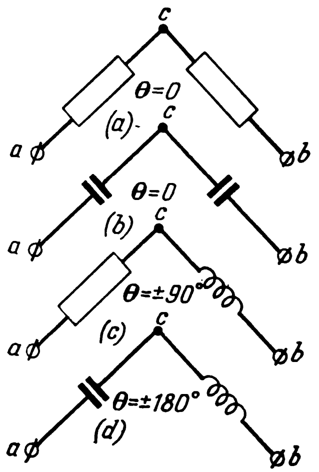


Fig. 6-6

As follows from the foregoing, because of the phase angles between the arm impedances, a.c. bridges can be made more sensitive than d.c. bridges.

For a given unknown quantity, a bridge network is synthesised as follows:

(1) a suitable circuit arrangement and standard arm are selected;

(2) several types of ratio arms are chosen, fulfilling the balance condition and providing for independent balance adjustments;

(3) from the ratio arms thus chosen, the one giving maximum sensitivity is selected.

The calculation of the output voltage of a bridge often calls for knowledge of its phase. Therefore, in addition to the magnitude of the ratio factor, it is essential to know its phase angle. It can be obtained from the expression:

$$\tan \gamma = \frac{(1-a^2)\sin\theta}{2a+(1+a^2)\cos\theta}. \tag{6-28}$$

The magnitude and argument of the ratio factor can also be taken from the charts of Figs. 6-7 and 6-8.

The curves of Fig. 6-7 relate the magnitude of the ratio factor k to the magnitude of the arm ratio a for various phase angles θ in degrees of arc. Since the set of curves $k=f(a)$ is symmetrical with respect to unity, i.e.,

$$f(a)=f(1/a),$$

the curves are only given for $a>1$. If $a<1$, use should be made of the reciprocal $1/a$. For convenience, the values of a are plotted on a logarithmic scale.

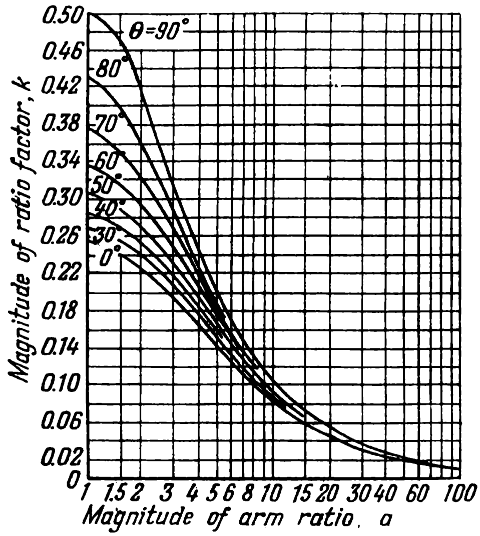


Fig. 6-7

The curves of Fig. 6-8 relate the argument of the ratio factor to the magnitude of the arm ratio $[\gamma=\varphi(a)]$ for the values of the phase angle of the arm ratio, θ, from zero to 90°.

Thus, if the ratio of the arm impedances

$$ae^{j\theta}=(|Z_1|/|Z_2|)e^{j(\varphi_1-\varphi_2)}$$

in a bridge at balance is known, the ratio factor and then the sensitivity can be found from either the charts of Figs. 6-7 and 6-8 or from Equations (6-27) and (6-28).

Here is an example of practical application of the theory.

Determine the vector of the change in the output voltage of the bridge shown in Fig. 6-9 for the following design data:

$V_{ab}=5$ volts; $C_1=0.1$ microfarad;

$R_1=79.5$ ohms; $R_2=1{,}590$ ohms;

$$R_3 = 318 \text{ ohms}; \; C_3 = 0.025 \text{ microfarad};$$
$$C_4 = 0.5 \text{ microfarad}; \; \omega = 2\pi f = 2\pi \times 1{,}000.$$

As can be easily seen, the bridge is at balance. Let the change in the tangent of the loss angle in the first arm be $\Delta \tan \delta = 0.01$. The change ΔV_{cd} can be found from Equations (6-12) and (6-19) and from the charts of Figs. 6-7 and 6-8.

From Eq. (6-19) we have

$$\delta Z_{\tan \delta} = \Delta \tan \delta_1, \; e^{-j\varphi_1} = 0.01e^{j87°}.$$

Since $a = \frac{|Z_1|}{|Z_2|} \approx 1$ and $\theta = \varphi_1 - \varphi_2 = \varphi_1 \cong -87°$, then from the charts we obtain

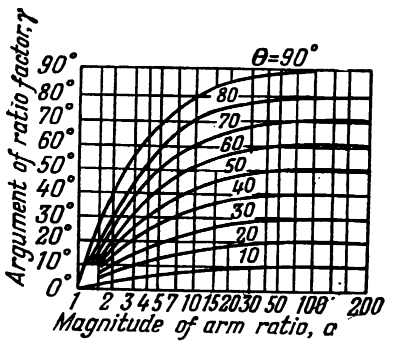


Fig. 6-8

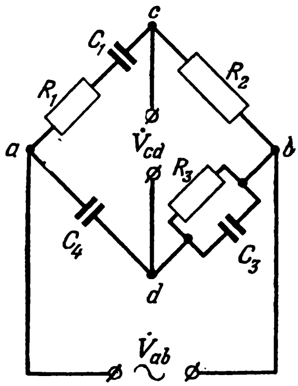


Fig. 6-9

that $\gamma = 0$ and $|\kappa| \cong 0.48$. Using Eq. (6-12), we find that the vector of the change $\Delta \dot{V}_{cd}$ in the output voltage is

$$\Delta \dot{V}_{cd} = \delta Z_{\tan \delta} \, | k | \, e^{j\gamma} \, V_{ab} = 0.01e^{j87°} \times 0.48 \times 5 = 0.024e^{j87°} \text{ volt}.$$

Since $a = 1$, the bridge in question is symmetrical and has maximum sensitivity.

In conclusion, it should be noted that for technical reasons the conditions of symmetry for a.c. bridges are satisfied as often as not.

6-3. Balancing and Circle Diagrams of A. C. Bridges

In measurements with a.c. bridges, the most crucial point is securing a balance of the bridge, i.e., obtaining a zero difference of potential at the terminals of the detector by adjustment of some of the component values. Therefore, a clear understanding of the balancing procedure is essential to the intelligent use of a.c. bridges and to the success of measurements in general.

As was stated in Sec. 6-1, obtaining a balance for an a.c. bridge calls for at least two adjustments. This, in a general case, gives 15

possible pair combinations. A closer study of the problem will, however, show that the choice of adjustments is not so broad as it appears. The choice of the bridge elements and, especially, adjustable ones affects very important properties of a bridge, such as balance convergence and independence of measurement. Therefore, one must not divorce examination of the balancing procedure from the selection of the fixed and adjustable elements of a bridge network.

Securing a balance for a bridge can most conveniently be presented by means of a circle diagram.

Figure 6-10 shows an inductance bridge, while Fig. 6-11 gives its diagram. The diagram has been plotted on the assumption that the unknown complex impedance is placed in the arm *ac* (R_1, L_1), while the adjustable com-

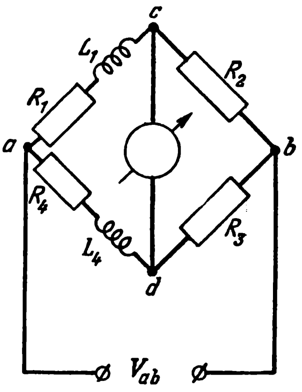


Fig. 6-10

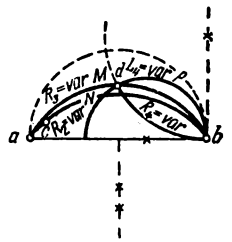


Fig. 6-11

ponent may be any of the remaining values: R_2, R_3, R_4 or L_4. The points *a*, *b*, *c* and *d* in the diagram give the potentials at the respective junctions of the bridge. The lines *M*, *N*, *R* and *Q* are the paths traced out by the potential points *d* and *c* when the respective value is varied (path *P* for L_4, path *M* for R_3, etc.).

For four-arm bridges, including the general form in which all the four arms are complex impedances, the paths of the potential points in the diagram will be circles. Here is one of the possible proofs that these paths, called locus or balancing lines, are circles.

The voltage $\dot{V}_{ad}$ is given by

$$\dot{V}_{ad} = \frac{\dot{V}_{ab} Z_4}{Z_3 + Z_4}. \tag{6-29}$$

Substituting the expressions for the complex impedances Z_3 and Z_4 in Eq. (6-29) we get:

$$\dot{V}_{ad} = \frac{\dot{V}_{ab}(R_4 + jX_4)}{(R_3 + jX_3) + (R_4 + jX_4)}.$$

For convenience, let us introduce the function

$$W = \dot{V}_{ad}/\dot{V}_{ab},$$

which gives the voltage drop across the arm in fractions of the supply voltage $\dot{V}_{ab}$:

$$\dot{V}_{ad}/\dot{V}_{ab} = W = \frac{R_4 + jX_4}{(R_3 + jX_3) + (R_4 + jX_4)} \quad .$$

With one of the quantities (say, R_4) in the above expression being a variable, the function W can be given a more convenient form:

$$W = \frac{jX_4 + R_4}{[R_3 + j(X_3 + X_4)] + R_4}. \tag{6-30}$$

Equation (6-30) is a linear rational function W of the variable R_4. This can be easily proved by comparing Eq. (6-30) and the general expression for a linear rational function:

$$W = \frac{e + fP}{g + hP}, \tag{6-31}$$

where e, f, g and h are constant complex quantities and P is a variable.

As will be recalled, one of the properties of a linear rational function is the simple relationship between the path described by the end of the vector W and the path of the change in the variable P, namely: if the path of the point P for a change in P is a straight line, the end of the vector W will describe a circle.

In our case, the variable is R_4 or any other component value of the bridge (R_i or X_i), a change in which in a plane Z_i can only be thought of as taking place along a straight line (the axis of reals or imaginaries); consequently, the end of the vector $W = \dot{V}_{ad}/\dot{V}_{ab}$ follows a circle.

It is not difficult to prove that the equations giving the voltages $\dot{V}_{ac}$ or $\dot{V}_{bc}$ can similarly be reduced to a linear rational function of the form of Eq. (6-31).

If an arm contains impedances connected in parallel, the expression for the voltage vector $\dot{V}_{ad}$ will have coefficients other than in Eq. (6-30), while remaining a linear rational function. This is based on a well-known observation that two linear rational functions likewise produce a linear rational function. With the impedances in an arm connected in parallel, the total impedance Z of the arm is a linear rational function of the variable which may be R or X, i. e.,

$$Z = f_{lr}(R) \text{ or } = f_{lr}(X).$$

Since in Eq. (6-29) it is shown that

$$W = f_{lr}(Z),$$

then

$$W = f_{lr}(R) \text{ or } = f_{lr}(X).$$

The expanded expression for W will naturally have coefficients other than in Eq. (6-31):

$$W = \frac{k + lP}{m + nP}.$$

From the last expression we may conclude that in any four-arm bridge the expressions for the paths of the points c and d can be reduced to a linear rational function. Accordingly, we may assume that a change in any component value of an a. c. four-arm bridge will cause the points c and d in the circle diagram to follow a circle. In other words, the balancing lines for a. c. four-arm bridges are circles. It should be added that the particular case in which the balancing circle is a straight line (for example, when $Z_3 = R_3$; $Z_4 = R_4$) does not disprove the general statement, if we regard the straight line as a circle of radius $r = \infty$.

We shall now apply the balancing procedure to the bridge of Fig. 6-10 which uses a null detector.

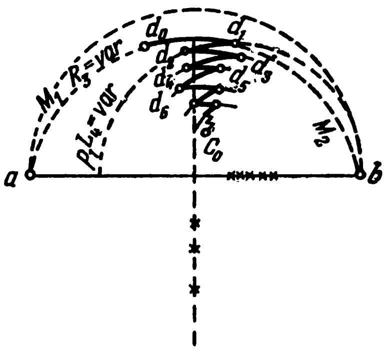


Fig. 6-12

L_4 and R_3 are the Variable Components. In this case, a balance will be secured in accordance with the circle diagram of Fig. 6-12. The bridge will be at balance when the potential point d_0 is moved to the point C_0. We adjust the variable R_3 until the point d_0 arrives at the point d_1, which gives the least deflection of the detector. At the same time, the line C_0d_1 makes a right angle with the tangent to the circle M at the point d_1.

Now L_4 is adjusted until the point d_1 arrives at d_2 which likewise gives the least deflection of the detector. So, going over back and forth between R_3 and L_4, we finally bring the point d_0 to the point C_0.

R_4 and R_3 are the Variable Components. The progress of balancing is shown in Fig. 6-13.

R_4 and R_2 are the Variable Components. In this case, the variable components are placed in different branches (*acb* and *adb*, respectively), and for a balance the points c and d must be brought to a common point k. Figure 6-14 shows the balancing procedure for an arbitrary, but quite possible, initial position of the points C_0

and d_0, which inevitably leads to a false balance at the point b, where $R_2=0$ and $R_4 \mp \infty$.

In such cases, R_2 should be adjusted for a maximum, and R_4 should be brought to zero. Then the points d_0 and C_0 will each shift to a new initial position at d_0' and C_0'. Then by adjusting the two components in turn, a true balance is obtained at the point k.

The above outline of the balancing procedure as applied to a bridge using an ordinary null detector suggests the following conclusions:

1. The balancing procedure involves alternate adjustments of two variables and is physically possible only when the two vari-

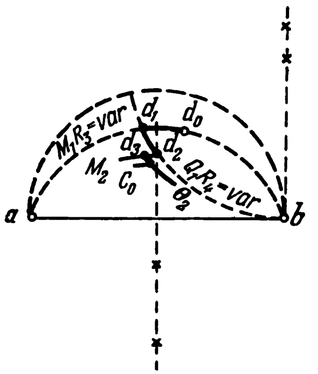


Fig. 6-13

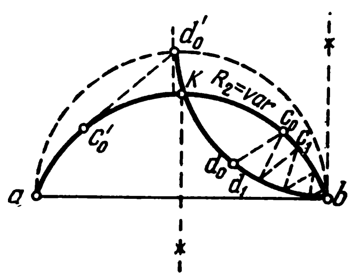


Fig. 6-14

ables have been so chosen that their balancing loci intersect (or touch each other) in the plane of the diagram in at least one more point in addition to the points a and b. Incidentally, the bridge of Fig. 6-10 cannot be balanced by adjusting R_2 and R_3 in accordance with the diagram of Fig. 6-11.

2. The balancing procedure of a bridge depends on the variable chosen and may be characterized by what is known as balance convergence, i. e., the ability of a bridge to approach balance.

3. The number of alternate adjustments necessary to secure a balance decreases with increasing angle of intersection between the balancing circles * (compare Fig. 6-12 and Fig. 6-13), the so-called angle of convergence, which gives a numerical value of a bridge's ability to converge towards a balance.

4. Where the variable components are placed in different branches of a bridge (acb and adb), balancing may lead to points of

* Or, rather, tangents at the point of intersection.

false balance, a and b. To avoid it, certain limits must be set for the variable components prior to balancing.

From the foregoing it seems clear that circle diagrams facilitate an analysis of the balancing procedure for a.c. bridges. This leads us to a very important practical problem of constructing such diagrams. Another problem of comparable importance is the determination of the line of centres for the balancing circles.

To begin with, we should arrive at a definition of the *line of centres*. In constructing a balancing circle, we proceed from the assumption that the branch in question contains only one variable quantity, and so the balancing circle is constructed for this variable, while the other component values of the bridge are deemed fixed and determine the coordinates of its centre. Practically, it boils down to a single adjustment of the bridge.

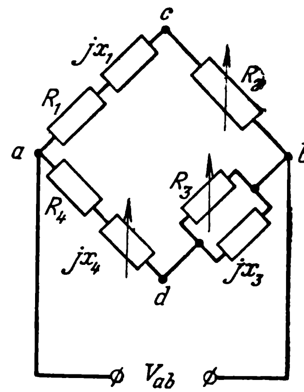


Fig. 6-15

In the case of a. c. bridges, however, two adjustments are necessary in order to obtain a balance. Therefore, should both be placed in the same branch, the centre of the balancing circle will be shifted, since one of the fixed parameters determining the coordinates of the centre has now become a variable, too.

Accordingly, the line which gives the consecutive positions of the centre of the balancing circle for one variable due to adjustment of another is called the line of centres.

From a comparison of the various methods for the construction of circle diagrams one may conclude that the most efficient way is to start from the coordinates of the centre and the length of the radius of the balancing circles.

The efficiency of this method can be proved thus. Firstly, the equation giving the coordinates of the centre at the same time describes the line of centres. For this, it suffices to deem one of the fixed quantities in the equation variable. Secondly, only the coordinates of the centre have to be determined, the radius being implicitly known because all the circles pass through the fixed points a and/or b on the circle diagram.

The latter is very simple to show. Figure 6-15 shows a bridge with arms containing various impedances. Any two component values of the bridge may be assumed to be variable. Thus, for the variable R_2 the balancing circle will pass through the point a when R_2=infinity and through the point b when $R_2=0$.

For the variable R_3, the balancing circle will pass through the point b when $R_3=0$, and for the variable X the circle will pass through the point a when $X_4=\infty$ etc.

All of these examples are easy to generalize, since all the arms of the bridge terminate either at the bridge junction a or at the bridge junction b. The impedances in any arm may be connected both in parallel and in series. Therefore, when the series-connected variable is made infinitely large, the junctions corners c or d and a or b turn out to be at the same potential. Similarly, the potentials at these junctions will be equal, when the parallel-connected variable is reduced to zero.

The equation giving the coordinates of the centre of a circle may be obtained in several ways (including those of analytical geometry). The simplest one is based on conformal representation. By this method, the vector of the centre, W_c, of a balancing circle is given by:

$$W_c=\frac{j}{2}\times\frac{\begin{vmatrix}\frac{e}{\bar{g}} & \frac{f}{\bar{h}}\end{vmatrix}}{\begin{vmatrix}\mathrm{Re}\,(g) & \mathrm{Im}\,(g)\\ \mathrm{Re}\,(h) & \mathrm{Im}\,(h)\end{vmatrix}}, \tag{6-32}$$

where e, f, g, h = constant complex quantities obtained by reducing the expression for the function W for the branch in question to the form of Eq. (6-31);

$\bar{g}$, $\bar{h}$ = conjugate values of the complex quantities g and h;

Re(g), Re(h), Im(g), Im(h) = real and imaginary parts of the respective complex numbers.

Taking a general four-arm impedance bridge as an example, let us examine the construction of a balancing circle with the aid of Eq. (6-32). It stands to reason that it will suffice to examine only one branch containing the variable component (say, adb), shown in Fig. 6-16, since the other branch is identical.

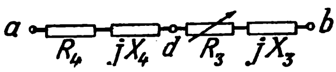


Fig. 6-16

Let the two variable components (the construction must include the line of centres) be in the branch adb. The variables must be designated distinctly. Let the variable for which the balancing circle is to be constructed be called an *independent variable*, and the quantity whose adjustment determines the line of centres for the balancing circles of the independent variable be called simply a *variable*.

In our case, R_3 will be the independent variable, and one of the three remaining quantities will be the variable.

For convenience, points on the circle diagram will give relative potentials, i. e., potentials with respect to the fixed potential V_{ab}. Formerly, this quantity was designated by W. Thus, in all cases $W_a=0$ (i. e., W for the junction a is zero), $W_b=1$, and for an arbitrary point k we have $W_k=\dot{V}_{ak}/\dot{V}_{ab}$.

The balancing circle is constructed in the following sequence:

1. Transform $W_d=\dot{V}_{ad}/\dot{V}_{ab}$ into a linear rational function, i. e.,

$$W=(e+fP)/(g+hP),$$

where P is the independent variable R_3:

$$W=\frac{\dot{V}_{ad}}{\dot{V}_{ab}}=\frac{R_4+jX_4}{R_4+jX_4+R_3+jX_3}=\frac{R_4+jX_4}{[R_4+j(X_3+X_4)]+R_3}.$$

2. Writing down the values of the coefficients e, f, g, h, Re(g), Re(h), Im(g) and Im (h) which affect the expression for the linear rational function, we find the coordinates of the centre from Eq. (6-32) and plot them on the diagram. In our cases, the above coefficients will have the values:

$$e=R_4+jX_4;\ f=0;\ g=R_4+j(X_3+X_4);\ h=1;\ \mathrm{Re}(g)=R_4;$$
$$\mathrm{Im}(g)=X_3+X_4;\ \mathrm{Re}(h)=1;\ \mathrm{Im}(h)=0,$$

whence

$$W_c=\frac{j}{2}\times\frac{\begin{vmatrix} R_4+jX_4; & 0 \\ R_4-j(X_3+X_4); & 1\end{vmatrix}}{\begin{vmatrix} R_4; & (X_3+X_4) \\ 1; & 0\end{vmatrix}}.$$

On clearing the brackets, we obtain:

$$W_c=\frac{X_4+jR_4}{2(X_3+X_4)}. \tag{6-33}$$

The coordinates of the centre, which are known to be equal to the real and imaginary parts of the vector respectively, will be

$$\frac{X_4}{2(X_3+X_4)};\quad -j\frac{R_4}{2(X_3+X_4)}.$$

3. Find the fixed point through which the sought-for circle should pass. From reference to Fig. 6-16, when $R_3\to\infty$, the potential point d coincides with the point a.

Thus, we now have all that is necessary for the construction of the balancing circle, namely: the coordinates of the centre of the circle and a point on the circle.

4. Determine the character and direction of the line of centres. As has been noted already, the line of centres is described by the same equation as the vector of the centre, i. e., Eq. (6-32), provided one of the terms on the right-hand side is made variable. Equation (6-32) is a linear rational function and, consequently, with the variable varying linearly the end of the vector W_c will slide along a circle (or, in a particular case, a straight line). The path followed by the end of the vector W_c is the sought-for line of centres.

In order to determine the character of the line of centres in our case, we should additionally analyze Eq. (6-33). As will be noted, when the variables take certain definite values, namely $R_4=\infty$ or $X_3=-X_4$, then $W_c=\infty$ which is characteristic of a straight line and not of a circle. To state it differently, Eq. (6-33) gives a straight line. Through a more detailed analysis it can be established that lines of centres in the form of a straight line are generally typical of all practical a. c. bridges.

Now that we have arrived at the conclusion about the character of lines of centres in general, we can determine their exact direction by finding two points on each of them.

Let us examine three possible lines of centres.

The Line of Centres for the Variable R_4. In this case the line of centres is a straight line passing through the centre of the circle parallel with the axis of imaginaries since the real part of the vector W_c, equal to $\frac{X_4}{2(X_3+X_4)}$, remains constant.

The Line of Centres for the Variable X_4. Determine two points on the line. When $X_4=-2X_3$ the first point will be

$$W_c=1+j\,(R_4/2X_3);$$

the second point may well be the centre of the circle already found. However, where it is desirable to determine the line of centres independently of the previous procedure, it is convenient to assume $X_4=\infty$; then $W_c=1/2$, and the line of centres is identified unambiguously, being a straight line making an angle with the coordinate axes.

The Line of Centres for the Variable X_3:

The first point: $W_c=0$ when $X_3=\infty$.

The second point: $W_c=1-j(R_4/X_4)$, when $X_3=-\frac{1}{2}X_4$.

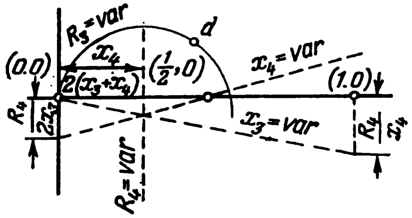


Fig. 6-17

As in the previous case, the line of centres makes an angle with the coordinate axes.

On the strength of the above data, we can construct a circle diagram (Fig 6-17) for the independent variable R_3 and three possible variables X_4, X_3 and R_4.

Using the above technique, it is possible to determine the coordinates of the centre and to construct lines of centres for any four-arm bridge network.

6-4. Balance Convergence of A. C. Bridges

A simple and rapid balancing procedure for a. c. bridges where a balance is secured by simultaneous adjustment of two component values is certainly a pivotal problem, both practically and theoretically.

From this point of view, of special importance are events which take place near the state of balance, since they mainly govern the ability of a bridge to approach a balance.*

* While at the beginning of the balancing procedure the bridge is far from balance, already the first adjustment, as can be seen from the circle diagram, brings it close to balance.

It was noted earlier (Sec. 6-3) that the ability of a bridge to approach the state of balance is referred to as *convergence*. It may be added now that by the convergence of an a. c. bridge is meant the rate at which a given bridge approaches the state of balance near balance.

Near balance, the relationships between the voltages of the network can be presented by a small area on the circle diagram, so that the arcs of the balancing circles may be approximated by straight lines with a sufficient degree of accuracy. With such approximation, an analysis of the balancing procedure for a. c. bridges is greatly

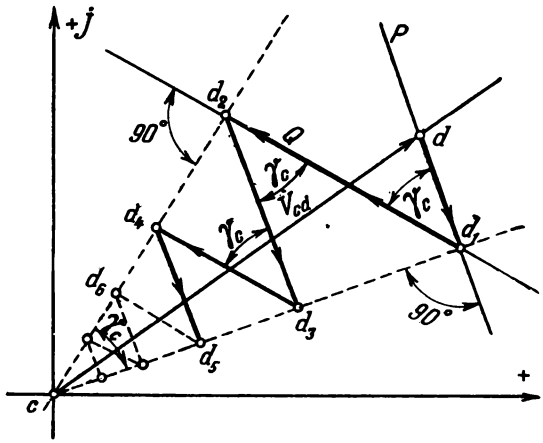


Fig. 6-18

simplified, and their convergence may well be expressed by certain definite quantities which lend themselves easily to mathematical treatment.

Let us examine the behaviour of a bridge near the balance point and determine the quantities characterizing its convergence.

Let the variable components p and q by whose adjustment the bridge is brought to balance are placed in the branch *adb*. Then, during the balancing procedure, only the potential of the point d will vary. In Fig. 6-18 the straight lines represent the balancing lines P and Q for changes in the variables p and q. The stationary potential point c, corresponding to the bridge junction c, gives the origin of the coordinate system.

It should be qualified from the outset that we shall examine bridges which use null detectors responding only to the magnitude of the unbalance voltage. For simplicity, we shall also assume that the threshold sensitivity of the null detector is zero, and the lowest

magnitude of the unbalance voltage can consequently be ascertained with ideal accuracy. The balancing procedure will then be as follows (Fig. 6-18). Let the vector of the unbalance voltage be originally equal to $\dot{V}_{cd}$. By adjusting the quantity p, we find the lowest magnitude of the unbalance voltage. It will be obtained when the point d has moved to the position d_1 and the line P is normal to the line cd_1. Then, the unbalance voltage will be equal to the vector $\dot{V}_{cd}$. As the component q is adjusted, the point d moves along the line d_1d_2. In the position d_2, the magnitude of the unbalance voltage will have reached another minimum equal to $\dot{V}_{cd2}$. Now the line cd_2 will be normal to the line Q. Similar reasoning applies to the further movement of the point d.

From Fig. 6-18 it will be seen that the magnitudes of the unbalance voltage vectors bear the following relations to one another:

$$\frac{V_{cd_1}}{V_{cd_2}}=\frac{V_{cd_2}}{V_{cd_3}}=\frac{V_{cd_3}}{V_{cd_4}}=\cdots\cdots=\frac{V_{cd_n}}{V_{cd_{n+1}}}=\frac{1}{(\cos\gamma_c)}, \tag{6-34}$$

where γ_c is the angle between the balancing lines near the balance point. In the subsequent discussion, the angle γ_c will be referred to as the *angle of convergence*.

From Eq. (6-34) it is possible to determine the number of consecutive adjustments, n, required in order to reduce the unbalance voltage by a factor m. Since

$$m=\frac{V_{cd_1}}{V_{cd_{n+1}}}=\frac{V_{cd_1}}{V_{cd_2}}\times\frac{V_{cd_2}}{V_{cd_3}}\times\cdots\cdot\times\frac{V_{cd_n}}{V_{cd_{n+1}}}=\left(\frac{1}{\cos\gamma_c}\right)^n,$$

he number of adjustments, n, will be

$$n=\frac{\log_{10} m}{\log_{10}\dfrac{1}{\cos\gamma_c}}. \tag{6-35}$$

As follows from Eq. (6-35), a measure of convergence is given by the ratio of the logarithm of the relative reduction in the unbalance voltage, m, to the required number of consecutive adjustments, n, or

$$k_c=\log_{10} m/n=\log_{10} 1/\cos\gamma_c. \tag{6-36}$$

Thus, the convergence factor, k_c, as found for idealized conditions (the threshold sensitivity of the null detector is assumed to be zero), depends solely on the constants of a given bridge network and is equal to the logarithm of the secant of the convergence angle γ_c. From Eqs. (6-35) and (6-36) it follows that the best convergibility and the maximum convergence factor will be obtained when

$$\gamma_c=\pi/2.$$

In practical a. c. bridges, the sensitivity of the null detector is always limited, and for any detector, however perfect, there is a limit of sensitivity. Therefore, slight changes in the unknown quantity equal to the threshold sensitivity will not be detected. A measure of this imperfection is given by the angle ε * which is the maximum unobservable increase in the magnitude of the vector $\dot{V}_{cd}$ above its minimum value, as the end of the vector $\dot{V}_{cd}$ moves along the balancing line. Figure 6-19 shows the region of a circle diagram for a

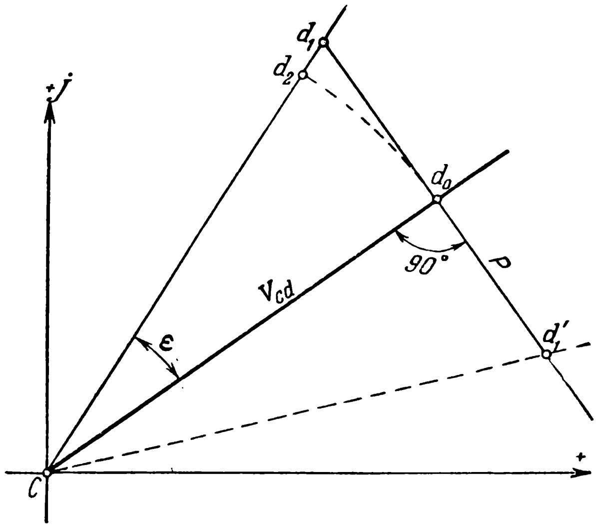


Fig. 6-19

bridge near the balance point. During the balancing procedure, the end of the vector $\dot{V}_{cd}$ moves along the balancing line P. Let the point d move from the position d_0, which corresponds to the smallest unbalance voltage, to the position d_1 so that the magnitude of the vector $\dot{V}_{cd}$ increases by $\dot{V}_{d_2d_1}$ equal to the threshold sensitivity, Δ, of the null detector. As can be seen from the figure, the deflection ε of the vector $\dot{V}_{cd}$ and the threshold sensitivity, Δ, are related thus:

$$\cos\varepsilon = 1/(1+\Delta/V_{cd_0}).$$

As follows from the above relationship, the angle ε, which is unambiguously defined by the relative threshold sensitivity Δ/V_{cd}, may well serve as the quality index of the null detector. The angle ε depends on the type of detector used. For example, for vibration

* The angle ε is often referred to as the insensitivity angle of a null detector.

(tuned) galvanometers it is 8° to 10°, while for telephones, when used as the null detector, it is 15° to 20°.

Now to go back to the balancing of the bridge. Because of the sensitivity threshold, the vector of the output voltage V_{cd}, during adjustment for, say, p (Fig. 6-19), may take up any position within the insensitivity angle equal to 2ε. In some cases, this may considerably impair the convergence of the bridge. Let us determine the convergence factor with allowance for the angle ε. Consider the

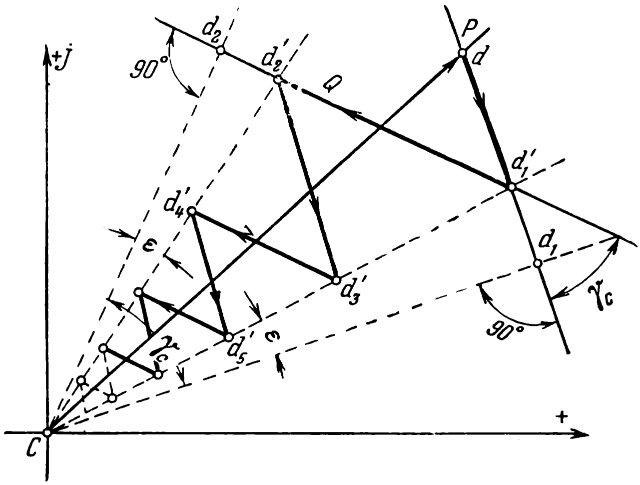


Fig. 6-20

balancing procedure under the most adverse conditions when at the end of each adjustment the vector of the output voltage $\dot{V}_{cd}$ takes up a position most unfavourable to convergence. Diagrammatically, this is shown in Fig. 6-20. The convergence angle is the same as in the previous case (Fig. 6-18). During adjustment for p, the point d of the vector $\dot{V}_{cd}$ fails to reach the position d_1 which corresponds to the true minimum of the magnitude of V_{cd}, and the vector of the output voltage becomes equal to $\dot{V}_{cd}$. The angle $d_1'cd_1$ is equal to the angle ε. During adjustment for q, the point d moves as far as d_2'' and again fails to reach the point d_2 corresponding to the minimum of the magnitude of V_{cd}, etc. From Fig. 6-20 it follows that

$$V_{cd_2'} = \frac{V_{cd_2}}{\cos \varepsilon}; \quad V_{cd_1'} = \frac{V_{cd_2}}{\cos (\gamma_c - \varepsilon)};$$

whence

$$\frac{V_{cd_1'}}{V_{cd_2'}}=\frac{\cos\varepsilon}{\cos(\gamma_c-\varepsilon)}.$$

It can be shown that generally for $n=1, 2, 3...$ the following relationship holds:

$$\frac{V_{cd_n'}}{V_{cd_{n+1}'}}=\frac{\cos\varepsilon}{\cos(\gamma_c-\varepsilon)}. \tag{6-37}$$

From Eq. (6-37) we get

$$m=\frac{V_{cd_1'}}{V_{cd_{n+1}'}}=\frac{V_{cd_1'}}{V_{cd_2'}}\times\frac{V_{cd_2'}}{V_{cd_3'}}\times\cdots\cdots\times\frac{V_{cd_n'}}{V_{cd_{n+1}'}}=\left[\frac{\cos\varepsilon}{\cos(\gamma_c-\varepsilon)}\right]^n \tag{6-38}$$

where m, as in the previous case, stands for the relative decrease in the output voltage and n denotes the requisite number of consecutive adjustments. From Eq. (6-38), the number of adjustments is

$$n=\frac{\log_{10} m}{\log_{10}\dfrac{\cos\varepsilon}{\cos(\gamma_c-\varepsilon)}}. \tag{6-39}$$

Thus, with due allowance for the sensitivity threshold of the null detector, the convergence factor will, according to Eq. (6-39) be

$$k_c=\log_{10}\frac{\cos\varepsilon}{\cos(\gamma_c-\varepsilon)}. \tag{6-40}$$

In deriving all the relationships for the balance convergence of a.c. bridges, we assumed, as had been qualified earlier, that the adjustments were placed in the same branch *adb* and the point c remained fixed during the balancing procedure. It may be shown that these relationships also hold when the adjustments p and q are placed in different branches and both points c and d are movable during the balancing operation.

As can be seen from Eq. (6-40), when $\gamma_c=2\varepsilon$ the convergence factor k_c is zero and the bridge will not be balanced. The same will happen when $\gamma_c<2\varepsilon$. Consequently, for a bridge to operate satisfactorily, it is essential that $\gamma_c>2\varepsilon$.

It should be noted also that the convergence angle γ_c affects not only the number of consecutive adjustments involved but also, because of the sensitivity threshold, the accuracy of bridge balance, i.e., in the final analysis, the accuracy of measurement.

From the foregoing it is clear that the convergence angle is a very important characteristic of bridge networks. Now we shall go over to a technique for determining it.

The unbalance voltage of a bridge is given by

$$\dot{V}_{cd}=\dot{V}_{ab}\frac{Z_1Z_3-Z_2Z_4}{(Z_1+Z_2)(Z_3+Z_4)}=\dot{V}_{ab}\frac{H}{D}, \tag{6-41}$$

where $H=Z_1Z_3-Z_2Z_4$. At balance, $H=0$. In the case of an unbalance due to p and q, the potential point d (or the potential points c and d, if p and q are placed in different branches) will move along the respective balancing line (or circle). The derivatives $\partial\dot{V}_{cd}/\partial p$ and $\partial\dot{V}_{cd}/\partial q$ determine the tangents to the balancing circles P and Q at the balance point. It is obvious that the difference in angle between these tangents will determine the angle of intersection between the balancing circles, i. e., the convergence angle γ_c.

Thus,

$$\gamma_c=\arg\frac{\partial\dot{V}_{cd}}{\partial p}-\arg\frac{\partial\dot{V}_{cd}}{\partial q}. \tag{6.42}$$

According to Eq. (6-41), we have

$$\frac{\partial\dot{V}_{cd}}{\partial p}=\frac{\dot{V}_{ab}}{D^2}\left(D\frac{\partial H}{\partial p}-H\frac{\partial D}{\partial p}\right);$$

$$\frac{\partial\dot{V}_{cd}}{\partial q}=\frac{\dot{V}_{ab}}{D^2}\left(D\frac{\partial H}{\partial q}-H\frac{\partial D}{\partial q}\right).$$

Since we examine the behaviour of a bridge network near balance we may assume that $H=0$. In view of this, from the last expressions we have

$$\frac{\partial\dot{V}_{cd}}{\partial p}=\frac{\dot{V}_{ab}}{D}\times\frac{\partial H}{\partial p};$$

$$\frac{\partial\dot{V}_{cd}}{\partial q}=\frac{\dot{V}_{ab}}{D}\times\frac{\partial H}{\partial q}. \tag{6-43}$$

Substituting Eq. (6-43) in Eq. (6-42) and noting that the argument of a product is equal to the sum of arguments, we finally obtain

$$\gamma_c=\arg\frac{\partial H}{\partial p}-\arg\frac{\partial H}{\partial q}. \tag{6-44}$$

To sum up, the convergence angle can be easily determined for any pair of adjustments, if the four impedances contained in the arms of the bridge are known. Knowledge of the convergence angle may be necessitated by the selection of adjustments when designing a new bridge or analyzing existing bridges.

In conclusion, we shall dwell upon the relationship between the convergence angle and the conditions for independent measurement.

At the beginning of this chapter we briefly outlined these conditions. Now we shall show that the conditions for independent measurement unambiguously define the convergence angle γ_c if a bridge is balanced by independent consecutive adjustments. *When the conditions for independent measurement are fulfilled, the convergence angle of an a.c. bridge is equal to the angle between the vectors of changes in the unknown complex impedance Z (or admittance Y) which correspond to the respective adjustments.* This is true of bridges

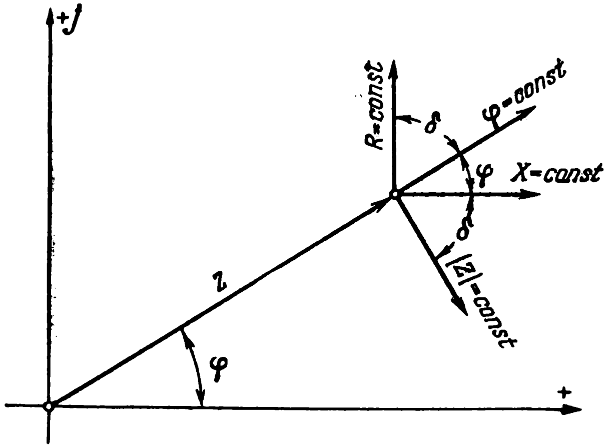


Fig. 6-21

with any number of arms and any impedance of the source and detector circuits. Indeed, in both independent measurement and adjustment, the balancing lines for the adjustments must coincide with the balancing lines for the respective components of the unknown complex impedance (or admittance). For any bridge, the vector of the unbalance voltage $\dot{V}_{cd}$ is related to the unknown complex impedance (or admittance) by a certain linear rational (analytical) function W which transforms the plane Z (or Y) conformally into the plane W. As will be recalled, in the case of a conformal transformation, the respective angles in both planes are equal. Therefore, the convergence angle γ_c will be equal to the angle between the vectors of changes in Z (or Y), corresponding to the components measured independently.

All possible unknown quantities can be reduced to six basic quantities: R, X, G, B, $|Z|$, and φ.

The respective changes in the complex impedance Z and admittance Y are shown in Figs. 6-21 and 6-22. Referring to the figures,

"ideal" convergence ($\gamma_c=90°$) is attained only in bridges where it is possible to measure independently R and X, or G and B, or $|Z|$ and φ. In the measurement of X and φ (L and Q, respectively) and also of B and φ (c and $\tan\delta$, respectively) the convergence angle for bridges capable of independent measurement will be $\gamma_c=\varphi$ (Figs. 6-21 and 6-22). As soon as the unknown components to be measured independently are defined for a bridge, this simultaneously defines its convergence angle. Conversely, where the best possible

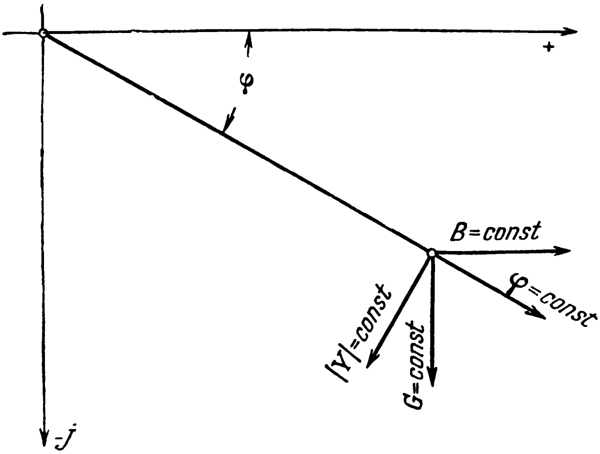


Fig. 6-22

convergence angle is desired, the bridge must give independent indication only for R and X or G and B, or $|Z|$ and φ. No bridge, however complex (including multiple-arm networks), can ensure ideal convergence in the case of independent measurement of, say, c and $\tan\delta$, or L and Q. By the way, it should be noted that the loss angles, δ, of capacitors are usually fairly small, i. e., the convergence angle in this case $\gamma_c=\varphi=90°-\delta$, is very close to 90° and varies relatively little. This is not the case with the Q-factor of inductors, which varies within broad limits and may sometimes be very low (φ is low). For these reasons, the convergence of capacitance bridges is ordinarily better than that of inductance bridges.

We have examined one of the main characteristics of a.c. bridges—convergence; we have derived an equation for the convergence angle and traced the relationship between the conditions for independent measurement and the convergence angle. Now we shall take up the relationship between independent adjustment and independent measurement.

6-5. Independence of Adjustments and Measurement

As has been shown, a balance is obtained for an a.c. bridge by alternate adjustments.

This is often a fairly lengthy and tiring procedure, calling for much skill and experience on the part of the operator. Indeed, in some cases an inexperienced operator may fail altogether because of poor convergence or a false balance.

Against this background, one cannot stress too strongly the importance of an orderly and rapid balancing procedure. To begin with, we shall examine the conditions under which a bridge can be balanced by single adjustment of each variable component. This is what may be called independent adjustment.

In principle, the possibility, or otherwise, of independent adjustment is clear from examination of the diagram (Fig. 6-23) which gives the balancing circles for the variable components R_3 and R_4 (as an example, we shall analyze the diagram of the inductance bridge shown in Fig. 6-10).

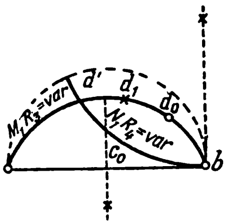


Fig. 6-23

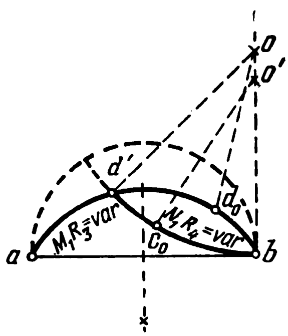


Fig. 6-24

Indeed, if we had a special indicator which would, during adjustment for R_3, show when the point d_0 arrives at the point d' and not the point d_1, as is the case with an ordinary null detector, then during adjustment for the other variable R_4 the point d' would slide along the circle N_1 directly to the point c_0 where the bridge is at balance.

In the case under examination, the balancing will involve the two main balancing circles M_1 and N_1 passing through the original positions of the points c_0 and d_0. It may be conveniently shown that even if the variable components are placed in different branches, i. e., when for a balance to be secured the points c_0 and d_0 must be brought to a common point, the method of independent adjustment described here will hold.

Thus, the only condition for independent adjustment to be possible is the provision of a special indicator showing the point of intersection between the main balancing circles. Independent adjustment is likewise applicable to any bridge having four impedance arms, since the character of the balancing line remains the same as in the particular case discussed.

In the light of the foregoing, the concept of independent adjustment as applied to a.c. bridges can be stated thus:

Independent adjustment is a procedure by which a balance is secured in two independent steps: in the first step, the point of intersection between the main balancing lines is determined; in the second step, the potential points of the detector terminals are brought to a common point. Practically, this implies obtaining a balance with the least possible number (two, in the limit) of adjustments of the variable components.

Thus, independent adjustment can be effected with the aid of an indicator capable of showing the point of intersection between the main balancing circles. Such an indicator can in principle be realized in a variety of ways. We shall describe what is known as the magnitude method which consists in the following.

The circle diagram of Fig. 6-24 shows two balancing circles M_1 and N_1 which are to be followed in independent adjustment, and the N-type line of centres *ob*.

Independent adjustment which boils down to moving the point d_0 to the point d' and then to the point c_0 consists of three operations:

1. *Determination of the Centre of the Circle* N_1. To perform this operation, the voltages $V_{o'b}$ and $V_{o'c_0}$ are applied to a differential voltmeter. The point o' is moved to the point o so as to obtain equality between the applied voltages, which will take place when the centre of the circle N_1 is attained. Naturally, in addition to a differential voltmeter, this operation calls for an auxiliary voltage which is in quadrature with the supply voltage V_{ab} (in the circle diagram this voltage is represented by the line of centres *ob*).

2. *Determination of the Point of Intersection* d'. The commutator is turned so that the voltages V_{ob} and V_{od0} are now applied to the differential voltmeter. The variable R_3 is adjusted until the point d_0 is shifted to the point d'; at this point the differential voltmeter should indicate the equality of the applied voltages.

Figure 6-25 shows how a differential voltmeter *DV* should be connected in this operation. At *M* is a centre-zero instrument; the phase-shifting network R_{ph}/C_{ph} is connected across the secondary of a transformer *T* in order to produce the requisite phase shift. Due to this network, the voltage applied to the voltage divider *VD* is in quadrature with the supply voltage V_{ab}.

3. *Securing a Balance.* The other variable R_4 is adjusted until the point d' is made to coincide with the point c_0, thereby securing a balance. This operation can be performed either with the same differential voltmeter as used in the previous operations or with the standard null detector connected across *dc*.

The differential voltmeter used in the first two operations introduces an error, since it has a shunting effect on the arms of the bridge. This error, which may incidentally be reduced by using a voltmeter

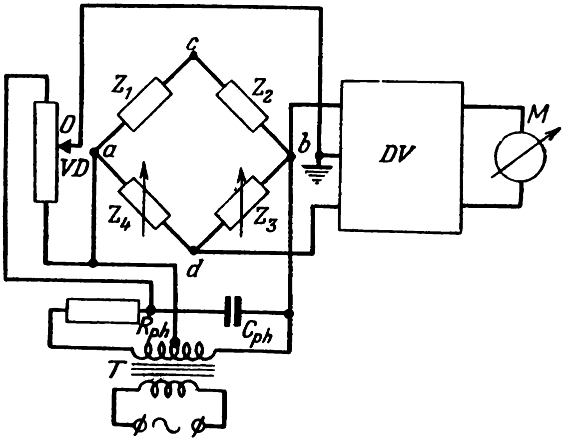


Fig. 6-25

with a high input impedance, results in that the third operation fails to secure a precise balance. Consequently, the variables have to be given one or two complimentary adjustments.

If the variable components are placed in different branches, the centre of the circle needs not be determined for each measurement, since the adjustment only involves the stationary circles, i. e., the number of operations is reduced to two.

The major advantages offered by indicators for independent adjustment are:

1. The whole balancing procedure is broken down into several simple operations. Furthermore, the direction of the adjustments is always known (from the deflection of the indicator to the right or left of zero).

2. Since the operations are simple, no special skills are required on the part of the operator.

3. The circuitry of an indicator for independent adjustment is no more complicated than that of an ordinary null detector using a valve amplifier.

4. Independent adjustment may be embodied in automatic a.c. bridges.

All this gives enough ground for recommending the use of independent-adjustment indicators instead of ordinary null detectors using valve amplifiers.

However, irrespective of the balancing procedure employed, the values of the variable components must be taken down at balance. Then, using the well-known relationship

$$Z_1 = Z_2 Z_4 / Z_3,$$

which can in turn be broken down into two equations for the real and pure imaginary parts of the complex number, the requisite components of the unknown impedance Z_1 are found. Thus, in each particular case, the final result can be obtained only after certain computations, fairly laborious sometimes.

In this connection, it seems worth while to study in greater detail such conditions of bridge operation under which the requisite components of the unknown complex impedance can be read directly from the respective bridge scales. This boils down to the independent measurement mentioned in Sec. 6-1.

The component values of interest are most often any pair of the three quantities: R_1, X_1 and φ_1, or a pair of simple functions, each related only to one of these quantities R_1 and L_1, C_1 and $\tan\delta_1$, L_1 and $\tan\varphi_1$, etc.

Consider the conditions that a bridge permitting of independent measurement must satisfy.

It was noted earlier (Sec. 6-1) that after the complex impedances Z_2, Z_3 and Z_4 have been substituted by R and jX the unknown impedance Z_1 is given by

$$R_1 + jX_1 = (R_2 + jX_2)(R_4 + jX_4)/(R_3 + jX_3).$$

This relationship can be given the form

$$R_1 + jX_1 = A + jB, \tag{6-45}$$

or, using admittance and its components,

$$G - jB_1 = A + jB_1.$$

From Eq. (6-45) it follows that independent measurement in a bridge will be obtained only when the terms A and B contain each at least one different component which may be made variable and calibrated in units of R_1 or X_1. This automatically satisfies the condition of independent measurement for the third quantity φ. Independent measurement can be obtained only in bridges with two complex arms in which the disposition and character of the complex

impedances, as can be easily proved with the aid of Eq. (6-45), should be as follows:

1. Where complex impedances are placed in adjacent arms, their arrangement must be the same, i. e., both must be made up of either series or parallel components. Thus, if we assume that the unknown complex impedance consists of parallel-connected components, the components of the complex impedance in the adjacent arm must be arranged in exactly the same way.

2. Where complex impedances are placed in opposite arms, these impedances must be different in character, i.e., their components must be connected in series in one arm and in parallel in the other.

Thus, in a general case the presence of two pure arms is not a sufficient condition for independent measurement. By the way, it may be noted that the assumed placement of the two adjustments in one and the same arm in this case is not a "must", either.

However, departures from the rules for the placement and connection of complex impedances formulated above are very rare. Therefore, we may, as we did in Sec. 6-1, place emphasis on the character of the ratio arms as being the overriding feature.

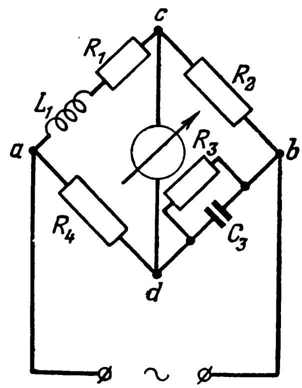


Fig. 6-26

The choice of variable components is also an important point. In bridges with independent measurement these may be the resistive and reactive components of the standard arm. Then, it is possible to determine directly the resistive (R_1) and the reactive (L_1 or C_1) components of the unknown impedance. However, the use of a reactive component as a variable element entails certain inconveniencies of technical nature *. Therefore, as often as not, the second variable element is the resistive component of another arm, which is simpler and cheaper to make. Then, independent measurement is possible for R_1 or X_1 (L_1, C_1) and the relationship between them giving the Q-factor of coils:

$$Q_1 = \omega L_1/R_1 = \tan \varphi_1,$$

or the tangent of the loss angle of capacitors:

$$\tan \delta_1 = \omega C_1 R_1 = 1/\tan \varphi_1,$$

whence it is an easy matter to determine the remaining component of the unknown complex impedance.

* Yet, from the view-point of convergence, this is the most advantageous arrangement, since the balancing lines intersect at right angles.

An example of a bridge with independent measurement is given by the circuit of Fig. 6-26.

The unknown components in this network can be given the form:

$$R_1 = R_2 R_4 / R_3;$$
$$L_1 = R_2 R_4 C_3;$$
$$Q_1 = \omega L_1 / R_1 = \omega C_3 R_3.$$

For independent measurement of L_1 and R_1, it is essential that R_3 and C_3 be made adjustable. On the other hand, L_1 and Q_1 can be measured separately when R_3 and R_4 are adjustable.

In conclusion, it should be noted that independent measurement is a convenient expedient. Therefore, in designing a bridge it should always be sought to fulfil the conditions for it.

6-6. Basic Types of A. C. Bridges

In the previous sections we dealt with the basic theory of a.c. bridges. Now we shall analyze several practical forms of bridge networks. The most commonly used forms are four-arm bridges, multiple-arm bridges, and mutual-inductance bridges.

Four-arm Bridges. It has been noted already that in present-day electrical measurements a fairly great variety of four-arm a.c. bridges is employed. For all the outward dissimilarity, however, they can be analyzed in about the same manner. Therefore, we shall limit ourselves to a few most typical examples.

One of them is the УМ-3 bridge manufactured in the Soviet Union for the measurement of capacitors and inductors at audio frequencies (100 and 1,000 c/s) to a relatively low accuracy (about 1 per cent).* Figure 6-27 shows the circuit for the measurement of capacitors, and Fig. 6-28 for the measurement of inductors.

The unknown capacitance is represented by a series combination C_1 and R_1. The resistance shown is the one inherent in the capacitor. The arm R_2 consists of seven resistance coils, so that the range of the bridge can be easily changed at will. In the measurement of capacitors, a balance is secured by adjustment of R_3 and R_4. The arm R_3 consists of four sections (which gives a coarse balance) and a continuous slide-wire. The arm R_4 provides the requisite loss angle and is also constructed as a slide-wire. The bridge gives independent indication of the capacitance and loss angle of capacitors.

Let us derive the balance equation for the bridge and the formulas for the components of the unknown impedance Z_1. Referring

* Provisions are also made in the bridge for the measurement of resistances under d.c. conditions.

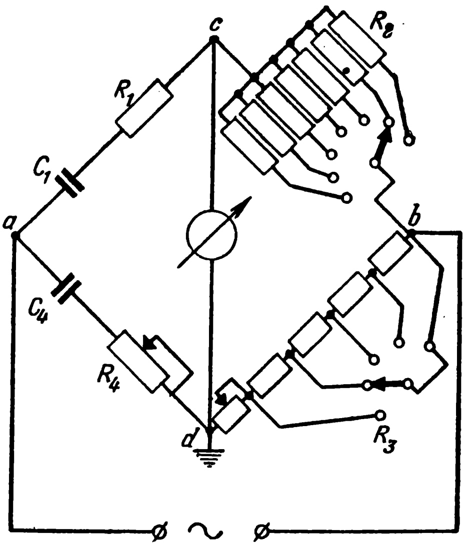


Fig. 6-27

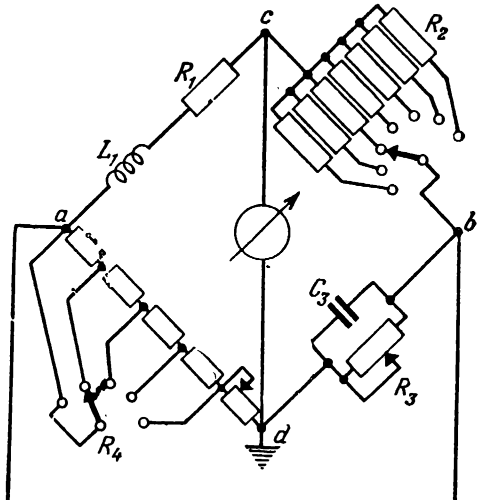


Fig. 6-28

to Fig. 6-27, the impedances of the arms are:

$$Z_1 = R_1 - j(1/\omega C_1);\ Z_2 = R_2;$$
$$Z_3 = R_3;\ Z_4 = R_4 - j(1/\omega C_4).$$

The balance equation is

$$H = Z_1Z_3 - Z_2Z_4 = \left(R_1 - j\frac{1}{\omega C_1}\right)R_3 - R_2\left(R_4 - j\frac{1}{\omega C_4}\right) = 0;$$

$$R_1R_3 - R_2R_4 - j(R_3/\omega C_1 - R_2/\omega C_4) = 0.$$

Equating the real and imaginary parts of the vector H separately we obtain

$$R_1 = R_2R_4/R_3;\ C_1 = C_4(R_3/R_2). \tag{6-46}$$

The tangent of the loss angle, δ_1, is given by

$$\tan\delta_1 = \omega C_1R_1 = \omega C_4R_4. \tag{6-47}$$

From Eqs. (6-46) and (6-47) it follows that when securing a balance by variation of R_3 and R_4, independent measurement is obtained for the unknown capacitance C_1 and the tangent of the loss angle, $\tan\delta_1$. R_3 can be calibrated directly in units of capacitance C_1 and R_4 in units of $\tan\delta_1$.

The convergence angle of the bridge, as found from Eq. (6-44), will be

$$\gamma_c = \arg\frac{\partial H}{\partial R_3} - \arg\frac{\partial H}{\partial R_4} = \arg\left(R_1 - j\frac{1}{\omega C_1}\right) - \arg R_2. \tag{6-48}$$

From Eq. (6-48) one may conclude that the convergence angle, γ_c, of the bridge is equal to the phase angle, φ_1, of the unknown impedance. Since the bridge gives independent indication of C_1 and $\tan\delta_1$, this is what should be expected in accordance with the definition of the convergence angle given in Sec. 6-4. Indeed, in Fig. 6-21 a change in the capacitance C_1 being measured and measured separately (when $\tan\delta_1$=const) corresponds to the line φ=const, while a change in $\tan\delta_1$ (when C_1=const) corresponds to the line X=const. Referring to the figure, the angle between the lines φ=const and X=const is equal to the phase angle of the impedance, i. e., the angle φ.

The relative sensitivity of the bridge (see Sec. 6-2) is given by

$$\dot{S}_V^0 = \dot{V}_{ab}\frac{A}{(1+A^2)} = \frac{\dot{V}_{ab}}{1/A + 2 + A}.$$

Substituting

$$A = Z_1/Z_2 = \frac{R_1 - j(1/\omega C_1)}{R_2}$$

into the previous relationship, we get

$$\dot{S}_V^0 = \frac{\dot{V}_{ab}}{R_2 | [R_1 - j(1\omega C_1)] + 2 + [R_1 - j(1/\omega C_1)] | R_2}. \qquad (6\text{-}49)$$

From Eq. (6-49) it follows that the sensitivity of the bridge is solely dependent on the relationship between Z_1 and R_2, the phase angle of the impedance being measured, and the supply voltage V_{ab}.

Thus, we have derived the basic relationships for the УМ-3 bridge for the measurement of capacitances.

Inductances are measured with the circuit shown in Fig. 6-28. The components of the unknown inductance are designated by L_1 and R_1. As in the previous case, R_2 serves to change the range of the bridge. The requisite phase relationships and independent measurement of L_1 and Q_1 are obtained owing to the fact that the standard capacitance and resistance are parallel connected in the arm Z_3.

The basic relationships for this circuit are given without proof. The inductance L_1 and the quality factor Q_1 are given by

$$L_1 = R_2 R_4 C_3; \quad Q_1 = \omega L_1 / R_1 \doteq \omega C_4 R_3. \qquad (6\text{-}50)$$

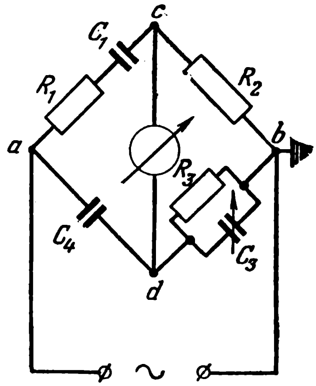


Fig. 6-29

From Eq. (6-50) it follows that balancing for R_3 and R_4 gives independent indication of Q_1 and L_1, respectively.

As in the previous case, the convergence angle γ_c is equal to the phase angle, φ_1, of the unknown impedance.

The sensitivity is given by

$$\dot{S}_V^0 = \frac{\dot{V}_{ab}}{R_2/(R_1 + j\omega L_1) + 2 + (R_1 - j\omega L_1)/R_2}. \qquad (6\text{-}51)$$

Another example is provided by the bridge network of Fig. 6-29, known as the Shering bridge, widely employed to measure capacitances and phase-defect angles under conditions of high voltage.

The arm impedances of the bridge are given by

$$\left.\begin{aligned} Z_1 &= R_1 - j(1/\omega C_1); \\ Z_2 &= R_2; \\ Z_3 &= 1/[(1/R_3) + j\omega C_3]; \\ Z_4 &= -j(1/\omega C_4). \end{aligned}\right\} \qquad (6\text{-}52)$$

Substituting Eq. (6-52) in the general balance equation

$$Z_1Z_3 = Z_2Z_4$$

and assuming the unknown placed in the first arm we obtain:

$$R_1 - j(1/\omega C_1) = -jR_2/\omega C_4(1/R_3 + j\omega C_3);$$

$$R_1 - j(1/\omega C_1) = R_2R_3/C_4 - jR_2/\omega C_4R_3. \qquad (6\text{-}53)$$

Equating the real and imaginary parts of Eq. (6-53) separately, we obtain two independent equations:

$$R_1 = R_2(C_3/C_4); \quad C_1 = C_4(R_3/R_2). \qquad (6\text{-}54)$$

Thus, Eq. (6-54) gives us the values of the unknown C_1 and R_1 simultaneously. Since, however, this bridge is mainly used for the testing of insulation, what interests us in the final analysis is not the effective resistance R_1, but the tangent of the loss angle:

$$\tan\delta_1 = \omega C_1R_1.$$

Substituting R_1 and C_1 from Eq. (6-54) we have

$$\tan\delta_1 = \omega C_4(R_3/R_2)R_2(C_3/C_4) = \omega C_3R_3.$$

Thus, the final form of the design formulas for the bridge are

$$C_1 = C_4(R_3/R_2) \text{ and } \tan\ \delta_1 = \omega C_2R_3.$$

It is obvious that the best way to obtain a balance is to keep R_3 and C_4* fixed and to adjust R_2 and C_3. Then, independent measurement is obtained. Indeed, C_3 does not enter the expression for C_1 and only defines tan δ_1, while the opposite is true of R_2. Ordinarily, the value of the fixed R_3 is

$$R_3 = 10{,}000/\pi = 3{,}183 \text{ ohms, approx.}$$

Then, if the bridge operates on commercial frequency (which practically is always the case) for which

$$\omega = 2\pi f = 100\pi,$$

and the capacitance C_3 is in farads, the expression for the loss angle will take the form:

$$\tan\delta_1 = 100\pi(10{,}000/\pi)C_3 = 10^6C_3;$$

if C_3 must be expressed in microfarads, then

$$\tan\delta_1 = C_3.$$

---

* C_4 is a standard high-voltage capacitor which is extremely difficult to make variable.

Since the Shering bridge measures independently C_1 and $\tan \delta_1$, we may conclude from the foregoing (see Sec. 6-4) that here, too,

$$\gamma_c = \varphi_1,$$

i. e., the convergence angle, γ_c, as in the previous case, is equal to the phase angle, φ_1, of the unknown impedance.

From a comparison of the upper branches in the networks of Figs. 6-27 and 6-29 we can see that the coefficient A in both cases is the same. Consequently, the sensitivity of the Shering bridge can be found from Eq. (6-49).

Exactly the same approach will yield the balance equation and design formulas for any of the 26 elementary four-arm a.c. bridge circuits shown in Table 6-1. For reference, only the final design formulas are given, arranged in a table similar to Table 6-1. For any circuit arrangement, the design formulas will be found in the same place of Table 6-2 as in Table 6-1.

Reference to Tables 6-1 and 6-2 will show that one and the same component can be measured with many circuit arrangements. It is essential, therefore, to establish criteria for the selection of a circuit for each particular case.

It should be qualified from the outset that the selection of a particular method and circuit arrangement is a fairly complicated matter affected by a variety of special factors. This is where the experience and skills of the operator count most, especially if the measurement is not a routine one. Therefore, we can only give a very broad outline of the possible approaches, leaving the final decision with the experimentor.

What can be said in this connection may in the main be reduced to two criteria which are in most cases decisive to the selection of the requisite circuit. The first criterion, rather practical and qualitative in character, applies to the selection of the standard and is often dependent on whether or not a given facility is available. The second criterion depends on the magnitude of the component to be measured, its order being usually known, at least approximately, in advance. More specifically, in some cases, when the magnitude of the unknown is fixed beforehand, a balance with certain circuit arrangements will be secured only if some special forms of standard, are used. Such standards may prove quite unusual or inconvenient say, too large or too small.

A suitable example is provided by the choice between the series and parallel connection of a capacitance and a resistance in an equivalent circuit in the measurement of losses in dielectrics by means of ratio-arm bridges. Indeed, if the specimen has low losses, the use of the parallel connection will require a resistor of the order of

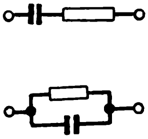
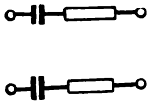
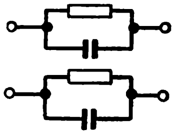
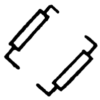
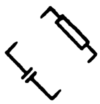
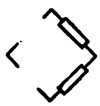
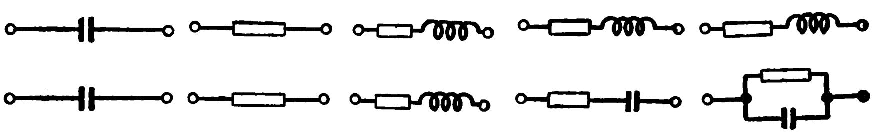
				Type and arrangement of balance-dete		
				Capacitive		
Balance-determining arms / Ratio arms				Seriesparallel	Series	Parallel
						
Type and arrangement of ratio arms	Product arm bridges	$\varphi_2+\varphi_4=$ $=0°$				
		$\varphi_2+\varphi_3=$ $=-90°$		$C_1=C_4\frac{R_3}{R_2}$ $R_1=R_2\frac{C_3}{C_4}$	$C_1=\frac{\omega^2C_3^2\times}{\omega^2C_3^2\times}$ $\frac{\times R_3^2+1}{\times R_2R_3}C_4$ $R_1=\frac{C_3\times}{(\omega^2C_3^2\times}$ $\frac{\times R_2}{\times R_3^2+1)C_4}$	$C_1=\frac{R_3\times}{(\omega^2R_3^2\times}$ $\frac{\times C_4}{\times C_3^2+1)R_2}$ $R_1=\frac{(\omega^2R_3^2\times}{^2R_3^2\times}$ $\frac{\times C_3^2+1)R_2}{\times C_3C_4}$
	Ratio arm bridges	$\varphi_2-\varphi_3=$ $=0°$		$C_1=R_3\times$ $\times\frac{\omega^2R_4^2C_4^2+1}{\omega^2R_4^2R_2C_4}$ $R_1=\frac{R_2R_4}{R_3(\omega^2\times}$ $\overline{\times C_4^2R_4^2+1)}$	$R_1=\frac{R_2R_4}{R_3}$ $C_1=\frac{R_3C_4}{R_2}$	$R_1=\frac{R_2R_4}{R_3}$ $C_1=\frac{R_3C_4}{R_2}$

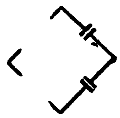
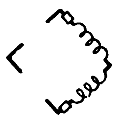
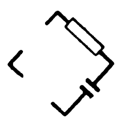
Table 6-2

rmining arms (Unknown and Standard)

Similar			Dissimilar	
Capacitive	Resistive	Inductive	Series	Seriesparallel



Capacitive	Resistive	Inductive	Series	Seriesparallel
	$R_1 = \frac{R_2R_4}{R_3}$		$L_1 = \frac{R_2R_4C_3}{1+\omega^2R_3^2C_3^2}$ $R_1 = \frac{\omega^2R_3C_3^2R_2R_4}{1+\omega^2R_3^2C_3^2}$	$R_1 = \frac{R_2R_4}{R_3}$ $L_1 = R_2R_3C_3$
$C_1 = \frac{R_3C_4}{R_2}$	$R_1 = \frac{R_2R_4}{R_3}$	$R_1 = \frac{R_2R_4}{R_3}$ $L_1 = \frac{R_2L_4}{R_3}$		

Balance-determining arms / Ratio arms			Type and arrangement of balance-deter: Capacitive — Seriesparallel	Series	Parallel
Ratio arm bridges	$\varphi_2-\varphi_3=0^\circ$		$C_1=\frac{C_2(\omega^2 C_4^2R_4^2+1)}{\omega^2R_4^2 C_3C_4}$ $R_1=\frac{C_3 R_4}{(\omega^2R_4^2 C_4^2+1)C_2}$	$C_1=\frac{C_2C_4}{C_3}$ $R_1=\frac{C_3R_4}{C_2}$	$C_1=\frac{C_2C_4}{C_3}$ $R_1=\frac{C_3R_4}{C_2}$
			when $\varphi_3=\varphi_2$ $C_1=\frac{\omega^2C_4^2R_4^2+1}{\omega^2R_4^2C_4}\times\frac{L_3}{L_2}$ $R_1=\frac{R_4}{\omega^2C_4R_4^2+1}\cdot\times\frac{L_2}{L_3}$	when $\varphi_3=\varphi_2$ $C_1=C_4\frac{L_3}{L_2}=C_4\frac{R_3}{R_2}$ $R_1=R_4\frac{L_2}{L_3}=R_4\frac{R_2}{R_3}$	when $\varphi_3=\varphi_2$ $C_1=C_4\frac{L_3}{L_2}=C_4\frac{R_3}{R_2}$ $R_1=R_4\frac{L_2}{L_3}=R_4\frac{R_2}{R_3}$
	$\varphi_2-\varphi_4=+90^\circ$				

(Continued)

mining arms (Unknown and Standard)				
Similar			Dissimilar	
Capacitive	Resistive	Inductive	Series	Seriesparallel
$C_1 = \frac{C_2 C_4}{C_3}$	$R_1 = \frac{C_3 R_4}{C_2}$	$R_1 = \frac{C_3 R_4}{C_2}$ $L_1 = \frac{R_3 L_4}{R_2}$		
$C_1 = \frac{R_3}{R_2} C_4 =$ $= \frac{L_3}{L_2} C_4$	$R_1 = \frac{R_2}{R_1} R_4 =$ $= \frac{L_2}{L_3} R_4$	$R_1 = \frac{L_2}{L_3} R_4 =$ $= \frac{R_2}{R_3} R_4$ $L_1 = \frac{L_2}{L_3} L_4 =$ $= \frac{R_2}{R_3} L_4$		
			$L_1 = R_2 R_4 C_3$ $R_1 = \frac{C_3}{C_4} R_2$	$L_1 = \frac{R_2 R_4 C_3}{\omega^2 C_4^2 R_4^2 - 1}$ $R_1 = \frac{\omega^2 C_3 C_4 R_2 R_2^2}{\omega^2 C_4^2 R_4^2 - 1}$

several hundred thousand ohms, which is obviously an inconvenience. It is not hard to see that the series connection will be more advantageous. If it is expected that the specimen has high losses, the opposite will be the case. Therefore, in selecting a circuit arrangement, it is a good plan to determine, at least approximately, the magnitude of the component values involved and whether or not the selected circuit is feasible.

The above two criteria may of course serve only as "rough guides". If they fail to give an unambiguous answer, resort has to be made to a comparison of the competing circuits for sensitivity, the possibility of independent measurement and accuracy. It should be noted that, whenever possible, preference should be given to a circuit whose balance condition is not, explicitly, dependent on frequency.

In any case, the goal in selecting a bridge network should be utmost simplicity. As the complexity and the number of components of a bridge increase, sources of errors also increase in number, and the measuring circuit is exposed to stronger stray disturbances. This is the reason why there is a well-established tradition in electrical measurements to make ratio arms as single and pure quantities. The most difficult thing is to construct a pure inductance. On account of this, inductance-containing arrangements are practically nonexistent (except, of course, cases where an inductance is to be measured). Capacitive circuits usually yield good results.

To sum up, it will be good practice to construct an a.c. measuring circuit out of resistances and capacitances, while keeping inductances in cases where they are to be measured or compared, or where it is impossible, for one reason or another, to get rid of them.

Our discussion would be incomplete without mention being made of some special forms of bridge networks employed on a limited scale and omitted from our classification. To begin with, there is a rather ingenious arrangement used for composite comparisons. Known as a resonance bridge, it has three purely resistive arms, the fourth being made up of a capacitance and an inductance series connected; either the capacitance or the inductance must be adjustable.

Diagrammatically, such a bridge is shown in Fig. 6-30. The variable component (which may be L_1 or C_1) is adjusted until a resonance is obtained. After that the first arm acts as a pure resistance, and the entire network is actually turned into a resistive four-arm Wheatstone bridge. Securing a balance of the bridge as a whole we may write on the basis of the balance equation for the bridge and the resonance condition for the first arm that

$$\omega L_1 = 1/\omega C_1; \quad R_1 = R_2 R_4 / R_3.$$

Of course, the inductance and the capacitance can also be connected in parallel, in which case current resonance may be utilized.

Resonance bridges have found quite a number of special applications: as sensitive frequency meters, in voltage and current waveform analysis, frequency stability measurements, etc. However, for the measurement of capacitances and inductances they are employed but seldom.

As a closing remark in our discussion of a.c. four-arm bridges, it should be noted that the space available has not permitted an

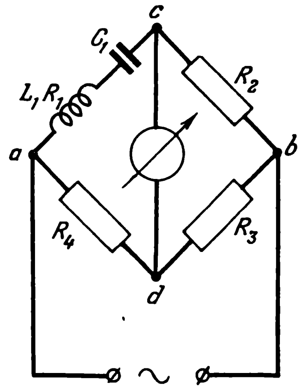


Fig. 6-30

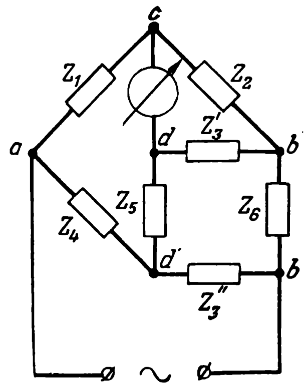


Fig. 6-31

analysis to be made of the properties and applications of actual bridge networks. The reader is advised to turn to the technical literature for more information on the subject.

Multiple-arm Bridges. In most cases, these are six-arm networks, the seven-arm variety being very rare.

For all dissimilarity, multiple-arm bridges may be regarded as special cases of ordinary four-arm bridges. By suitable mesh-to-star conversions (also known as wye-delta transformations) they can be reduced to the four-arm type.

Figure 6-31 gives the connections of a seven-arm bridge. A seven-arm bridge can be reduced to a six-arm one by making the impedance Z_5 or Z_6 equal to zero. The six-arm bridges thus obtained are shown in Figs. 6-32 and 6-33.

The general circuit arrangements of Figs. 6-31, 6-32 and 6-33 are embodied in actual seven- and six-arm networks, such as shown in Figs. 6-34 through 6-39.

An analysis of a multiple-arm bridge (determination of the balance condition, sensitivity, convergence, etc.) can be performed either on the basis of the theory of four-terminal networks or by reducing a given bridge to a four-arm arrangement which can be then treated

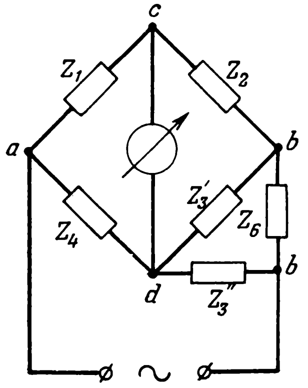


Fig. 6-32

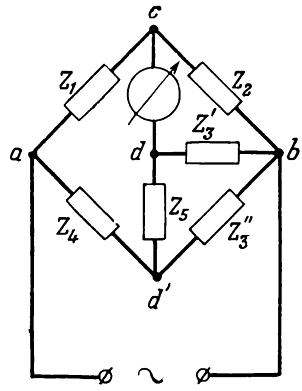


Fig. 6-33

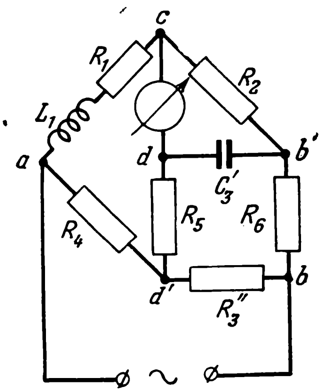


Fig. 6-34

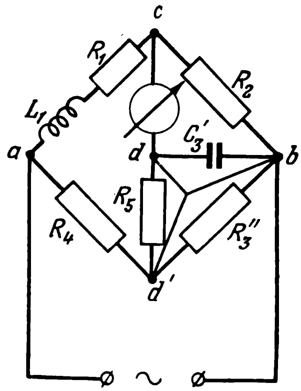


Fig. 6-35

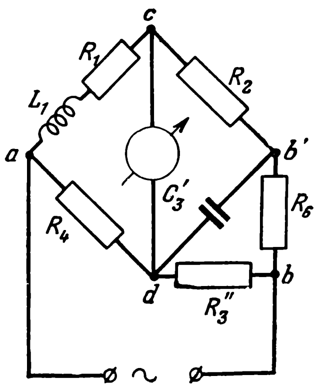


Fig. 6-36

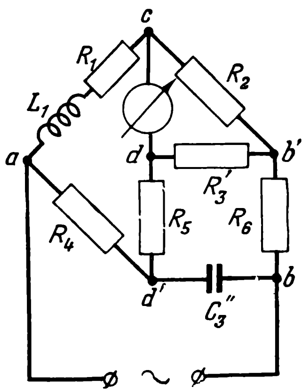


Fig. 6-37

by any of the classical methods. Other methods are used relatively seldom.

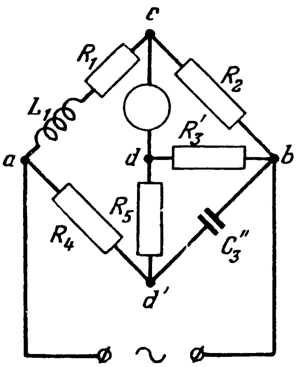


Fig. 6-38

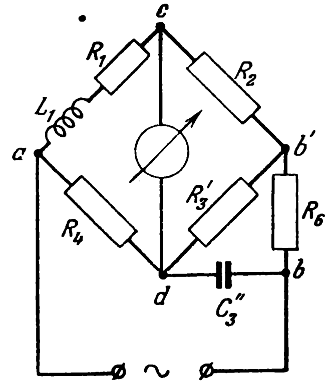


Fig. 6-39

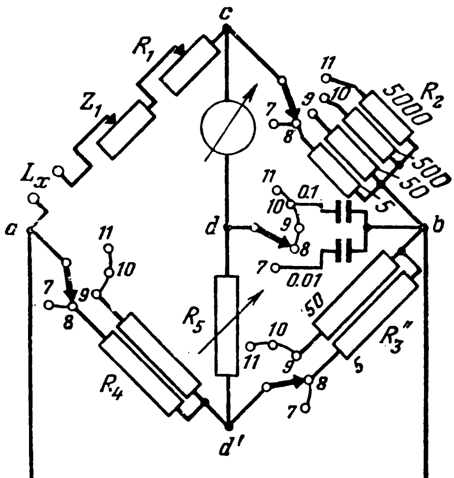


Fig. 6-40

As an example, we shall analyze the six-arm arrangement employed in the Type P50-1 bridge for the measurement of inductances. This arrangement is shown in Fig. 6-40 * and is similar to the one shown in Fig. 6-35 (it is often referred to as the Anderson bridge). The inductor (L_x, R_x) to be measured is connected across the terminals L_x. The bridge is balanced by adjustment of R_1

* The reference numbers are the same as on the switch panel of the actual P50-1 bridge.

and R_5. The range of the bridge is selected by means of a switch which brings in circuit R_2, R_3'', R_4 and C_3', whenever necessary.

First, determine the balance condition for the bridge. To this end, we shall transform the mesh C_3', R_3'', R_5 (Fig. 6-35) into a star. By analogy with the mesh junctions d, d' and b, let the branches of the star be designated by Z_d, $Z_{d'}$, and Z_b. After the conversion, it will be easily noticed from the circuit of Fig. 6-35 that the balance condition is given by

$$H = R_2 (R_4 + Z_{d'}) - Z_1 Z_b = 0. \qquad (6\text{-}55)$$

Substituting for the impedances of the star branches in Eq. (6-55):

$$Z_b = \frac{-j\,(1/\omega C_3')\,R_3''}{R_3'' + R_5 - j\,(1/\omega C_3')}\,; \quad Z_{d'} = \frac{R_3'' R_5}{R_3'' + R_5 - j\,(1/\omega C_3')},$$

we get

$$H = R_2 R_4 + \frac{R_2 R_3'' R_5}{R_3'' + R_5 - j\,(1/\omega C_3')} + \frac{Z_1\,(j/\omega C_3')\,R_3''}{R_3'' + R_5 - j\,(1/\omega C_3')} = 0.$$

Rewriting, we have:

$$H = R_2 R_4 R_3'' + R_2 R_4 R_5 + R_2 R_3'' R_5 + j\,(Z_1 R_3''/\omega C_3') - j\,(R_2 R_4/\omega C_3') = 0. \qquad (6\text{-}56)$$

Noting that for the circuit of Fig. 6-40, $R_3'' = R_4$ in all cases and also that $Z_1 = j\omega L_x + R_x + R_1$ we obtain from Eq. (6-56) the following expressions for L_x and R_x*:

$$L_x = R_2 C_3'\,(2R_5 + R_4); \qquad R_x = R_2 R_4 / R_3'' - R_1. \qquad (6\text{-}57)$$

Now determine the convergence angle of the bridge. Since R_1 and R_5 are adjustable, we have according to Eq. (6-44) from Eq. (6-56):

$$\gamma_c = \arg\,[j\,(R_3''/\omega C_3')\,(\partial Z_1/\partial R_1)] - \arg\,(R_2 R_4 + R_2 R_3'') = \pi/2$$

$$\gamma_c = \pi/2. \qquad (6\text{-}58)$$

From Eq. (6-58) it follows that the bridge has a good and unvarying convergence over the entire range.

The relative sensitivity of the bridge in terms of the unknown quantity is given by

$$\dot S_V^0 = \dot V_{ab}\,A/(1 + A)^2.$$

Substituting for A:

$$Z_1/Z_2 = (j\omega L_x + R_x + R_1)/R_2,$$

we obtain

$$\dot S_V^0 = \dot V_{ab}\,\frac{1}{R_2/(j\omega L_x + R_x + R_1) + 2 + (j\omega L_x + R_x + R_1)/R_2}\,. \qquad (6\text{-}59)$$

From this it may be concluded that the analysis technique is the same for both the four- and multiple-arm bridges.

* The P50-1 bridge has no provision for the measurement of the loss-balancing resistance R_x.

Mutual-inductance Bridges. This type of bridge is rather common in electrical measurements. By this we mean not so much the conventional applications where mutual inductances are measured (such as in the testing of mutual inductors, mutual-inductance transducers for the measurement of nonelectrical quantities, etc.), as the special types which are superior to existing conventional models as far as stray-effect control, stability and other performance characteristics are concerned. For some time past, they have been winning ever more ground.

We shall examine several forms of mutual-inductance bridges. To begin with, here is a brief outline of two forms employed for the

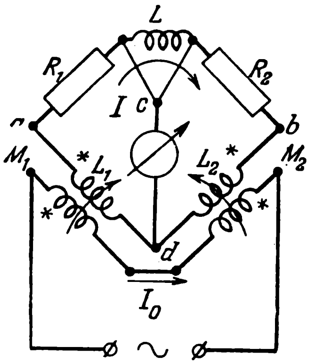


Fig. 6-41

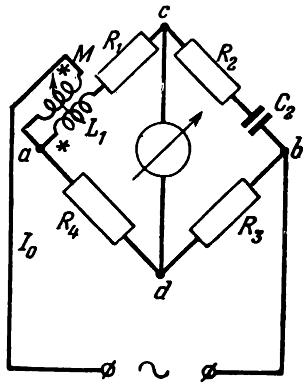


Fig. 6-42

measurement of mutual inductances. One of them is the Maxwell mutual inductance bridge shown in Fig. 6-41. It contains two mutual inductances M_1 and M_2 whose secondaries have self-inductances L_1 and L_2. The primaries and secondaries in both inductors must be connected identically (i.e., either aiding or opposing) in order that the same effect due to the e.m.f.s of self and mutual inductance can be obtained in both coils. Balance can be obtained by variation of either R_1 or R_2. The bridge also contains a small inductor L which serves to balance the inductances L_1 and L_2. The balance condition for this bridge can be conveniently obtained by the circulating-current method. Assuming that the bridge is at balance, we have, using the notation of Fig. 6-41:

$$(R_1 + j\omega L_1)\,\dot{I} - j\omega M_1 \dot{I}_0 = 0;$$

$$(R_2 + j\omega L_2)\,\dot{I} - j\omega M_2 \dot{I}_0 = 0.$$

From these equations it follows that

$$\frac{R_1+j\omega L_1}{R_2+j\omega L_2}=\frac{M_1}{M_2},$$

and finally

$$M_1/M_2=R_1/R_2=L_1/L_2.$$

Where a standard mutual inductor is not available, use may be made of the bridge of Fig. 6-42.* In this bridge, the mutual inductance is measured in terms of a known capacitance. The balance condition for this bridge can be derived on the assumption that at balance the voltage drops across the arms are equal, or $\dot{V}_1=\dot{V}_4$ and $\dot{V}_3=\dot{V}_2$.

In our case,

$$\dot{I}_1(R_1+j\omega L_1)-\dot{I}_0 j\omega M=\dot{I}_4R_4;$$

$$\dot{I}_1\left(R_2-j\frac{1}{\omega C_2}\right)=I_4R_3.$$

Substituting

$$\dot{I}_0=\dot{I}_1+\dot{I}_4$$

and eliminating the currents, we get

$$[R_1+j\omega(L_1-M)]R_3=\left(R_2-j\frac{1}{\omega C_2}\right)(R_4+j\omega M).$$

Equating the real and imaginary components separately and solving the respective equations, we obtain

$$R_1R_3-R_2R_4=M/C_2;$$

$$L_1R_3+(1/\omega^2C_2)R_4=(R_2+R_3)M.$$

If we now assume $R_4=0$ (as is usually done), the above expressions will be simplified to the form:

$$R_1R_3=M/C_2;$$

$$L_1R_3=(R_2+R_3)M.$$

Consequently, when $R_4=0$, the balance of the bridge will be independent of supply frequency. It is interesting to note that because of the effect of mutual inductance the impedance

$$Z_2=R_2-j(1/\omega C_2)$$

of the arm opposite to R_4 does not disappear from the balance condition even when $R_4=0$.

---

* Known as the Heydweiller or Carey-Foster bridge.-*Tr.*

This bridge can be used to measure both M (in which case balancing is done by variation of R_1, R_2 or C_2) and C_2. In the latter case, balance must be secured by adjustment of M and R_2.

Now we shall examine the most interesting form of mutual inductance bridges—those with tight inductive coupling between adjacent arms. Sometimes, individual types of these bridges are referred to as inductive-ratio bridges.

The inductive coupling between two coils (L_n and L_m) is said to be tight when the coupling factor k_c given by the relationship $k_c = M/\sqrt{L_n L_m}$ differs but little from unity, i.e., when we may assume $M = \sqrt{L_n L_m}$.

The underlying principle of inductive-ratio bridges has been known since long. However, their characteristics and the possibility of constructing relatively simple circuits of extremely high performance have been investigated only recently.

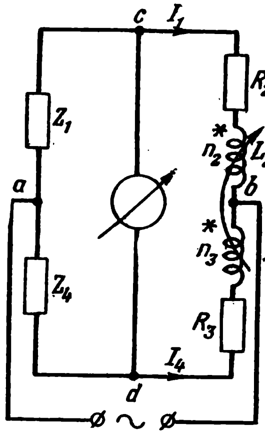


Fig. 6-43

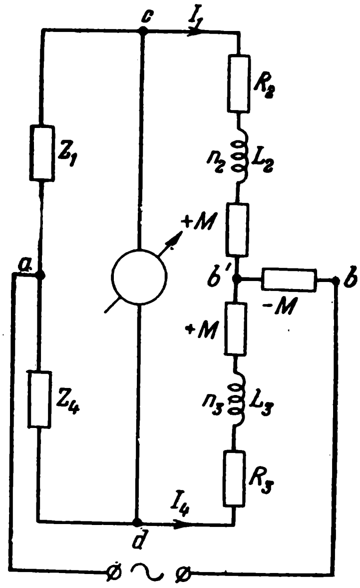


Fig. 6-44

The principal advantages of inductive-ratio bridges are the stable ratio of the inductively-coupled arms, its independence of temperature variations, no ageing, and also simple shielding against stray coupling between the components of the measuring circuit.

We shall examine three basic circuits of inductive-ratio bridges, namely parallel, series and series-parallel.

Figure 6-43 gives the connections of a bridge with tightly coupled arms connected in parallel. Z_1 is the unknown complex impedance,

Z_4 is a standard, L_2 and L_3 tightly coupled inductors wound on a common ferromagnetic core, R_2 and R_3 are the resistances inherent in the coil windings (the core losses are disregarded). The windings are connected in parallel opposition, i.e., the e.m.f.s of mutual induction oppose the e.m.f.s due to L_2 and L_3. The bridge is balanced by variation of Z_4. The requisite range is selected by varying the number of turns, n_2 and n_3, of the coils.

Determine the balance condition for the bridge. To this end, we shall use the conversion of the mesh comprising L_2, L_3 and M into a star. Noting the connection of the coils and the direction of the currents $\dot{I}_1$ and $\dot{I}_4$ with respect to the nodal point b, the voltage drops $\dot{V}_{cb}$ and $\dot{V}_{db}$ across the circuit of Fig. 6-43 will be given by

$$\left.\begin{aligned} \dot{V}_{cb} &= \dot{I}_1(R_2+j\omega L_2)-j\omega M\dot{I}_4; \\ \dot{V}_{db} &= \dot{I}_4(R_3+j\omega L_3)-j\omega M\dot{I}_1. \end{aligned}\right\} \tag{6-60}$$

Denoting the impedances of the partial circuits cb', db' and bb' in the star cbd of the equivalent circuit (Fig. 6-44) by Z_{cb}', Z_{db}', and Z_{bb}' the equations for the voltage drops $\dot{V}_{cb}$ and $\dot{V}_{db}$ in this circuit will be

$$\left.\begin{aligned} \dot{V}_{cb} &= \dot{I}_1Z_{cb'}+(\dot{I}_1+\dot{I}_4)Z_{bb'} = \dot{I}_1(Z_{cb'}+Z_{bb'})+\dot{I}_4Z_{bb'}; \\ \dot{V}_{db} &= \dot{I}_4Z_{db'}+(\dot{I}_1+\dot{I}_4)Z_{bb'} = \dot{I}_4(Z_{db'}+Z_{bb'})+\dot{I}_1Z_{bb'}. \end{aligned}\right\} \tag{6-61}$$

The conditions for the equivalence of the circuits cbd of Figs. 6-43 and 6-44 will be fulfilled, if the coefficients affecting the identical currents in Eqs. (6-60) and (6-61) are equal. Then

$$\begin{aligned} Z_{bb'} &= -j\omega M; \\ Z_{cb'} &= R_2+j\omega L_2-Z_{bb'} = R_2+j\omega L_2+j\omega M; \\ Z_{db'} &= R_3+j\omega L_3-Z_{bb'} = R_3+j\omega L_3+j\omega M. \end{aligned}$$

Thus, the equivalent circuit of the inductive-ratio bridge of Fig. 6-43 is in effect an ordinary four-arm bridge (Fig. 6-44) with an equivalent negative mutual inductance, $-M$, placed in the source circuit. The second and third arms, in addition to L_2, L_3, R_2 and R_3, also contain equivalent positive mutual inductances M. Referring to Fig. 6-44, the balance condition for the bridge is:

$$H = Z_1(R_3+j\omega L_3+j\omega M)-Z_4(R_2+j\omega L_2+j\omega M)=0. \tag{6-62}$$

For the tightly coupled L_2 and L_3 we have:

$$M=\sqrt{L_2L_3}. \tag{6-63}$$

Substituting Eq. (6-63) in Eq. (6-62) gives

$$H = Z_1(R_3 + j\omega L_3 + j\omega\sqrt{L_2L_3}) - Z_4(R_2 + j\omega L_2 + j\omega\sqrt{L_2L_3}) = 0. \tag{6-64}$$

Different relationships may exist between the resistances R_2 and R_3 and the inductances L_2 and L_3 of the coils. If so desired, it would be possible to connect some additional noninductive resistances in series with the coils. Assume that the resistances R_2 and R_3 are so adjusted that the relationships

$$\left.\begin{aligned} R_2 &= a\sqrt{L_2}, \\ R_3 &= a\sqrt{L_3}, \end{aligned}\right\} \tag{6-65}$$

where a is a proportionality factor, are always fulfilled.

Substituting Eq. (6-65) in Eq. (6-64) gives

$$H = Z_1\sqrt{L_3}(a + j\omega\sqrt{L_3} + j\omega\sqrt{L_2}) - Z_4\sqrt{L_2}(a + j\omega\sqrt{L_2} + j\omega\sqrt{L_3}) = 0.$$

Eliminating and rationalizing, we obtain

$$H = Z_1\sqrt{L_3} - Z_4\sqrt{L_2} = 0. \tag{6-66}$$

Since, in the case of tight coupling, we may also assume that

$$\sqrt{L_2}/\sqrt{L_3} = n_2/n_3,$$

where n_2 and n_3 are the turns of the coils L_2 and L_3, respectively, we finally obtain the following balance condition:

$$H = Z_1 n_3 - Z_4 n_2 = 0. \tag{6-67}$$

The unknown complex impedance Z_1 is given by

$$Z_1 = Z_4(n_2/n_3), \tag{6-68}$$

i.e., it is equal to the complex impedance of the standard arm Z_4 times the real factor n_2/n_3 which gives the turns ratio of the coils L_2 and L_3. It is obvious that Z_4 can most conveniently be used as an adjustable element for securing a balance, while the range can be selected by varying the number of turns, n_2 and n_3. It should be noted that variation of the ratio n_2/n_3 can successfully be employed for balancing some special bridge networks. Referring to Eq. (6-68), the bridge can be used only for comparison of impedances having the same phase angle.

The coupling between the bridge arms can be made sufficiently tight, if the toroidal cores for the coils L_2 and L_3 are made of Permalloy or any other ferromagnetic material with a high residual permeability and low losses, and also if the coils are suitably placed

with respect to each other. The coupling factor, k_c, can then be brought very close to unity (k_c=0.999 to 0.9999). In certain cases, the discrepancy can be made even smaller. With such coupling, Eq. (6-68) is fulfilled to a very close approximation. Z_1 and Z_4 in this case can be compared accurate to within a few thousandths of one per cent. Neither variations in ambient temperature nor the ageing of the coil cores affect the accuracy. Another interesting feature about the bridge of Fig. 6-43 is that at balance the difference of potentials between the points b, c and d is practically zero (in fact it is equal to the voltage drops across the effective resistances R_2 and R_3 of the coils, which are very small). If the point b is earthed, the detector circuit has a practically zero potential with respect to earth—a factor which greatly simplifies the shielding of the bridge.

The relative sensitivity of the bridge is given by

$$\dot{S}_V^0 = \dot{V}_{ab'} \frac{A}{(1+A)^2},$$

where A, according to Fig. 6-44, is

$$A = \frac{Z_1}{R_2 + j\omega (L_2 + M)}.$$

The voltage $\dot{V}_{ab}'$ depends on the supply voltage $\dot{V}_{ab}$ and the ratio of $-j\omega M$ connected in the source circuit of the bridge to the input impedance of the equivalent circuit of the four-arm bridge of Fig. 6-44, as measured relative to points ab'.

The convergence angle is given by

$$\gamma_c = \arg \frac{\partial H}{\partial p} - \arg \frac{\partial H}{\partial q}, \tag{6-69}$$

where p and q are the adjustable components of the impedance Z_4. Substituting Eq. (6-67) in Eq. (6-69), we obtain

$$\gamma_c = \arg \frac{\partial Z_4}{\partial p} - \arg \frac{\partial Z_4}{\partial q}.$$

Thus, the convergence angle is equal to the angle between the vectors of changes in the impedance Z_4 for changes in p and q, which agrees well with the statement in Sec. 6-4.

Consider series inductive-ratio bridges. The connections of one such bridge are given by Fig. 6-45. Z_1 is the unknown impedance, Z_2 is a standard, L_3 and L_4 are tightly coupled inductors wound on a common ferromagnetic core, while R_3 and R_4 are the resistances of the coils (as before, the core losses are disregarded). The coil windings are connected in series aiding, i.e., the e.m.f. of mutual inductance adds to the e.m.f.s due to L_3 and L_4. The bridge is

balanced by variation of the components of Z_2. The range is selected by varying the numbers of turns, n_3 and n_4.

Determine the balance condition for the bridge. To this end, as in the previous case, use will be made of the conversion of the mesh

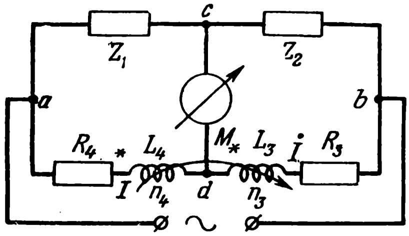


Fig. 6-45

comprising L_3, L_4 and M into a star. Noting the connection of the coils L_3 and L_4, the conversion produces the equivalent circuit of Fig. 6-46, i.e., an ordinary four-arm bridge with a negative mutual inductance $-M$ placed in the detector circuit. The third and fourth arms of the circuit, in addition to L_3, R_3, L_4 and R_4, also

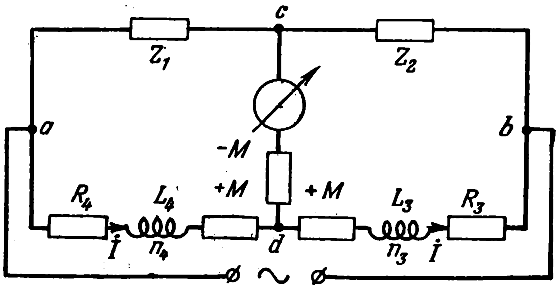


Fig. 6-46

contain equivalent positive mutual inductances of the value M. Referring to Fig. 6-46, the balance condition for the bridge is given by:

$$H = Z_1(R_3 + j\omega L_3 + j\omega M) - Z_2(R_4 + j\omega L_4 + j\omega M) = 0. \quad (6\text{-}70)$$

Since Eq. (6-70) is analogous with Eq. (6-12), then for tight inductive coupling, i.e., when $M = \sqrt{L_3 L_4}$ and also when $R_3 = a\sqrt{L_3}$ and $R = a\sqrt{L_4}$, the result will be identical with Eq. (6-67). Consequently, rationalizing and noting the above conditions, we obtain:

$$H = Z_1 n_3 - Z_2 n_4 = 0. \quad (6\text{-}71)$$

The unknown complex impedance is given by

$$Z_1 = Z_2 (n_4/n_3). \tag{6-72}$$

The impedance Z_2 is made adjustable. The ranges are selected by varying the turns ratio n_4/n_3. As follows from Eq. (6-72), the bridge is only suitable for comparison of impedances having the same phase angle. As for overall performance, this bridge is similar to the parallel inductive-ratio bridge. Shielding is also easy to accomplish, since the stray capacitances, etc., between the points *a* and *d*, *b* and *d*, as distinct from ordinary bridges, have but a very negligible effect on the accuracy with which Eq. (6-72) is fulfilled.

The relative sensitivity of the bridge of Fig. 6-45 is given by the same expression as that of an ordinary four-arm bridge:

$$\dot{S}_V^0 = \dot{V}_{ab} \frac{A}{(1+A)^2}.$$

Since $A = Z_1/Z_2 = n_4/n_3$, then:

$$\dot{S}_V^0 = \dot{V}_{ab} \frac{1}{Z_1/Z_2 + 2 + Z_2/Z_1} = \dot{V}_{ab} \frac{1}{n_4/n_3 + 2 + n_3/n_4}. \tag{6-73}$$

From Eq. (6-73) it follows that the maximum sensitivity will be obtained when $n_3 = n_4$. When this condition is fulfilled,

$$\dot{S}_V^0 = \dot{V}_{ab}/4.$$

As in the previous case, the convergence angle, γ_c, of this bridge is given by

$$\gamma_c = \arg \frac{\partial Z_2}{\partial p} - \arg \frac{\partial Z_2}{\partial q},$$

where p and q are the adjustable components of the complex impedance Z_2. Consequently, the convergence angle, γ_c, is equal to the angle between the vectors of changes in Z_2 for changes in p and q.

It should be emphasized that for all outward similarity, parallel inductive-ratio bridges differ from series-connected in the state of the core at balance. In the former (Fig. 6-43), the core is practically demagnetized, since the fluxes due to mutual and self-induction oppose each other. In the latter (Fig. 6-45), where these fluxes are aiding each other, the core is always magnetized (including when the bridge is at balance).

A few words should be said about double inductive-ratio bridges with a series-parallel combination of the tightly coupled arms. One

such bridge is shown in Fig. 6-47. For simplicity, it is assumed that the resistances of all the inductance coils are equal to zero. In most cases such an assumption does not introduce any appreciable error in the final result, since inductors are always made so as to keep their resistive component to a minimum. Z_1 is the unknown; Z_4, which is made adjustable, is a standard. The third and fourth arms of the bridge are tightly coupled inductors L_2 and L_3. The coils L_3' and L_4' serve the same purpose as L_3 and L_4 in the bridge of Fig. 6-45 in which the tightly coupled arms are series-connected: they act as a precise voltage divider with a division factor practically independent of the load on the individual arms within certain limits.

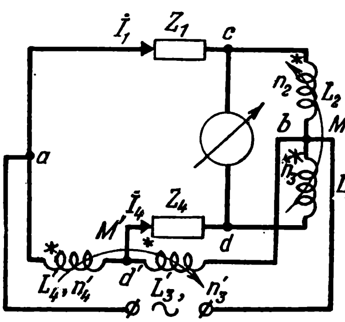


Fig. 6-47

Determine the balance condition for the bridge. At balance, the difference of potential between the points c and d (Fig. 6-47) is zero. It may be shown that in this case (with L_2 and L_3 tightly coupled and their resistance being zero), the voltages $\dot{V}_{cb}$ and $\dot{V}_{db}$ are also equal to zero. On account of this, at balance the currents $\dot{I}_1$ and $\dot{I}_4$ passing through the impedances Z_1 and Z_4 are conditioned solely by these impedances and the voltages $\dot{V}_{ab}$ and $\dot{V}_{a'b}$, i.e.,

$$\dot{I}_1 = \dot{V}_{ab}/Z_1; \quad \dot{I}_4 = \dot{V}_{d'b}/Z_1,$$

whence

$$Z_4/Z_1 = (\dot{I}_1/\dot{I}_4)(\dot{V}_{ab}/\dot{V}_{d'b}). \tag{6-74}$$

Also, since $\dot{V}_{cb}$ and $\dot{V}_{db}$ are equal to zero when L_2 and L_3 are traversed by definite currents $\dot{I}_1$ and $\dot{I}_4$, this implies that (with the resistances of the inductors equal to zero) at balance the magnetic fluxes in the coils balance each other and, consequently, $\dot{I}_1 n_2 = \dot{I}_4 n_3$, or

$$\dot{I}_4/\dot{I}_1 = n_2/n_3. \tag{6-75}$$

Substituting Eq. (6-75) in Eq. (6-74) gives

$$Z_1/Z_4 = (n_2/n_3)(\dot{V}_{ab}/\dot{V}_{d'b}). \tag{6-76}$$

When L_3' and L_4' are tightly coupled (as in the previous case, their resistances may be disregarded), the ratio of $\dot{V}_{ab}$ to $\dot{V}_{d'b}$ will

be equal to the respective turns ratio, or

$$\dot{V}_{ab}/\dot{V}_{d'b}=(n_3'+n_4')/n_3'. \tag{6-77}$$

It should be noted that in the case of "ideal" coupling ($M'=\sqrt{L_3'L_4'}$) the fulfilment of Eq. (6-77) is independent of the load due to Z_1 and Z_4. It is this factor that makes series-parallel inductive-ratio bridges at least a theoretical possibility. In practice, however, "ideal" coupling can be obtained only with a certain degree of approximation. Now that new magnetic materials with an extremely high permeability have been developed, and also special methods of winding and forms of cores, coupling can be made so tight that Eq. (6-77) is fulfilled in most cases with an accuracy sufficient for all practical purposes.

Substituting Eq. (6-77) in Eq. (6-76) gives

$$Z_1/Z_4=(n_2/n_3)\cdot[(n_3'+n_4')/n_3'].$$

Substituting n_2' for $(n_3'+n_4')$, we finally obtain the balance condition:

$$Z_1/Z_4=(n_2/n_3)\,(n_2'/n_3'), \tag{6-78}$$

whence

$$Z_1=Z_4(n_2/n_3)\,(n_2'/n_3'). \tag{6-79}$$

From Eq. (6-79), it can be noted that the unknown impedance is given in terms of the standard impedance times the double turns ratio n_2/n_3 and n_2'/n_3'. Consequently, a standard of the same impedance used in a series-parallel inductive-ratio bridge will give a broader range than when used in a single bridge. This is the motive behind the popularity of the series-parallel variety especially where low capacitances are involved. Using quite reasonable standard capacitors they can reliably measure capacitances of a few thousandths or even ten-thousandths of a picofarad.

As for shielding, series-parallel bridges do not differ in principle from the bridges with parallel and series tightly coupled arms described earlier.

6-7. Quasi-balanced A.C. Bridges

For all the advantages, balanced a.c. bridges suffer from a major drawback, namely, the necessity of securing a balance, which involves ancillary operations.

The balancing procedure is sometimes complicated, calling, as it does, for certain skills on the part of the operator, and is always

time-consuming. Therefore, every encouragement must be given to any attempt to simplify it.*

Also, there are cases where instead of two components of an impedance it will suffice to determine only one, which may be magnitude, resistive or reactive component. Under such circumstances, it is desirable to have a simplified method which would measure the components independently, i.e., in such a manner that the value not measured cannot affect the state of the bridge at measurement. It may be noted in parenthesis that balanced bridges do not permit of this possibility, since a change in any of the components of the unknown inevitably upsets the balance.

The desire to simplify the measuring procedure has spurred the search, concurrently with work on the theory and design of balanced bridges, for new arrangements, perhaps not so precise, but as simple as d.c. bridges and enabling the various components to be measured independently. These new arrangements have come to be known as quasi-balanced or conditionally-balanced bridges.** As distinct from balanced bridges, at quasi-balance none of the voltages in the bridge is equal to zero. Yet, the supply voltage does not affect the state of quasi-balance.

In a quasi-balanced bridge, adjustments of the variable values give a certain relationship between the magnitudes of two voltages or a certain phase angle, most often 0, $\pi/2$ or π. As a corollary, use is made of differential detectors registering the moment when the magnitudes of two voltages are equal, or phase-sensitive detectors noting a certain phase angle between the two voltages applied to the input of the indicator.

For a better understanding of the principle and operation of such networks, we shall examine the simplest quasi-balanced bridge which can measure independently the magnitude, phase angle and resistive component of a complex impedance. The connections of the bridge are given by Fig. 6-48 where Z_1 is the unknown inductive impedance represented by a series equivalent circuit; R_2 is a variable resistance; R_3 and R_4 are two equal ratio arms. The detector is a phase-sensitive detector showing when the voltages applied to its input terminals are in quadrature.

It is obvious that, since the detector is connected in parallel with the various circuit components, its input impedance must be very high. Therefore, use is made of valve-type detectors.

The circuit of Fig. 6-48 cannot be balanced in principle, since the phase condition of balance cannot be fulfilled for the reason that there is no inductance in the second or fourth arm.

* One such simplified technique, independent adjustment, was, it will be recalled, discussed in Sec. 6-5.

** The term "semi-balanced", used at one time, is perhaps less adequate.

Figure 6-49 gives the circle diagram of the bridge. Adjustment of the variable resistance R_2 causes the point c to move along the arc acb.

For the measurement of the magnitude of the complex impedance Z_1, the voltages $\dot{V}_{dc}$ and $\dot{V}_{db}$ are applied to the input terminals

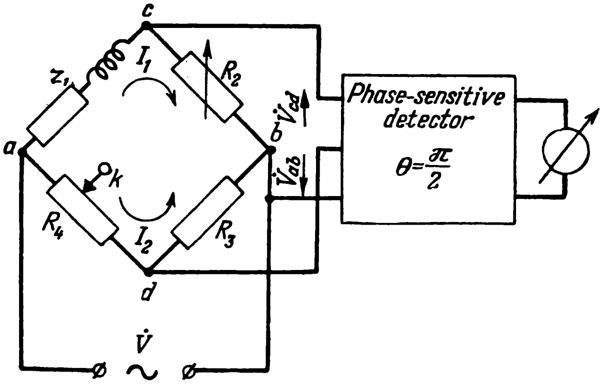


Fig. 6-48

of the phase-sensitive detector. When adjustment of R_2 brings the bridge to a quasi-balance, i.e., causes one voltage to be in quadrature with the other, it can be noted that the point c has moved to the position c' in the diagram. Since we have assumed $R_3=R_4$

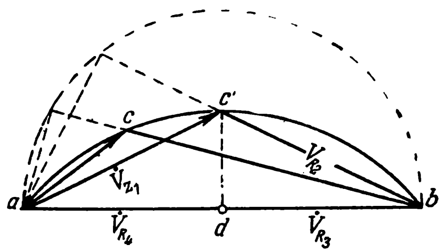


Fig. 6-49

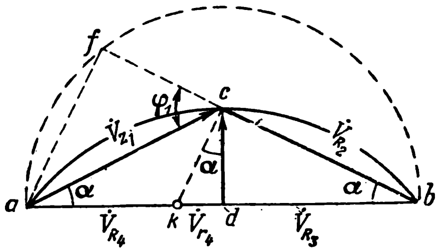
(the input current of the detector may be neglected), then the magnitude of the voltage vector $\dot{V}_{ac}$ is equal to that of the voltage vector $\dot{V}_{cb}$:

$$I_1 Z_1 = I_1 R_2$$

and

$$Z_1 = \sqrt{R_1^2 + X_1^2} = R_2.$$

Thus, using a phase-sensitive detector, it is possible to secure a quasi-balance by independent adjustment of a single component quantity, so that the magnitude of Z_1 can be obtained in terms of R_2. The polarity of the phase-sensitive detector also serves to ensure the unambiguity of the result. At a phase shift of 90°, the pointer of the meter which is of the centre-zero type, stays at zero. For shifts less than 90°, the pointer will deflect to the right, and for shifts greater than 90° to the left of the zero division—a feature which materially simplifies the circuitry of the bridge.



. Fig. 6-50

The circuit of Fig. 6-48 can also be employed for the measurement of the phase angle of a complex impedance Z_1. For this purpose, one of the resistances, R_3 or R_4, should be made adjustable (in the form of a potentiometer, say), and the phase-sensitive detector fed with the voltages $\dot{V}_{ck}$ and $\dot{V}_{cb}$. Assume that the measurement has already been taken and the vector $\dot{V}_{dc}$ makes a right angle with the supply voltage vector $\dot{V}_{ab}$. Then with $\dot{V}_{cb}$ and $\dot{V}_{ck}$ in quadrature (obtained by shifting the contact arm K of the potentiometer R_4), the circle diagram will have the form of Fig. 6-50. Referring to the diagram, when $Z_1 = R_2$, the phase angle φ_1 is twice the angle α:

$$\varphi_1 = 2\alpha.$$

In turn, the angle α can be found from the expression:

$$\tan\alpha = V_{r_4}/V_{dc},$$

and since

$$V_{dc} = V_{R_4}\tan\alpha,$$

then

$$\tan^2\alpha = V_{r_4}/V_{R_4}.$$

Noting that a steady current flows in the circuit *akd*, we finally obtain

$$\tan^2\alpha = \tan^2(\varphi_1/2) = V_{r_4}/V_{R_4} = r_4/R_4.$$

The last relationship shows that the scale of the potentiometer R_4 may be calibrated directly in units of phase angles φ.

The above arrangement is also convenient for the measurement of the resistive component R_1 of the unknown impedance represented by the series equivalent circuit. Indeed, as follows from Fig. 6-50, when $\dot{V}_{ck}$ and $\dot{V}_{cb}$ are in quadrature, the distribution of the voltages in the network is such that two meshes, *afb* and *kcb*, are formed. Consequently, the voltages can be related thus:

$$V_{R_1}/V_{R_2}=(V_{R_4}-V_{r_4})/(V_{R_3}+V_{r_4}),$$

or

$$I_1R_1/I_1R_2=I_2(R_4-r_4)/I_2(R_3+r_4),$$

whence

$$R_1=R_2(R_4'/R_3'),$$

where R_4' is the resistance between the points *ak*, and R_3' between the points *kb*.

It should be noted that while the phase angle can be measured with the network discussed only after the magnitude of Z_1 has been determined, the resistive component can be found independently, and R_4 need not be constructed as a potentiometer. In the measurement of the resistive component of Z_1, any of the three resistances R_2, R_3 or R_4 may be made adjustable.

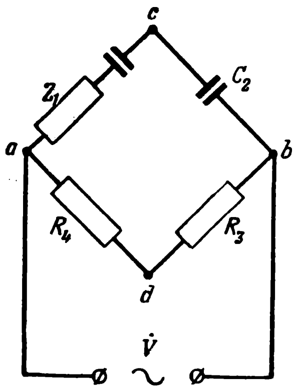


Fig. 6-51

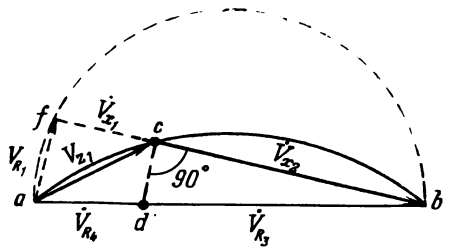


Fig. 6-52

The above method of measuring the resistive component of Z_1 shows that the measuring procedure with a quasi-balanced bridge does not differ in the least from the measurement of resistances with d. c. bridges. Assuming $R_2 = R_3$ or $R_3 = R_4$, the unknown will be directly obtained in terms of the adjustable resistance. The range of the adjustment of the variable component can be changed by changing the ratio R_2/R_3 or R_4/R_3.

In the case of a capacitive unknown, the resistance of the second arm can be replaced with a standard capacitor C_2, and the reactive component X_1 of the impedance Z_1 can then be measured (Fig. 6-51).

With $\dot{V}_{cd}$ and $\dot{V}_{cb}$ in quadrature, the distribution of the voltages in the network will be as shown in Fig. 6-52, and the condition

$$V_{x_1}/V_{x_2}=V_{R_4}/V_{R_3}$$

will be fulfilled. Hence

$$X_1=X_2\,(R_4/R_3),$$

or

$$C_1=C_2\,(R_3/R_4).$$

However, this simple method and the circuit of Fig. 6-52 are good for the measurement of only capacitive reactance for the reason that the above condition applies solely where the second arm contains a reactance free from losses. Since "pure" inductances are practically nonexistent, this method cannot be used for inductance measurements.

The reactance of inductive components can be measured with a series-parallel equivalent circuit in the standard arm, i.e., by connecting the standard in series with the unknown so that the impedance of the first arm is predominantly capacitive. Then the second arm will have also to contain a capacitance, and the above condition will be fulfilled.

So far we have been discussing quasi-balanced bridges using a quadrature phase-sensitive detector. It can be shown, however, that independent measurement can also be accomplished with a differential voltmeter capable of showing the moment when two voltages are equal in magnitude.

Suppose that instead of a quadrature phase-sensitive detector the circuit of Fig. 6-51 uses a differential voltmeter. The voltages $\dot{V}_{dc}$ and $\dot{V}_{db}$ are applied to its input terminals so that when they are equal in magnitude, the voltmeter reads zero. Then at quasi-balance

$$V_{dc}=V_{db}.$$

Using the notation of the circuit, these voltages are given by:

$$V_{dc}=V\left|\frac{Z_1R_3+jX_2R_4}{(Z_1-jX_2)\,(R_3+R_4)}\right|;$$

$$V_{db}=V\left|\frac{R_3}{R_3+R_4}\right|,$$

and the condition for quasi-balance may be written thus:

$$\left|\frac{R_1R_3+j\,(X_2R_4-X_1R_3)}{(R_3+R_4)\,[R_1-j\,(X_1+X_2)]}\right|=\left|\frac{R_3}{R_3+R_4}\right|.$$

Finding the moduli of the right- and left-hand sides, we obtain

$$\sqrt{(R_1R_3)^2+(X_2R_4-X_1R_3)^2}=\sqrt{(R_1R_3)^2+R_3^2(X_1+X_2)^2}\,,$$

whence we finally get:

$$X_1=X_2\,\frac{R_4/R_3-1}{2}\,.$$

Thus, using the circuit of Fig. 6-52 and a differential voltmeter connected as shown, independent measurement is possible of the reactive component X_1 corresponding to the series substitution circuit of the unknown Z_1, in terms of the known impedances of the bridge.

From the expression for X_1 it follows that when $R_4/R_3 = n = 3$, the unknown reactance of X_1 can be determined directly in terms of X_2. In a general case, the impedance ratio for the lower branch is chosen so as to obtain ratios convenient for practical use. For example, when $n = 1.2$, $X_1 = 0.1X_2$; when $n = 21$, $X_1 = 10X_2$; etc. It will be noted that when $n \leqslant 1$, the circuit is unsuitable for measurements.

It is obvious that the method of magnitude measurement can likewise be employed for the measurement of resistance with the same circuit, provided the resistance R_2 is placed in the second arm.

From a more detailed analysis it follows that with the magnitude method of measurement the circuit has a steadier sensitivity, depending on the ratio of the resistance to the reactance of the unknown impedance, than the same circuit in the phase method of measurement.

To sum up, we have examined several concrete examples of measuring the magnitude, resistive and reactive components, quality factor and losses of the unknown impedance. From the examination it may be concluded that securing the desired phase or magnitude relationship does not differ from obtaining a balance for a d.c. bridge.

The measurement of a component of a complex impedance is a fairly common task in practical measurements. Apparently, such circuits can be widely employed for the measurement of nonelectrical quantities, as percentage bridges for quality control in the quantity production of radio components, where the various characteristics of the part are to be determined at different stages of the manufacturing process and at different times. The flexibility of quasi-balanced bridges and the possibility of measuring various characteristics make them especially promising to different divisions of engineering.

In conclusion, it should be noted that such circuits are a sufficiently reliable and simple means for the rapid measurement of circuit

constants. As analysis and experiments show, quasi-balanced bridges, when used at audio and higher frequencies, can be accurate to within 0.5-1.0 per cent, which is within the limits specified for most technical measurements.

6-8. A. C. Percentage Bridges

In Sec. 5-6 we laid down the principles of d.c. percentage bridges and established that they are a convenient means of commercial quality control in the quantity production of resistances. Obviously, a similar problem—the checking of complex impedances—may arise in the case of a. c. bridges as well. A. c. percentage bridges can be (and are already being) used for checking capacitors and coils. Although convenient as independent measuring devices they are especially attractive as the measuring circuits of automatic grading machines which, when used on a large scale, could enhance the reliability of radio components. There seems to be enough ground for expecting that in the near future wider use will be made of a. c. percentage bridges; therefore, it appears worth while to examine the basic principles underlying them.

It should be noted that an a. c. percentage bridge is more difficult to construct than a d. c. percentage bridge. In a d. c. bridge even a small deviation in the unknown resistance from its rating upsets the balance,and the current through the detector gives a linear measure and the sign of the deviation. In an a. c. bridge, a deviation in the resistive and reactive values, either separately or together, also upsets the balance, but the voltage across (or the current in) the detector circuit cannot give an unambiguous measure of the deviation. Therefore, a. c. percentage bridges are based on deflection-type circuits.

Since the measurement of capacitors and coils involves two characteristics, the need most often arises for devices capable of determining the percentage deviation from two ratings. On the other hand, in some cases it will suffice to measure only one parameter, say, capacitance.

Alternating-current percentage bridges can be either of the limit or of the comparison type.

Limit bridges are based on the idea that the position of a definite potential point of a bridge is a function of changes in the unknown values. If this point is located within the given area of a complex plane, the unknown values are within the tolerance limits. If, on the other hand, the point lies outside the area, one or both unknown values exceed the tolerance limits. The bridge network is so constructed as to make it possible to ascertain which of the component variables has exceeded the tolerance limit.

For a better understanding of the principles underlying such bridges, we shall first examine a branch consisting of, say, the inductance coil to be measured (having a resistance R_1 and a reactance X_1) connected in series with a resistance R_2 (Fig. 6-53*a*), to which a voltage $\dot{V}$ is applied. As will be recalled (see Sec. 6-3), changes in R_1 and X_1 will cause the potential point of the bridge junction to move along definite circles (Fig. 6-53 *b*).

Let the circle M be the locus of $\dot{V}_{ac}$ ($\dot{V}_{bc}$), when X_1 is varying (X_1 = var) and R_1 is the lowest (R_1 = min), and the circle N be the locus of $\dot{V}_{ac}$ ($\dot{V}_{bc}$), when X_1 = var, and R_1 = max.

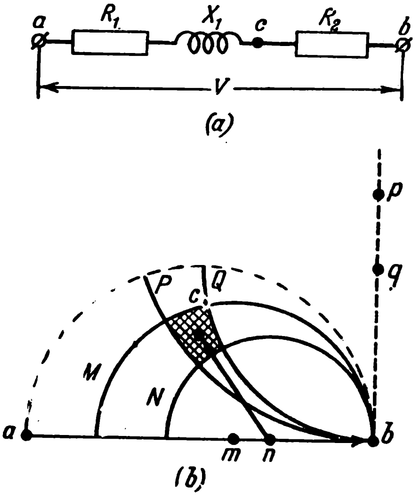


Fig. 6-53

The circles P and Q are the loci of the same voltage vectors R_1 = var., and $X_1 = X_1$ min and $X_1 = X_1$ max, respectively.

Thus, the circles M and N define the area of the permissible values of R_1, and the circles P and Q are the boundaries of the permissible values of X_1. It is obvious that the area bounded by the four circles (the shaded area in Fig. 6-53*b*) contains the potential points for which the values of R_1 and X_1 are within the tolerance limits. Therefore, if the point c is within the shaded area, the component values being measured will also be within the tolerance limits.

The fact that the point c lies within the shaded area can be ascertained by comparing the voltages corresponding to the radii of the boundary circles with the voltages between the centres of these circles and the point c.

The voltage ratios can be measured with a centre-zero differential voltmeter. The potential points required for the comparison can be obtained with the aid of voltage dividers and a phase-shifter.

Figure 6-54 gives the connections of an a. c. percentage bridge with a differential detector (*DD*), employed for testing inductance coils.

The bridge network incorporates the unknown coil (R_1, X_1) and resistances R_2, R_3, R_4 and R_5. The resistances in the lower branch

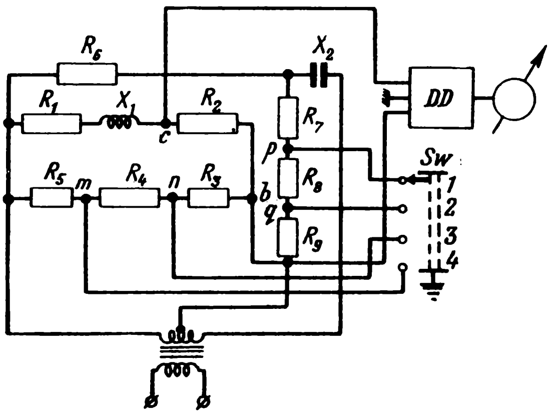


Fig. 6-54

have been chosen so that the voltage drop across R_3 is equal to the voltage across *nc* when R_1 = max, i. e., equal to the radius of the circle *N*. The voltage drop across $R_3 + R_4$ should be equal to the radius of the circle *M* (R_1 = min).

The resistor R_6 and the capacitor X_2 make up a phase-shifting network, while R_7, R_8, and R_9 constitute a voltage divider included to produce the potential points corresponding to the centres of the circles *Q* and *P*.

In finding the deviation in X_1 from the rated value, the voltage V_{pc} is compared with V_{pb} (the switch *Sw* is in position 1) and V_{qc} with V_{qb} (*Sw* in position 2). If $V_{pc} < V_{pb}$ and $V_{qb} < V_{qc}$, the reactance of the coil being tested is within the tolerance limits. When finding the deviation in R_1 from the rated value, V_{nc} is compared with V_{nb} and V_{mc} with V_{mb} (*Sw* in positions 3 and 4, respectively).

Since the measurements thus taken are of relative nature, the supply voltage does not affect the operation of the bridge, which is a major advantage. On the other hand, variations in the supply frequency are intolerable, since they may introduce considerable errors because of the presence of a frequency-dependent phas-

shifting network in the bridge and because of the fact that resistances and reactances have to be compared.

Comparison percentage bridges are mostly of the quasi-balanced type since it permits independent measurement of the component values of a complex impedance.

For determining the deviation of a single parameter, use may be made of the circuit of Fig. 6-55. In the diagram, R_1 and C_1 make up

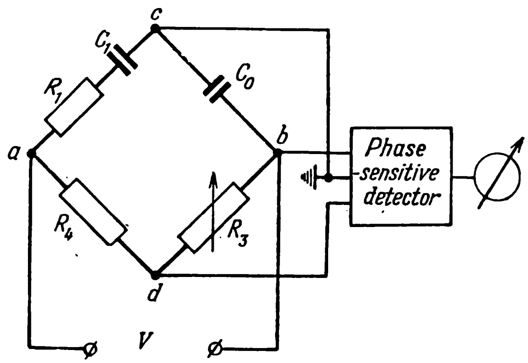


Fig. 6-55

the series equivalent circuit of the capacitor under test; C_0 is the capacitance of the standard capacitor; R_3 is a resistance box, and R_4 is a fixed resistor.

The presentation device is a phase-sensitive detector with a high input impedance, capable of indicating when the voltages $\dot{V}_{cd}$ and $\dot{V}_{cb}$ are in quadrature.

The voltages $\dot{V}_{cd}$ and $\dot{V}_{cb}$ are given by

$$\dot{V}_{cd}=\dot{V}\left(\frac{Z_1}{Z_1-jX_0}-\frac{R_4}{R_3+R_4}\right)=\dot{V}\frac{Z_1R_3+jX_0R_4}{(R_3+R_4)(Z_1-jX_0)};$$

$$\dot{V}_{cb}=\dot{V}\frac{-jX_0}{Z_1-jX_0},$$

and their ratios are:

$$\frac{\dot{V}_{cd}}{\dot{V}_{cb}}=\frac{Z_1R_3+jX_0R_4}{-jX_0(R_3+R_4)}=\frac{R_3X_1-X_0R_4+jR_1R_3}{X_0(R_3+R_4)}.$$

When $\dot{V}_{cd}$ and $\dot{V}_{cb}$ are in quadrature, the resistance in the above expression is zero, and the following condition will be fulfilled:

$$X_1=X_0(R_4/R_3) \text{ or } C_1=C_0(R_3/R_4).$$

When $C_1 = C_0$ the ratio $R_3/R_4 = 1$.

When C_1 deviates from C_0 by ΔC (i.e., $C_1 = C_0 \pm \Delta C$), R_3 is varied until $\dot{V}_{cd}$ and $\dot{V}_{cb}$ are again in quadrature and the detector gives zero deflection (i.e., until $R_3 = R_4 \pm \Delta R$). Then

$$C_1 = C_0 \pm \Delta C = C_0 (R_4 \pm \Delta R)/R_4 = C_0 \pm C_0 \Delta R/R_4.$$

Noting that $R_3 = R_4$, we obtain

$$\Delta C/C_0 = \Delta R/R_3.$$

Consequently, the fractional deviation in the capacitance C_1 from the rated value C_0 is equal to the fractional change in R_3 which can of course be calibrated directly in terms of percentage deviation.

Thus, the above circuit can measure independently the component values of a complex impedance and may therefore be used as a percentage bridge for the same purpose.

Where it is required to measure the deviation in resistances from the rated value, it will suffice to replace the capacitor in the second arm with a standard resistor R_2 having the same nominal value as R_1. Then the ratio $\dot{V}_{cd}/\dot{V}_{cb}$ will be given by

$$\dot{V}_{cd}/\dot{V}_{cb} = (Z_1 R_3 - R_2 R_4)/R_2 (R_3 + R_4),$$

and, with the two voltages in quadrature, the relationship

$$R_1 = R_2 (R_4/R_3)$$

will apply.

In the above circuits, a quasi-balance for any one component value can be secured both manually and automatically. In the latter case, the output voltage of the phase sensitive detector is fed into a servomechanism incorporating a reversible servo motor. The shaft of the servo is coupled to the variable element of the bridge network.

A somewhat different technique can be employed for measuring the deviation in both component values of a capacitor simultaneously. If, in the circuit of Fig. 6-55, C_0 is made adjustable, and $R_3 = R_4$, the magnitude of the voltage across dc at quasi-balance will give measure of tan δ of the capacitor under test.

Indeed, when the conditions $R_3 = R_4$ and $C_1 = C_0$ are fulfilled, the voltage $\dot{V}_{dc}$ is given by

$$\dot{V}_{dc} = \dot{V}\left(\frac{Z_2}{Z_1 - jX_0} - \frac{1}{2}\right) = \dot{V}\frac{R_1 - j(X_1 - X_0)}{2[R_1 - j(X_1 + X_0)]},$$

where

$$X_1 = 1/\omega C_1, \quad X_0 = 1/\omega C_0.$$

The magnitude of $\dot{V}_{dc}$ is

$$V_{dc} = V \frac{R_1}{2\sqrt{R_1^2 + 4X_1^2}},$$

and since

$$R_1/X_1 = \tan\delta_1,$$

we have

$$V_{dc} = V \frac{\tan\delta_1}{2\sqrt{\tan^2\delta_1 + 4}}.$$

Noting that for capacitors $\tan\delta$ is very small, the last expression may be given the form:

$$V_{dc} = V(\tan\delta/4).$$

Thus, in order to determine the deviation in the characteristics of a capacitor from their rated values, the standard capacitor C_0 is varied until a quasi-balance is obtained, and the sought deviation is determined in terms of C_0. At quasi-balance, the magnitude of V_{cd} gives a measure of $\tan\delta$ of the capacitor under test.

In conclusion, it should be noted that if one of the characteristics of the unknown impedance may be neglected, the current flowing through the detector when the bridge is out of balance will, as in the case of a d. c. bridge (see Sec. 5-5), be a function of the change in the variable resistor. When this change is small and the supply voltage is maintained constant, the detector current may be taken as being proportional to this change.

The forms of percentage bridges examined show the fundamental possibility of constructing a variety of test circuits for the rapid checking of radio components.

Chapter Seven

THE D. C. POTENTIOMETER METHOD

7-1. Basic Principles and Development of the Potentiometer Method

The basic idea of the potentiometer method is this. The unknown e. m. f. E_x (or voltage V_x) is applied in opposition to a known e.m.f. or balancing voltage V_k which is the voltage drop due to a definite (supply) current I_{sup} across a known resistance R_{bal} fed by a standard e. m. f. E_N (Fig. 7-1). V_k is varied until no current flows through the

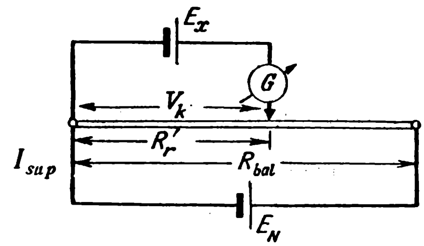


Fig. 7-1

detector. Obviously, this will happen when the unknown e. m. f. E_x and the known voltage V_k (which is a fraction of E_N) are made equal. Variation of $V_k = I_{sup}\ R_r'$ can be effected by two methods: be regulating R_r' or by regulating I_{sup}. The salient feature of the potentiometer method is that at zero deflection no current is taken from the source of the unknown e. m. f. Thus, we have every reason to state that the e. m. f. of the source under test, and not its voltage, is measured. Practically, there exists no other method to make such measurements (except for an approximate method by

means of instruments drawing very little energy, such as electrometers, valve voltmeters, etc.).

Another specific feature of the potentiometer method, due to the absence of current in the detector circuit, is that the resistance of the leads does not affect the result.

Thus, the potentiometer method is a measuring procedure which makes it possible to directly measure e. m. f. s (and also voltages). It should be recalled (see Sec. 1-1) that this feature is inherent only to the potentiometer method, since potentiometer circuits have two sources of energy, one of which may be a standard source. Bridge networks using one source are generally unsuited for this purpose, although by adding another energy source any bridge network can be converted into a potentiometer circuit without any radical changes.

As an example, we shall consider the conversion of a four-arm bridge into a potentiometer circuit. For this purpose, the unknown e. m. f. should be connected in series with the galvanometer. It will be easily noted that in this case the bridge can still be balanced. Balance will be obtained when the difference in potential between the terminals of the detector circuit due to the supply source is made equal in magnitude and opposite in sign to the unknown e. m. f. connected in series with the galvanometer. Thus, while retaining the appearance of a four-arm bridge, we have obtained a potentiometer circuit with all of its characteristic features by simply adding another energy source. Jumping a little ahead, we may add that this circuit is the basic form of the so-called bridge potentiometer.

Going back to the fundamental principle, we may note still another feature of the potentiometer method. It is self-evident (and we have had it in mind) that the potentiometer method can measure not only e. m. f. s, but also voltages in general, say the voltage drop across a resistance. Owing to this, the potentiometer method can also be employed for the measurement of current (in terms of the voltage drop across a known resistance) and of resistance (in terms of the ratio of the voltage drops across the known and the unknown resistor connected in series). Also, power can be found in terms of known current and voltage.

Because of this versatility, the potentiometer method is to electrical measurements more than simply a perfect method for voltage measurements. It will be no exaggeration to say that at present the potentiometer method is the basis of all electrical measurements, since it is an acknowledged tool for the comparison of working sources of e. m. f. with e. m. f. standards.

Technically, the key component of a potentiometer circuit is a known adjustable balancing voltage. It may be the voltage drop produced by a known current across a known resistance. As has been noted, the balancing voltage can be varied by regulating either the

known resistance or the known current. In effect, both methods are employed in practical measurements, preference being given to the former. Current regulation is resorted to in relatively rare, special cases; as a rule, the known current is maintained accurately constant, equal to a certain definite value.

The continuity and accuracy of adjustment of the balancing voltage determines to a great extent the accuracy of the method as such.* It is obvious that this accuracy depends on the accuracy of adjustment of the variable resistance and on the accuracy of determination and maintenance of the supply or working direct current in the variable resistance. Existing manufacturing methods produce resistances to very close tolerances. Therefore, taking the requisite precautions, it will be safe to consider the resistances used in potentiometer circuits sufficiently reliable and accurate. As for the adjustment of the supply current, the special techniques described below, based on the direct use of standard cells, provide a happy solution to this problem as well. On account of this, the potentiometer method today is among the most accurate and perfect. The error in the measurement of voltages as low as 1 volt can be reduced to a very low order of magnitude, being anywhere between 0.001 and 0.003 per cent in special cases.

Practically, the potentiometer method is embodied in a variety of potentiometers of fairly complicated design.

Let us review, at least in brief, the development of the potentiometer method, placing emphasis on the features and elements which, once introduced, have been retained till now.

The elementary form of potentiometer, as it was originally used, is shown in Fig. 7-1. The moving contact of the slide-wire is adjusted until the galvanometer shows no deflection. Then the unknown e. m.f. is given by

$$E_x = IR_r' = E_N (R_r'/R_{bal}).$$

An obvious drawback of this circuit is that the e. m. f. of the standard source is allowed to give a current in the slide-wire, and so the standard e. m. f. cannot be relied upon for the comparison of the two e. m. f. s. This is because the internal resistance of the standard source E_N is difficult to determine, and a considerable error is introduced in the final result (incidentally, this is the reason why another method has been developed, based on the adjustment and measurement of the current). It stands to reason that the smaller the current in the slide-wire, the smaller the error. Accordingly, the original form of potentiometer was very soon modified to include resistance boxes, connected in series with the slide-wire, thereby increasing the resistance of the circuit and extending the range of

* It should be noted that the accuracy of the balancing voltage is above all dependent on the e.m.f. standard, E_N.

the potentiometer. However, this measure could only reduce, and not eliminate, the error due to the load current taken from the standard source E_N.

A more radical approach, which has led to the modern form of potentiometer and which has proved the only way of improving the performance of potentiometers in general, was the inclusion of a separate working or supply battery for the slide-wire, whose e. m. f. would not affect the result. Then, the unknown and known e. m. f. s were in turn connected across the E_x terminals (Fig. 7-1), and a balance was secured twice. Naturally, both e.m.f.s are under identical conditions. At zero deflection, none of them gives any current, and E_N can be safely used for comparison. Denoting the balancing resistances, analogous to R'_r in Fig. 7-1, with R'_x and R'_N and the voltage of the supply battery with V, we obtain:

$$E_N = V\,(R'_N/R_{bat});\quad E_x = V\,(R'_x/R_{bat}).$$

whence

$$E_x = E_N\,(R'_x/R'_N). \tag{7-1}$$

This variety of the potentiometer method changes it from a method of direct concurrent comparison into one of consecutive substitution. As noted in Sec. 1-3, this method is widely employed in order to eliminate systematic errors (in our case, due to the internal voltage drop across the source of standard e. m. f., E_N).

Soon it was found that securing a balance twice for each measurement was unnecessary (where several successive measurements were to be made). Indeed, if the voltage of the supply battery is sufficiently steady, and, consequently, the supply current in the measuring circuit is constant, the balance point (i. e., the value of R'_N) will not change either. Accordingly, in serial measurements it will suffice to check the balance point from time to time so as to make sure that the supply current has not changed.

This idea has naturally developed into direct-reading potentiometers calibrated directly in volts. In them, it is essential to maintain the supply current equal to a certain definite value specified in advance. Then, obviously, the voltage drops across the definite sections of the regulating resistance will likewise be definite and always the same. Furthermore, the regulating resistances can be calibrated in terms of the respective voltage drops and not in units of resistance. Usually, in present-day potentiometers of high accuracy the supply current is made a multiple of ten to some negative power. Then, the values of resistances and voltage drops across them will only differ by a factor of the form 10^n, where n is a positive integer.

To make a potentiometer direct-reading, the current from the supply battery must first be made equal to the nominal one. This

is a very important operation, since any error in the setting of the supply current will be wholly added to the error of measurement. For this reason, ordinary indicating instruments cannot be used for a check on the supply current, because their accuracy is limited and insufficient. Instead, the supply current is adjusted likewise by the potentiometer method. For this purpose, instead of the unknown e. m. f., a standard cell, usually of the Weston type, is connected in the potentiometer, and the moving contacts of the balancing resistances are set so as to obtain the actual value of E_N. Obviously, if the supply current is adjusted properly, no current will flow through the galvanometer, and zero deflection will be obtained. A deflection is an indication that the actual supply current differs from the nominal one, and it must be adjusted until zero deflection is obtained. This operation is called standardization.

After it is standardized, a potentiometer practically becomes a constant-setting instrument, and the standard cell is no longer necessary, in principle. This is, however, true only of ideal conditions, i. e., when there is complete confidence that the supply current or, which is the same, the voltage of the supply battery, is absolutely steady. Naturally, this never happens, so a standard cell is always included as an accessory to any potentiometer system, to be used for regular checks on the initial standardization.

For convenience in making checks on the initial standardization, practically all existing potentiometers have a special switch with which the standard cell is connected to terminals the voltage drop across which is always equal to E_N, irrespective of the setting of the balancing resistances. This method of standardization is very convenient, since it makes redundant resetting the balancing resistances for the voltage of the standard cell. To take care of small variations in the standard e. m. f. E_N (due to, say, temperature variations), it is customary to include a small variable resistor which does not take part in balancing the unknown e. m. f. The value of this resistor is adjusted, depending on the e. m. f. of the standard cell employed and on its changes due to variations in ambient temperature.

A modern form of potentiometer is shown in Fig. 7-2. Sometimes, R_x' and R_N' are partly combined for economy. For example, the fixed portion of R_N' can be replaced by a portion of R_x', by providing a suitable tap independent of the setting of the sliding contacts.

The circuit of Fig. 7-2 is embodied in all practical d. c. potentiometers. They only differ in the design of the balancing resistance R_x', modified in each particular case in order to enhance the accuracy of voltage adjustment. Omitting minor improvements and forms many of which are now of only historical interest, we shall examine the

ways and means of enhancing the accuracy of adjustment, placing emphasis on those which have won general acceptance.

To begin with, it should be noted that by "accuracy of adjustment" we above all mean the continuity and flexibility of adjustment of the balancing voltage, leaving out instrumental errors and sensitivity. This is because instrumental errors are mainly determined by the quality of manufacture and adjustment of resistances and by the

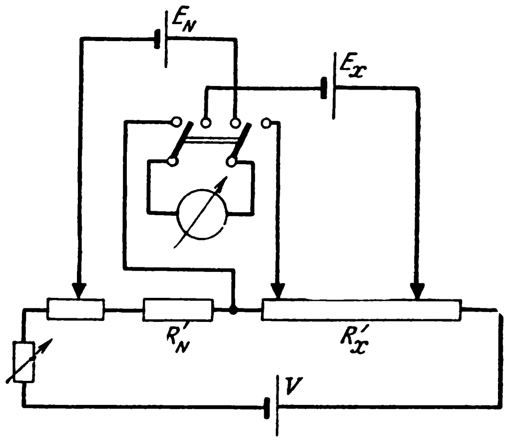


Fig. 7-2

standardization of the supply current. With modern manufacturing methods, resistances may be taken as fully meeting the relevant specifications. As for sensitivity (see Sec. 7-3), present-day moving-coil galvanometers are, in most case, quite satisfactory. Therefore, the performance of a potentiometer is, in fact, dependent on how continuously the balancing voltage can be adjusted or, in the final analysis, on the design of the balancing resistance. So, this is a task of prime importance.

Apparently, the simplest way to enhance the continuity and, consequently, the accuracy of adjustment is to increase the geometrical dimensions of R'_x. Where a slide-wire is used for this purpose, a longer wire of increased cross-section may be employed, usually wound on a rotating cylinder, with a suitably profiled roller sliding over the winding. Where decade boxes are employed (consisting of separate dial-type resistors), the total resistance should be divided into a great number of coils of low resistance each. As an alternative, a combination of both can be used. At one time, for example, potentiometers were manufactured for 1.5 V in which the regulating resistance consisted of 149 coils with a dial switch for as many positions and of

an additional slide-wire the full voltage drop across which was 0.01 V.

However, the slide-wire type of potentiometer has proved unsatisfactory; its very sensitive galvanometer readily responds to the thermal e. m. f. generated at the sliding contact, thereby introducing additional errors. Therefore, although several types of slide-wire potentiometers (such as the low-accuracy Types ПП, П-4 and П-5) are still in use, the tendency for precise potentiometers has for some time been towards a more reliable balancing resistors of the dial type. The requisite continuity and flexibility of adjustment has been retained through certain improvements in connections. Among other things, wide use is made of shunting and double decades. With them, the balancing voltage can be adjusted to any degree of continuity.

The shunting decade principle consists in that a coil of the main decade is bridged by another resistance which has a greater value than the bridged coil and is also divided into a number of coils. By adjustment, it can be made so that the voltage across one coil of the shunting decade is one-tenth that across one coil of the main decade, thereby enabling the balancing voltage to be adjusted to the next decimal place. Physically, the shunting decade can be connected across any portion of the main decade by a switch carrying two contacts always moving together. By this device, coarse adjustment of the balancing voltage is effected by shifting the twin switch of the main decade, and fine adjustment by moving the switch of the shunting decade.

This principle is realized in two ways. In the first*, two coils of the main decade consisting of eleven coils are continuously bridged by another ten-coil decade of a resistance equal to that of the two coils it bridges. Since the two coils are shunted by a resistance equal to their own, the resistance of the resultant parallel combination is the same as the resistance of one coil. Consequently, the voltage drop across it will remain unchanged, being equally distributed among the ten coils of the shunting decade. The voltage drop across each coil of the shunting decade will be one-tenth that across the main decade. This arrangement can be extended to include several consecutive shunting decades, i. e., two coils of one shunting decade are in turn shunted by a third resistance, and so on. In such a case, however, all the shunting decades (except the last one) should have eleven, and not ten, coils, each one-fifth of the resistance of one coil in the preceding decade.

In the other method,** only one coil of the main decade is shunted (Fig. 7-3). If the shunting resistance is made nine times as great as

* The Varley slide arrangement.—*Tr.*
** The Raps arrangement.—*Tr.*

the shunted unit and is divided into nine coils, the resistance of the parallel combination and, consequently, the voltage drop across it will be one-ninth of the resistance of, and the voltage drop across, one coil of the main decade. This voltage drop is equally distributed among the nine coils of the shunting decade, being one-tenth of the total across the main decade, as in the previous case.

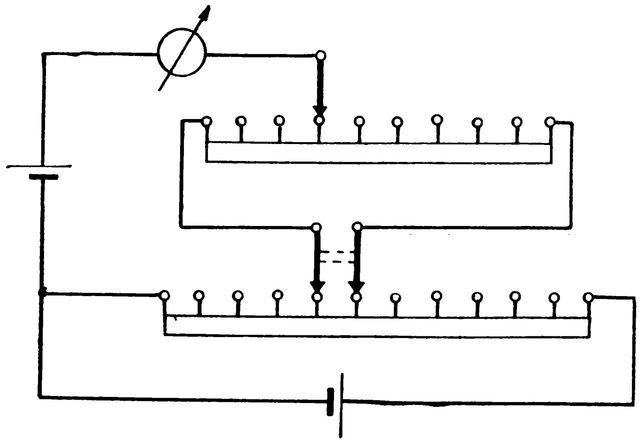


Fig. 7-3

The second method is more attractive than the first, because all the coils in both the main and the shunting decades are of the same resistance—a factor which simplifies adjustment and alignment of the potentiometer. Unfortunately, this advantage of the second method is only present in a potentiometer using two decades (one shunting and one shunted). Where several consecutive shunting steps are employed, only the coils of the two last decades may be made of the same resistance, the coils of the remaining decades being different, as in the case of the first method.

Where consecutive shunting is employed by the second method, all shunting decades (except the last one) should consist of ten coils each. The resistance of one coil in any decade (including the last one, consisting of nine coils, each of the same resistance as the coils of the last decade but one) will then be given by

$$r_m = 10^{\frac{m(2n-m-1)}{2}} r_0,$$

where r_0 is the resistance of one coil in the main decade (being shunted); n is the number of the shunted decades; and m is the ordinal number of the shunting step in question (the first step follows imme-

diately the main decade). As can be easily seen, the consecutive steps differ in resistance more than in the first method, the difference increasing with increasing number of decades and with decreasing ordinal number of a given step.

Of course, other relationships may be chosen for the resistance of the shunting decades. For example, the voltage drop across one coil of a shunting decade can be made equal not to 0.1 of that across the shunted decade, but to 0.01, 0.001 and, generally, to 10^{-k}, where k is any positive integer. Then, where two coils of the main decade are shunted, the shunting resistance should include a decade consisting of coils each of 2×10^{-k} resistance of one coil of the shunted decade,

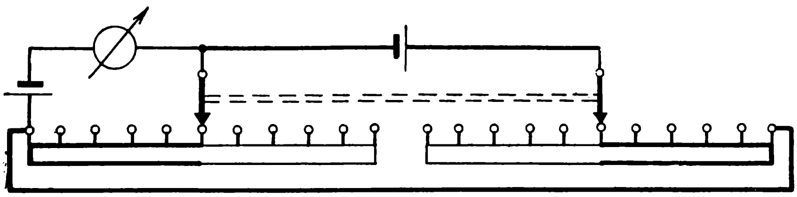


Fig. 7-4

and a series resistor so chosen as to make the total shunting resistance equal to the total resistance of the shunted decade. Where one coil of the main decade is shunted, the shunting decade consisting of equal-resistance coils and the coils of the shunted decade must be connected in series with an additional resistor so chosen as to make the total shunting resistance 99,999 and, generally, 10^k—1 times as great as the resistance of the shunted coil.

The main idea of the double-decade principle* is that the resistance of a portion of the circuit is adjusted without affecting the total resistance and, consequently, the current of the circuit (with the applied voltage being constant). Practically, this is accomplished by means of two equal resistances whose contact arms are mechanically coupled so that an increase in one brings about a decrease in the other, and vice versa. In other words, each of the two resistances can be varied from zero to a maximum, while their sum remains constant in all cases. The usual connection of a double decade is given by Fig. 7-4, in which the bold lines show the current paths; obviously, their total resistance will be always the same. This scheme can likewise use a number of consecutive decades whose resistance is a multiple of ten. A certain drawback of this arrangement is that twice as great resistance has to be provided as is actually required, only half of it carrying the current.

* The Feussner scheme.—*Tr.*

As compared with the double-decade method, the shunting-decade principle suffers from a number of drawbacks. For one thing, the sensitivity of the potentiometer varies significantly with the absolute value of the unknown (see Sec. 7-3). For another, while a double-decade potentiometer can be easily corrected for inaccuracies in the manufacture of the resistances (the total correction is practically equal to the algebraic sum of the corrections for the individual decades), this is impossible to do with a shunting-decade potentiometer (since the voltage read from the shunting decade should be corrected for inaccuracies not only in the coils of this decade but also in the coils being shunted).

Still another drawback of the shunting-decade principle is that variations in ambient temperature may introduce additional errors. This is because a shunting-decade potentiometer uses an external standardizing resistor. Therefore, should the standardizing resistor and the decades (usually the first one) differ in the temperature coefficient of resistance, an additional error may be introduced into the setting of the supply current and into the result of the measurement. In a double-decade potentiometer, on the other hand, the standardizing resistor (its greater portion) includes ten coils of the first decade, and inaccuracy in the standardization of the supply current due to temperature variations will not introduce any appreciable error in the final result. At the same time, both principles suffer from a common shortcoming: the presence of moving contacts in the detector circuit. This may result in additional errors due to variations in contact resistances and to the generation of thermo-e. m. f. s at the contact arms.

There are several ways of avoiding moving contacts in the detector circuit. This can be accomplished by connecting the same double decades somewhat differently. Referring to Fig. 7-4, the balance condition of a double-decade potentiometer will not change, if the detector is connected between a battery terminal and the free end of the decade, instead of between a battery terminal and a moving contact. Then we obtain an arrangement in which the double decades are reverse-connected, as shown in Fig. 7-5*a*. The circuit of Fig. 7-5*a* differs from that of Fig. 7-4 in that it has no moving contacts in the detector circuit. Two more arrangements using the reverse connection of double decades are given by Figs. 7-5*b* and *c*.

The circuits of Figs. 7-5*a* and *b* do not call for additional explanation. As for the circuit of Fig. 7-5*c*, where a certain fixed number of coils connected by a link (whose resistance is zero or has any fixed finite value) are shunted by a fixed resistance, moving the contact arms one stud will produce a voltage change equal to the resistance of one coil in the decade times the current in the shunt (and not the current in the coil, as this was in all previous cases). In other

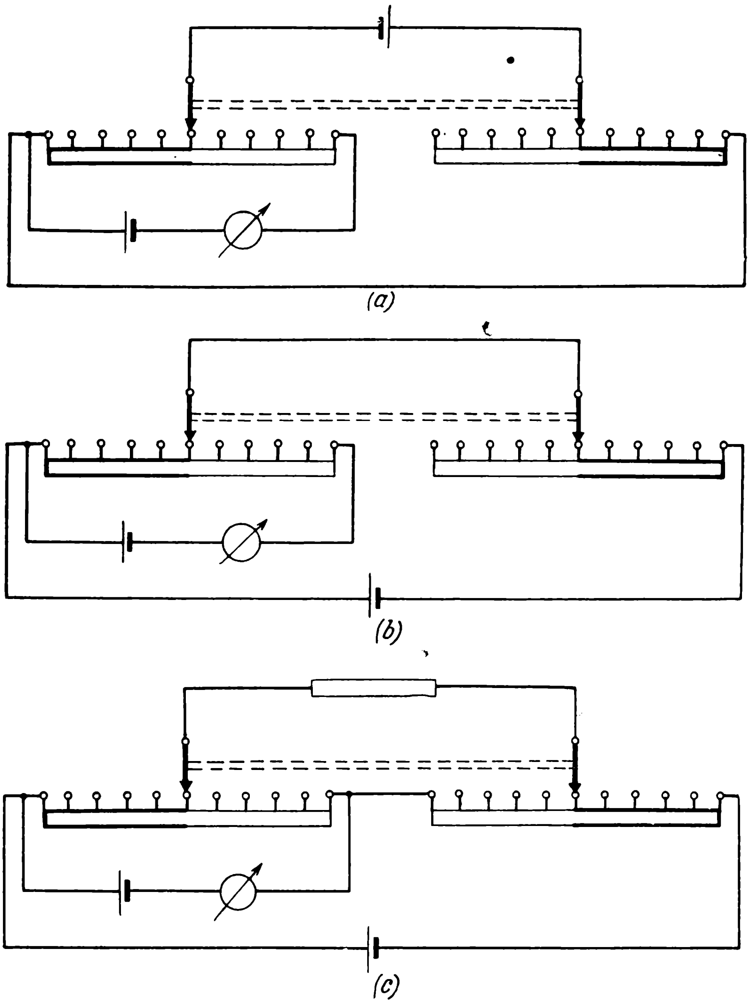
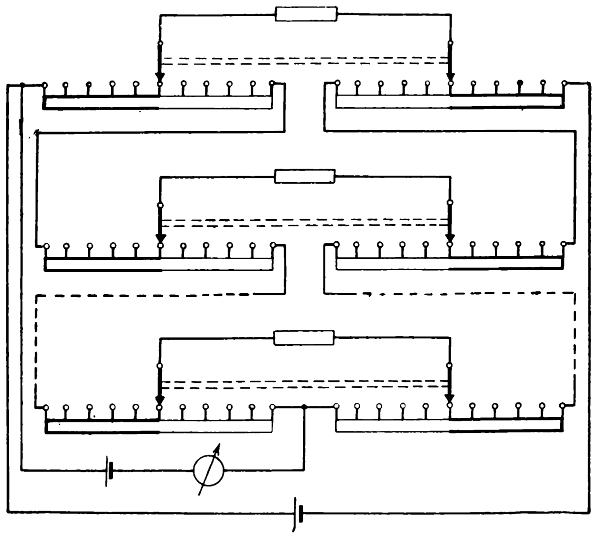


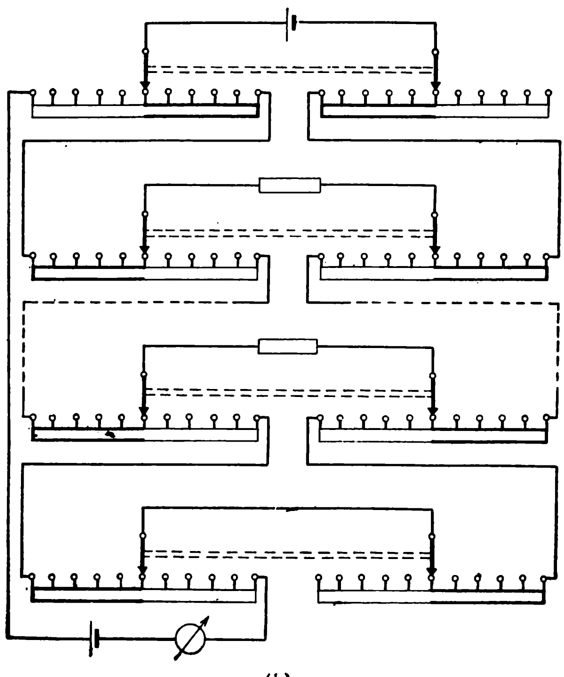
Fig. 7-5

words, the requisite voltage change can here be obtained by selection of the coils and shunt.

A disadvantage of the circuits given by Figs. 7-5*a* and *b* is that they can give only two steps of adjustment (since the number of steps is limited by the number of points where the detector can be connected to the resistance from which the regulated voltage is taken). The circuit of Fig. 7-5*c* can give any number of adjustment steps, because



(*a*)



(*b*)

Fig. 7-6

the resistance connected in the detector circuit can be divided into any number of double decades and fixed shunting resistances, Fig. 7-6*a*.

Multiple-step adjustment of the balancing voltage can also be obtained by combining the circuits of Figs. 7-5*a* and *b* and the circuit of Fig. 7-5*c*, as given by Fig. 7-6*b*. Since the voltage drops across the coils of different decades depend on the resistance of the coils and the currents of the respective shunts and not on the currents in these

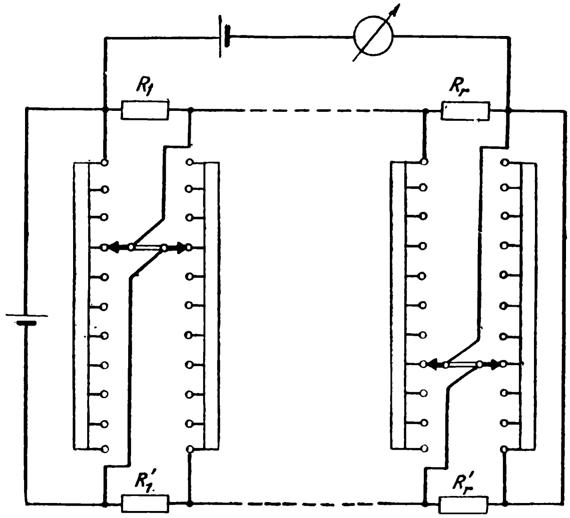


Fig. 7-7

coils, the ratios between the resistance of the coils in different decades are not necessarily decimal, as they were in the previous arrangements. A disadvantage of all the methods for multiple-step adjustment of the balancing voltage, using double decades and fixed shunts, is that, beginning with the third decade, the lowest balancing voltage obtainable is other than zero. However, this drawback can be easily eliminated by an arrangement examined later.

Another way of avoiding moving contacts in the detector circuit is the use of double shunting decades. The connections for this method are given by Fig. 7-7 where two equal fixed resistances (for example $R = R_1'$) are shunted by two equal decades consisting each of ten unequal coils. The coils are so constructed that moving the contact arm of the shunting decade one stud changes the voltage drop by one-tenth the maximum voltage across the decade. The coils are brought in circuit in a reverse sequence. Double shunting decades give any

number of adjustment steps without any fundamental difficulty. An obvious disadvantage of this method is the necessity of using a wide range of resistances whose values may be nonintegral numbers.

Now we have come to the bridge-type potentiometer. The basic idea of the bridge-type potentiometer consists in using a network of two parallel branches in which the unknown e.m.f. is connected in the detector circuit in series with the galvanometer; it is obvious that such an arrangement can be balanced. Since a balance can be secured in a variety of ways, it is possible to place the contact resistances so that they are outside the detector circuit. In an elementary case, they should be included with the supply circuit, as shown in Fig. 7-8. Referring to the figure, the balance condition is given by $E_x = I_1R_1 - I_2R_2$. In other words, the unknown e. m. f. is equal to the difference between two voltage drops produced across two different resistances by two different currents. This somewhat complicates the calculation of the resistances included in the network, the more so that the total resistance of the network must be made constant, and so must be the supply current (at the same time, there is an advantage in this requirement which will be discussed later on). It is also evident that (as was the case with the previous arrangements) the coils of different decades will greatly differ in resistance.

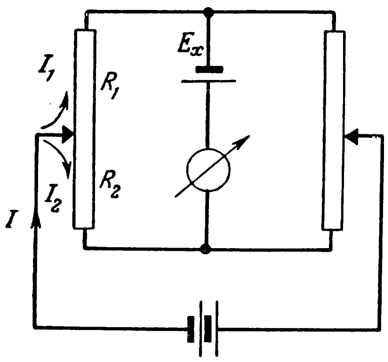


Fig. 7-8

Practically, the supply current in the bridge-type potentiometer is maintained constant mainly by using double reverse-connected and also double shunting decades. Since the result depends on the two currents flowing in opposite directions in the two parallel branches of the potentiometer, it is possible, without much difficulty, to make the initial reference voltage equal to zero—a thing unattainable in networks using double reverse-connected decades and multiple-step voltage adjustment. This can be done by balancing the voltage drops across the decades in one branch, with the contact arms in the zero position, by the voltage drops across the respective decades (also with the contact arms in the zero position) or across the series resistances in the other branch. The currents flowing in the two parallel branches can be made to bear any relation to each other (for example, 1 to 10, 1 to 9, etc.). The requisite ratio is obtained by connecting suitable series resistances in the parallel branches.

Figure 7-9 gives the connections of an elementary three-dial bridge-type potentiometer using three reverse-connected double decades. Decade *I* is arranged as at (*a*) and decades *II* and *III* as at (*b*) in Fig. 7-5. The series resistors placed in the supply current circuit between decade *I* and decades *II* and *III* are so chosen as to secure the requisite current ratio between the potentiometer branches. It

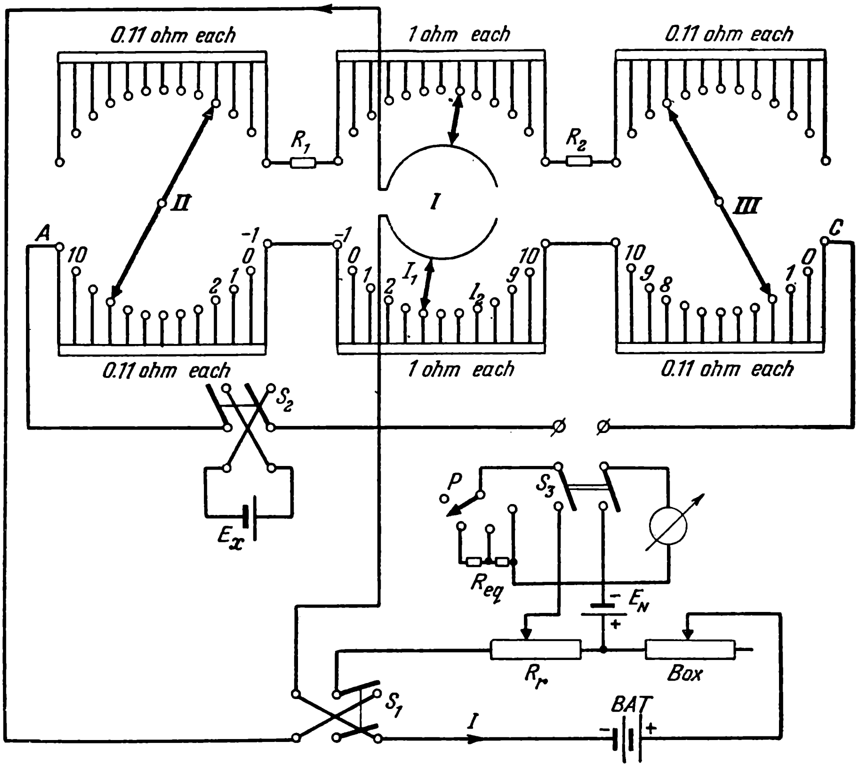


Fig. 7-9

may be added that these series resistors also serve to maintain the sensitivity practically constant over the entire range of measurement—a feature distinguishing the bridge type of potentiometers (see Sec. 7-3).

All of the methods for increasing the accuracy of adjustment with shunting and double decades involve a great number of resistance coils (more or less different in value) in the respective decades. This is of course an obvious disadvantage. It is nonexistent, however, in a method by which adjustment can be made to several

decimal places (in principle, there may be as many decimal places as one may care to have) with a single decade consisting of equal resistance coils. Based on current superposition, the method is presented diagrammatically in Fig. 7-10. Obviously, if currents I_1, $I_2 ... I_k$ are in the proportion of $1:0.1...10^{-k}$, where k is the number of adjustment steps, the balancing voltage will be $V_k = I_1 r(n_1 + 0.1\, n_2 + + ... + 10^{-k} n_k)$, where n_1, n_2, ... , n_k are the settings of the moving

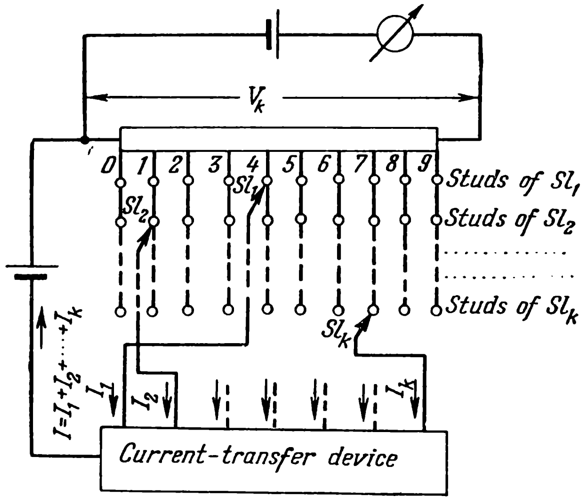


Fig. 7-10

arms through which the currents I_1, I_2, ... I_k are supplied; and r is the resistance of a coil in the decade. Naturally, the contact arms of the coils can be set so that any multiple reading of the balancing voltage can be obtained with only ten coils of the same resistance value.

In existing makes of potentiometers, the accuracy of adjustment is ensured by any one of the above methods, very often by a combination of them. Types ППТВ and ПВ-2, for example, use four shunting decades and one double normal (Feussner-type) decade (Fig. 7-4). Although these potentiometers can still be found in laboratories, they are not produced any longer (because of the fundamental and practical disadvantages of shunting-decade potentiometers mentioned earlier).

The normal connection of double decades is employed in Types ПВ-6, ПВ-7, ППТВ-1 and Р375. The bridge arrangement with reverse-connected double decades and double shunting decades is used in the Type КЛ-48 potentiometer.

The bridge arrangement with reverse-connected double decades and fixed shunting decades is embodied in the Type ПМС-48 potentiometer. The current-superposition principle is utilised in Types ППТН-1 and Р330.

The maximum range of modern potentiometers is 1 to 2 volts. This, however, does not imply that they cannot be used for the measurement of higher voltages: their range can be extended by means of a *measurement voltage divider.* A measurement voltage divider (also known as a volt-box) is a precise resistance fairly large in value (several ten thousand ohms and more) with tapping points placed usually at 0.1, 0.01 and 0.001 of the the total resistance from one of the ends, the common terminal. The unknown voltage is applied to the total resistance, and the voltage drop taken between the common terminal and a suitable tap is fed to a potentiometer. Since voltage is always distributed in proportion to resistances, the voltage measured by the potentiometer will be a definite, known fraction of the total voltage (equal to 0.1, 0.01 or 0.001 of the total, depending on which tap has been used).

Volt-boxes can only be employed in potentiometer work, since at balance a potentiometer has an infinitely high resistance (the current through the potentiometer is zero) and so it does not upset the voltage distribution in the volt-box.

It should be noted that a volt-box constitutes a definite, even though small, load for the circuit being measured. Therefore, when one is used, the measurement of e. m. f. s is out of the question.

In some designs of potentiometers use is made of devices reducing the range of measurement, i. e., acting in a manner opposite to that of a volt-box. The requisite reduction of the supply current in the known resistance is obtained by shunting it. At the same time, in order to avoid a new current standardization, a series resistor is inserted, of a value sufficient to keep the total resistance of the entire network unchanged.

Such are, in a broad outline, the salient features of present-day d. c. potentiometers.

In conclusion, mention should be made of a method for measuring differences of potentials, which is particularly advantageous when nearly equal potentials are to be compared. This method is often employed for the measurement of the e. m. f. s of standard cells and for instrument transformer testing. Its idea is that the unknown and known voltages or e. m. f. s are connected in an auxiliary circuit so that they oppose each other. The resultant difference is measured on a potentiometer. Obviously, since the difference is very small in comparison with the unknown, the fractional or percentage error will also be insignificant. In the main, the accuracy of meas-

rement depends on the accuracy of the standard used to obtain the difference. Customarily, this method is referred to as the differential measurement of potential differences.

7-2. Practical D. C. Potentiometers

As noted already, practical potentiometers are solely factory-made units less the detector and the battery supply. The circuitry may be arranged in one box or two. The series resistors employed for current standardization are not usually supplied, though in some makes they are added to the main circuit. Thus, a modern potentiometer is a fairly complicated piece of apparatus calling for care in its manufacture.

Before we examine practical potentiometers, it seems worth while to establish the features according to which they can be differentiated.

According to the principle embodied in the circuitry we might speak of series potentiometers (using double decades), parallel potentiometers (using shunt decades), current-superposition potentiometers, etc. However, that would be an artificial classification, since (as has been stressed) existing potentiometers often use combinations of the various circuit arrangements. A more practical approach (though, perhaps, not particularly scientific) is to divide all potentiometers into two groups depending on their total resistance: high-resistance (of the order of several ten thousand ohms) and low-resistance (of the order of several hundred ohms). Since the voltage range is more or less the same for all potentiometers (1-2 volts) the difference in the supply current will be proportional to the difference in the total resistance. Hence, it is much easier to maintain the supply current constant in high-resistance potentiometers (where it is about 0.1 mA) than in low-resistance potentiometers (where it is 10 to 1,000 times greater).

On the other hand, low-resistance potentiometers are more sensitive than high-resistance units. Therefore, where very small e. m. f. s are to be measured and, consequently, the sensitivity is a critical factor, use should be made of low-resistance potentiometers. If the unknown voltage is of the order of one volt (which is often the case when testing standard cells), a high-resistance potentiometer will be more suitable. Incidentally, if a low-resistance potentiometer were used for this purpose, the current flowing in the detector circuit even at small deviations from balance would prove dangerous to both the galvanometer and the test specimen (especially, if it is a standard cell). Although not dangerous to the operator, this may entail certain inconveniences to the experiment. It is usual, therefore, to place a protective resistor in series with the galvanometer

and to cut it out as a balance is approached. Also, the operator has to exercise utmost care. In high-resistance potentiometers, the possibility of damage to the circuit components is a very remote one, since the current through the detector is limited by the high value of the standard resistor. Yet, a protective resistor must be used all the same.

Potentiometers may furthermore, be classed into manual (both balanced and unbalanced), semi-automatic and automatic. Manually operated balanced potentiometers are the most perfect and commonly used type for laboratory work. Unbalanced potentiometers, produced commercially at one time, have dropped out of use, since they are inferior to the balanced type. As a matter of fact, such potentiometers are termed 'partial-deflection', since the bulk of the unknown voltage is coarsely balanced in the usual manner and only the last decimal place is read from a galvanometer. Owing to certain circuit refinements, the resistance of the detector circuit remains always constant, and so the galvanometer deflection is proportional to the unbalanced difference of potentials. All in all, unbalanced potentiometers have to be balanced by the operator, though to a lesser degree of accuracy than usual.

In semi-automatic potentiometers the bulk of the unknown voltage is balanced manually, and the remaining portion automatically. Thus, they are fully balanced potentiometers in which a balance is secured (i. e., a measurement is made) faster and simpler.

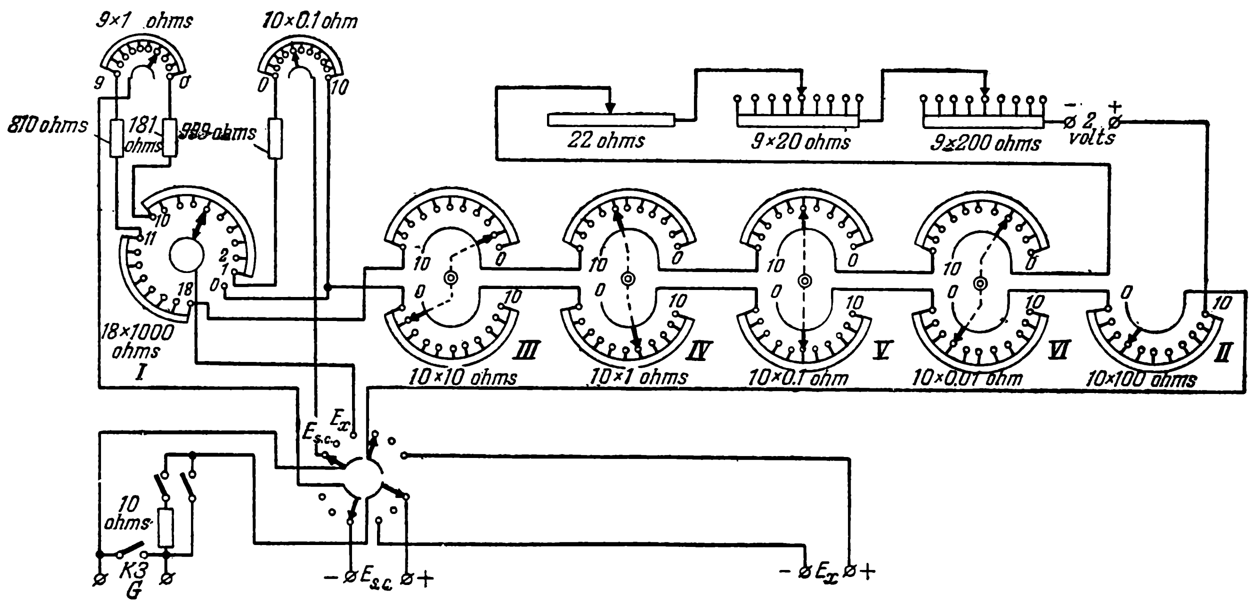
Automatic potentiometers are fully-balanced units in which a balance is secured without the operator's participation. They, however, lie outside the scope of this book.

Here are a few examples of practical d. c. potentiometers in use at the present time.

Figure 7-11 gives a diagram of the connections of the Type ПВ-7 potentiometer. It is a six-dial potentiometer (decades I through VI) and reads down to the sixth decimal place (i. e., down to 1 μV). The voltage range (without a volt-box) is 1.9 V. The potentiometer is of the high-resistance type (10,000 ohms per volt of the unknown voltage). The supply current is 0.1 mA; use is made of an external supply battery of 2 ± 0.1 volts. Standardization is obtained with rheostats series connected with the main decades. The galvanometer can be connected alternately across the standard cell and the unknown voltage by means of a double-throw switch E_s—E_x. The controls of the potentiometer include three push-buttons marked "10^5 ohms", "0" and "K3" (galvanometer shunt).

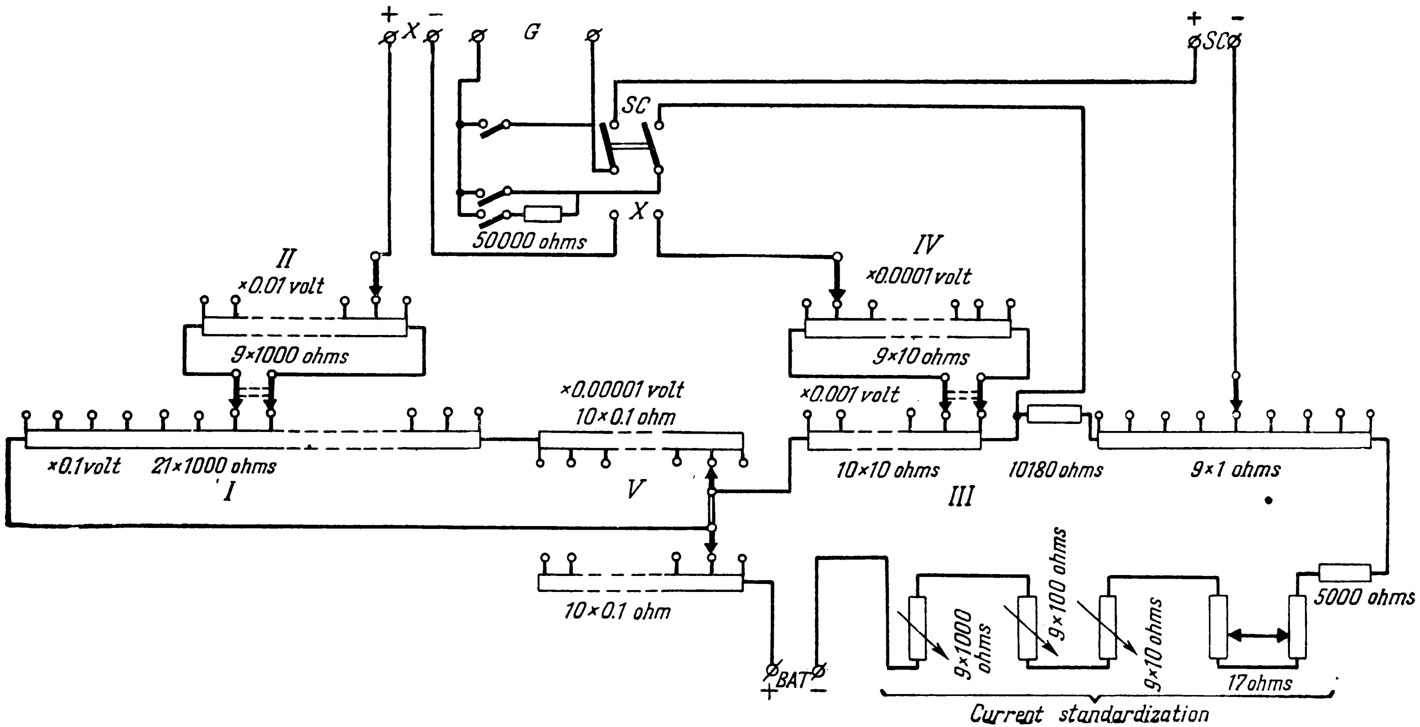
The circuit of the potentiometer consists of two single decades (dials I and II) permanently connected in the supply current circuit, and four (Feussner-type) double decades. The unknown voltage is applied to the compensating resistance by the contact arms of dials I and II.

The potentiometer is standardized by connecting a standard cell across the standardizing resistance (R'_N = 10,180 to 10,190 ohms) which enables standard cells with an e.m.f. of 1.0180 to 1.0190 volts to be employed. The main portion of the standardizing resistance is made up of the first ten coils of dial I. Small variations in the e. m. f. of the standard cell due to temperature changes



9×1 ohms
10×0.1 ohm
810 ohms
181 ohms
989 ohms
22 ohms
9×20 ohms
9×200 ohms
2 volts
18×1000 ohms
I
III
IV
V
VI
II
10×10 ohms
10×1 ohms
10×0.1 ohm
10×0.01 ohm
10×100 ohms
E_x
$E_{s.c.}$
10 ohms
K3
G
$E_{s.c}$
E_x

Fig. 7-11



+ X −
G
SC
X
50000 ohms
II
×0.01 volt
9×1000 ohms
IV
×0.0001 volt
9×10 ohms
+ SC −
×0.00001 volt
10×0.1 ohm
×0.001 volt
×0.1 volt
21×1000 ohms
I
V
10×10 ohms
10180 ohms
9×1 ohms
III
10×0.1 ohm
BAT
+ −
9×1000 ohms
9×100 ohms
9×10 ohms
17 ohms
5000 ohms
Current standardization

Fig. 7-12

or other causes are allowed for accurate to 0.001 per cent by inclusion of two compensating dials (x1 ohm and x0.1 ohm), one in the first coil and the other in the eleventh coil of dial I. The standard cell is connected across the standardizing resistance through the moving arms of the compensating dials.

As can be seen, when a standard cell is inserted in the potentiometer circuit, the galvanometer should read zero at any settings of the main dials, provided the supply current has been adjusted correctly. This arrangement, known as the standardizer, is employed in practically all existing forms of potentiometer. With it, it is unnecessary, when standardizing the potentiometer or when checking this standardization during the measurements, to reset the main dials, so there is no interference with the measurement proper.

Figure 7-12 gives a diagram of connections of the Type ППТВ high-resistance potentiometer which is still in common laboratory use. Built along the lines of the Tinsley vernier potentiometer, it has five adjustment steps and reads down to the fifth decimal place (i. e., down to 10 μV). Four steps are obtained by the double use of shunt dials I/II and III/IV. The last, fifth, decimal place in the reading is given by a normally connected double dial (V). The voltage range of the potentiometer is 2.1 volts (without a volt-box). The measuring resistance is up to 21,000 ohms. The supply current is 0.1 mA. The standardizing resistance (10,180 ohms plus the temperature-compensating dial 9×1 ohms) is made independent of the main decades.

Types ПВ-2, ППТВ-1, ПВ-6 and P375 are also high-resistance potentiometers. They are unsuitable where, in addition to high sensitivity, it is essential to reduce the effect of thermal e. m.f.s and variations in the contact resistance of the contact arms, i. e., in applications such as the measurement of small e. m. f. s and voltages (the testing of thermocouples and millivoltmeters).

The latest designs of low-resistance potentiometers are constructed so that there are no moving arms in the detector circuit, which might introduce some uncertainty in the final result.

Figure 7-13 gives the connections of the Type КЛ-48 low-resistance potentiometer manufactured by the Teplokontrol Factory in the city of Lvov, Ukraine. Of the bridge arrangement, it uses five steps of adjustment for the balancing voltage. It differs from the one shown in Fig. 7-9 in that in addition to reverse connected double decades it also uses double shunting decades (dials IV and V) in which the coils are of unequal resistance. These dials bridge two extreme coils in dial I (coil —1 and coil 11). Their coils have been designed so that moving the arm of the shunting dial one stud changes the voltage drop by 0.0001 V across coil —1 (including the shunt) and by 0.00001 V across coil 11 (including the shunt). When the unknown e. m. f. or voltage is connected with the correct polarity, the voltage drop across the right-hand portion is subtracted from the voltage drop across the left-hand portion of the potentiometer (Fig. 7-13). For this reason, the zero points on the shunting resistors are placed the other way round. The resistance increases to the left of zero (dial IV), thereby increasing the voltage drop, and decreases to the right of zero (dial V), thereby decreasing the drop. In both cases, the balancing voltage is increased. Since similar shunting decades (with the reverse connection of the coils) are also included in the duplicate section of dial I, the total resistance of the potentiometer at the source terminals remains unchanged.

Referring to Fig. 7-13, the potentiometer has no built-in standardizing device. Instead, use is made of an external standardizer built into a separate case. It consists of a fixed resistor of 10,180 ohms, two variable resistors of 12×1 ohms and 10×0.1 ohm, and also a series resistor of 7 ohms which makes up the balance for 10,200 ohms. Thus, as in the ПВ-7 potentiometer, the standardizing resistance and, consequently, the supply current can be set accurate to the sixth decimal place. The standardizer also contains a four-range shunt for supply currents of 0.0001, 0.001, 0.01 and 0.1 A. It should be noted, however,

that a change-over from range to range calls for a new standardization, since the resistance of the circuit changes.

The potentiometer has four voltage ranges, namely 1.11110, 0.111110, 0.0111110 and 0.00111110 V, and reads, respectively, down to 10, 1, 0.1 and 0.01 μV. The resistance across the galvanometer terminals is not over 14 ohms. The supply d. c. voltage is about 12 V on the range of 1.11110 V, and about 2 V on the other ranges.

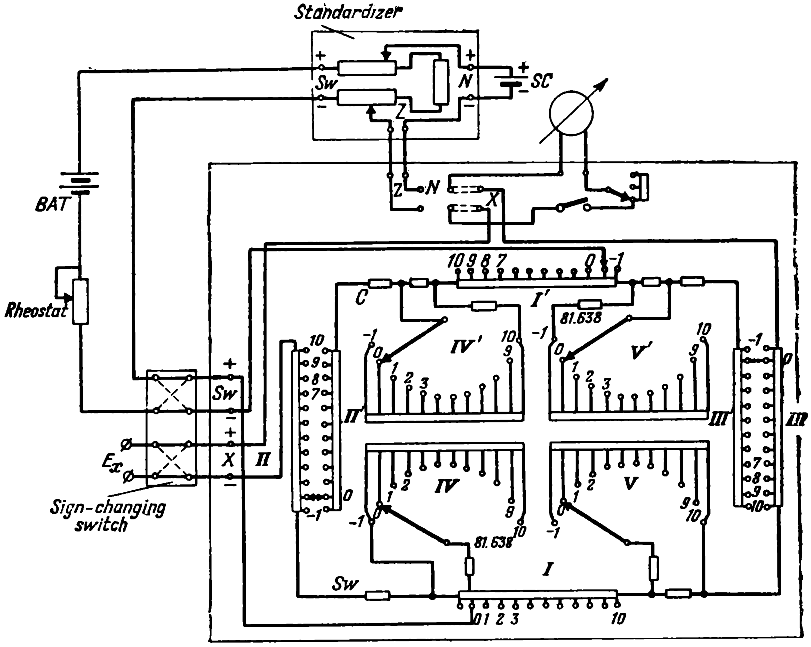
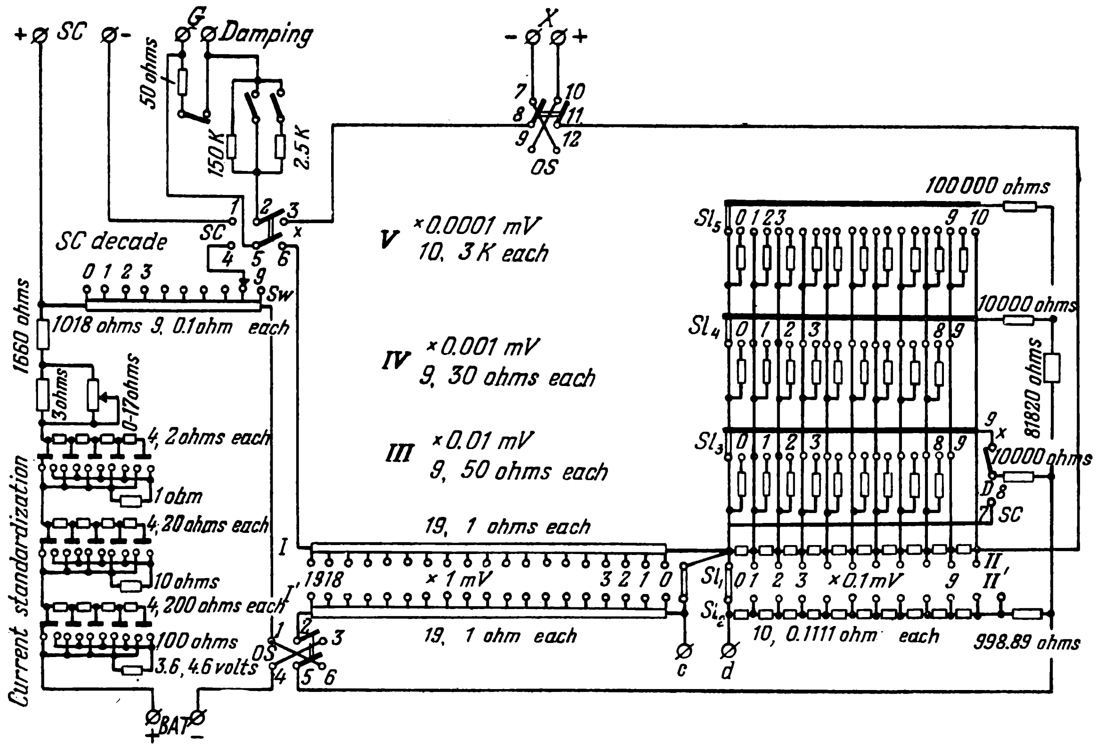


Fig. 7-13

Figure 7-14 gives the connections of the ППТН-1 low-resistance potentiometer manufactured by the ZIP Works in the city of Krasnodar and intended for the measurement of small e. m. f. s. The potentiometer reads the unknown voltage to the fifth decimal place. The first two places are given by two reverse-connected double decades, while the third, fourth and fifth places are obtained by current superposition on the main coils of the second decade. The range is 20 mV. The setting of the contact arm Sl_1 on the first decade gives units of millivolts, and the settings of the contact arms Sl_2, Sl_3, Sl_4 and Sl_5 give, respectively, tenths, hundredths, thousandths and ten-thousandths of one millivolt. The supply current of the potentiometer is 1 mA. It slightly varies (within 0.002 per cent), depending on the setting of the contact arms Sl_3, Sl_4 and Sl_5. Standardization is done with an external source with a voltage of 3.6 to 4.6 V. By the use of ballast resistors 998.89 ohms, 10,000 ohms, 81,820 ohms and 100,000 ohms the currents in the second, third, fourth and fifth decade are set in the ratio of 1 : 0.1 : 0.01 : 0.001. The sum of these currents is equal to the current in the first decade, or the working current of the potentiometer. The



+ SC -
G Damping
50 ohms
150 K
2.5 K
X
- +
7 8 9 10 11 12
OS
SC decade
0 1 2 3
Sw
1660 ohms
1018 ohms 9, 0.1ohm each
3ohms
0-17ohms
4, 2ohms each
1ohm
4, 20ohms each
10ohms
4, 200ohms each
100ohms
3.6, 4.6 volts
Current standardization
BAT
V ×0.0001 mV
10, 3 K each
IV ×0.001 mV
9, 30 ohms each
III ×0.01 mV
9, 50 ohms each
19, 1 ohms each
× 1 mV
19, 1 ohm each
Sl_5
Sl_4
Sl_3
Sl_1
Sl_2
100 000 ohms
10000 ohms
81820 ohms
10000 ohms
D
SC
× 0.1mV
10, 0.1111 ohm each
998.89 ohms
II
II'
c
d

Fig. 7-14

moving contacts Sl_3, Sl_4 and Sl_5 have each two contact studs between which resistors of 50, 300 and 3,000 ohms, respectively, are placed. Their purpose is to prevent current surges which would otherwise occur due to the shorting of the coils in the second decade as the arms Sl_3, Sl_4 and Sl_5 move from stud to stud.

The standardizer is built into a common unit with the main circuit. It consists of a three-dial box with steps of 100, 10 and 1 ohm for coarse current adjustment, and a shunted slide-wire of 17 ohms connected in series with a box of 3 ohms for fine current adjustment. There also is a series resistor of 1,660 ohms and a standard-cell resistance, both connected in series with the standardizing resistance. The standard-cell resistance consists of a fixed resistor of 1.018 ohms and a series-connected row of nine resistors of 0.1 ohm each, making up the standard-cell decade. When the current through the standard-cell resistor is 1 mA, the voltage drop across it is exactly equal to the e. m. f. of the standard-cell, which should be anywhere between 1.0180 and 1.0189 volts.

For standardization, the switch *Sw* is so turned that its contacts *1*, *2*, *4*, *5* place the galvanometer in the standard-cell circuit, while its contacts *7* and *8* connect the resistor of 10,000 ohms of dial III to the zero stud of the latter. For this reason, the settings of the contact arms Sl_1, Sl_2 and Sl_3 do not affect the resistance of the circuit during standardization (the effect of the contact arms Sl_4 and Sl_5 is negligibly small).

The potentiometer also includes resistors which protect the galvanometer against overloads without upsetting the condition of normal damping (which is close to critical). There are also oil "sign-changing" switches (*OS*) with which the direction of the potentiometer current and the polarity of the unknown e.m.f. can be reversed simultaneously.

It has been stated already that modern low-resistance potentiometers have no moving contacts in the detector circuit. Therefore, no contact resistances are included with the unknown e. m. f., which is the case with high-resistance potentiometers. Should a thermal e. m. f. be generated at any of the dial contacts, the path of the thermo-electric current is such that the voltage drop due to it across the decades will be but a negligible fraction of this e. m. f. In the КЛ-48 potentiometer, for example, the voltage drop is about 1.3 per cent of the total parasitic e. m. f. which amounts to several microvolts in the worst of cases. It is obvious that the resultant error may be safely neglected and that the parasitic thermal e. m. f. s will have practically no effect. (Of course, measures are usually taken in potentiometer design to prevent the generation of thermo-electric e. m. f. s or to reduce their value).

Another advantage common to all low-resistance potentiometers is that the total resistance of the detector circuit remains practically constant. In the КЛ-48 potentiometer it is about 13.31 ohms, remaining practically unchanged at any settings of the contact arms (there are negligibly small changes, though, due to the shunting effect of the other halves of decades II, III, IV and V). In the ППТН-1 potentiometer it is about 20 ohms, changing by not more than 0.25 per cent at the various settings of the moving arms. This implies that the sensitivity of potentiometers is practically constant, and the

galvanometer has the same amount of damping and the same calibration accuracy. Without going into greater detail (see Sec. 7-3), it may be noted that for this reason it is possible to obtain the last decimal place from the galvanometer without securing exact balance, when it is difficult to attain for one reason or another.

In addition to the advantages common to all low-resistance potentiometers, the ППТН-1 potentiometer is superior to the bridge type in that at balance the supply current flows in no portion of the circuitry. This extremely simplifies the zero adjustment of the instrument and permits the use of resistance materials (manganin) of lower stability than in other types of low-resistance potentiometers. Also, the ППТН-1 uses fewer precision resistances (since the decade currents are superposed on the same resistances).

A major disadvantage of all existing multidial potentiometers is that balance is secured manually and usually takes much time to obtain. Furthermore, they use reflecting galvanometers which (for the reason of high sensitivity) must be set up on substantial walls or foundations, because of which the potentiometer becomes a stationary apparatus. In recent years, much headway has been made in the design of portable multiple-reflection galvanometers with the result that the measurement procedure with potentiometers has been greatly simplified and a great number of portable potentiometers have been developed for laboratory and plant use.

The above drawbacks have to a great extent been eliminated in the P2/1 semi-automatic potentiometer manufactured by the ZIP Factory in Krasnodar. Shown diagrammatically in Fig. 7-15, it consists in effect of two series-connected potentiometers. One, nonautomatic, serves to balance the bulk of the unknown voltage, while the other, of automatic design, measures the unbalanced difference. The manual unit has two dials: D_1 with fifteen resistances of 10 ohms each and D_2 with ten resistances of one ohm each. The potentiometer current is 10 mA. The automatic section is a simplified photo-electric potentiometer amplifier fitted with a galvanometer G whose mirror illuminates two differentially connected photo-cells F_1 and F_2—a feature which permits the measurement of voltage differences of opposite signs. The voltage difference is proportional to the photo-electric current measured by a centre-zero microammeter A.

The P2/1 potentiometer is intended for instrument testing and a variety of precision measurements. Since it is a multirange instrument, it has shunts R_3 to vary the value of the potentiometer current in the nonautomatic section, and change resistors R_0 in the automatic section. The ranges of both the manual and automatic units are changed simultaneously and in the same proportion by means of a built-in switch Sw_3. At all settings, the range of the automatic section is ± 0.75 of the single graduation of the dial D_2.

The potentiometer current is adjusted by means of rheostats R_8 (for simplicity, only one rheostat is shown in the diagram), through a comparison of the voltage drop across R_5 with the e.m.f. of the standard cell connected across the terminals SC. The standard-cell circuit is operated by push-buttons PB_3 or PB_4. Small changes in the e.m.f. of the standard cell due to temperature variations and other causes are allowed for by varying R_5 with a switch Sw_4.

Range selection is effected without upsetting the initial standardization since the range-change switch is fitted with resistors R_4 which take care of the resultant variations in the circuit. Balance during standardization is indicated by the automatic portion of the instrument; with the push-button PB_1 released, it is always connected across the resistor R_6 in the standard-cell circuit. When the push-button PB_1 is pressed, the two units of the potentiometer are

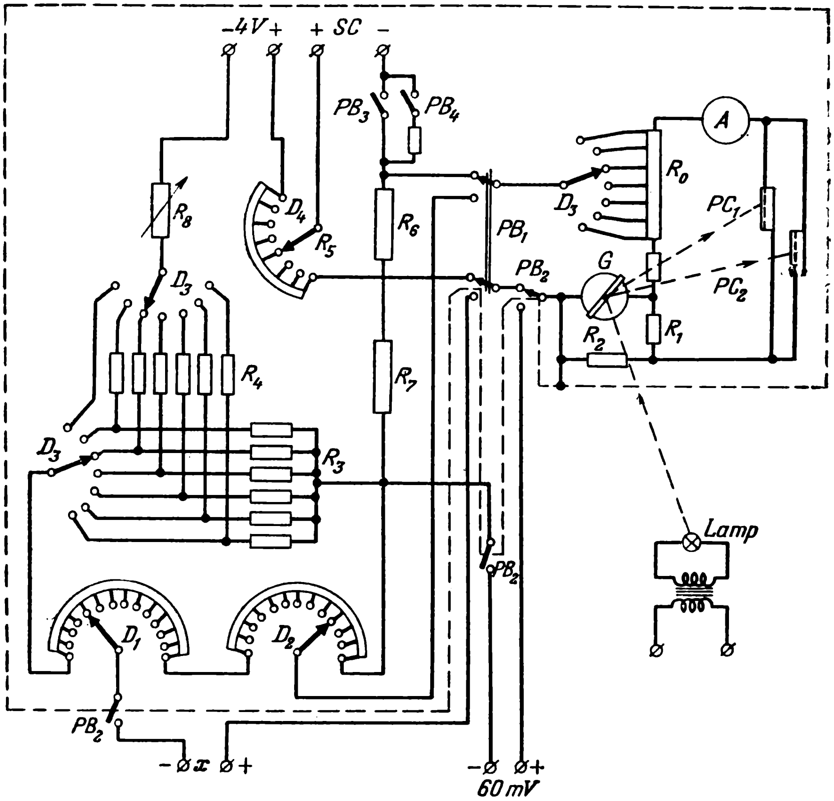


Fig. 7-15

connected in series and measure the voltage applied across the terminals marked X.

The potentiometer has six ranges: 30, 37.5, 45, 60, 75 and 150 mV. Accordingly it can test instruments reading down to multiples of 1, 2, 2.5, 3, 4 and 5. Corrections for the readings of the instrument under test can be taken directly from the scale in fractions of its least scale division. Also, the potentiometer has a range of 1,500 mV used for ordinary measurements of e.m.f.s and voltages.

In addition to ammeter and voltmeter testing the potentiometer can be used for the testing of wattmeters. For this purpose, the P-5 volt-box furnished as part of the potentiometer system has additional tappings corresponding to

the ranges usually adopted for wattmeters. The output voltage of the volt-box is always 60 mV. When the push-button PB_2 is pressed, this voltage is compared with the voltage drop across R_7. Balance is indicated by the automatic section of the potentiometer, while the terminals X are disconnected from the potentiometer circuit. The wattmeter current can be measured by the use of standard resistors with the voltage drop across them applied to the terminals X.

In order to avoid leakage between adjacent parts of the potentiometer circuit, a shield (shown by dots in Fig. 7-15) is provided. It is connected to a galvanometer terminal.

Of course, the potentiometers described above do not cover the multitude of makes and designs employed in measuring practice. For example, there are circuits using two and even three independent sources of supply; this offers some special advantages and also simplifies the circuitry. One of the arrangements, for example, consists in effect of three simple resistances each of which is supplied by a separate battery. The voltage drop across a portion of a resistance is taken by two adjustable contact arms; the three drops are then connected in series, giving the balancing voltage readable to the sixth decimal place. Although the operating principle of this arrangement is very simple, it is not at all easy to maintain the three supply currents constant.

There also exist special-purpose potentiometer circuits, such as "step" potentiometers employed for instrument testing. As their name implies, they cannot compare any voltage within a given range. Applicable only to a certain type of instrument, a step potentiometer will measure voltage deviations both ways from a certain step alone. Accordingly, readings of step potentiometers are in terms of the error of the instrument under test, rather than in units of voltage.

In conclusion, here is a short resumé of the requirements that potentiometers are to meet under the relevant USSR State Standard.

Table 7-1

Accuracy class	Max. error, volts	Temperature range within which no corrections are required, °C
0.005	$\pm(0.5\times10^{-4}V+0.2\Delta V)$	19-21
0.01	$\pm(10^{-4}V+0.2\Delta V)$	18-22
(0.015)	$\pm(1.5\times10^{-4}V+0.4\Delta V)$	18-22
0.02	$\pm(2\times10^{-4}V+0.4\Delta V)$	17-25
(0.03)	$\pm(3\times10^{-4}V+0.5\Delta V)$	17-25
0.05	$\pm(5\times10^{-4}V+0.5\Delta V)$	15-30
0.1	±0.1 per cent V_H	15-30
0.2	±0.2 per cent V_H	10-40
(0.5)	±0.5 per cent V_H	10-40

All potentiometers are grouped into accuracy classes: 0.005, 0.01, (0.015), 0.02, (0.03), 0.1, 0.2 and (0.5). The accuracy indices given in brackets ought not to be used in new designs of potentiometers, though instruments of previous makes are allowed for practical use. Whether or not a given potentiometer can be identified with a certain accuracy class depends on the design and principle it embodies. The actual accuracy limits are determined by tests against the relevant specifications or the standard, under specified conditions of temperature. The formulas used to determine the maximum permissible errors of potentiometers and also the temperature ranges within which the instruments may be used without corrections for temperature variations are given in Table 7-1 where the following notation is used: V is a potentiometer reading in volts; V_H is the range of the potentiometer in volts; and ΔV is the value of one step in the lowest decade or the least division of the slide-wire, also in volts.

7-3. The Sensitivity of D. C. Potentiometers

By definition (see Sec. 1-2), the sensitivity of the balancing circuit of a potentiometer, S_{bal}, is the ratio of the change in the detector (galvanometer) current ΔI_D to the ΔE_x in the e. m. f. E_x across the detector circuit, or

$$S_{bal} = \Delta I_D / \Delta E_x. \tag{7-1'}$$

Obviously, the change in the current is inversely proportional to the total resistance of the detector circuit $R_0 = R_D + R_x + R'$, i. e.,

$$\Delta I_D = \Delta E_x / R_0 = \Delta E_x / (R_D + R_x + R'), \tag{7-2}$$

where

R_D = resistance of the galvanometer;
R_x = resistance of the source of the unknown e. m. f.;
R' = balancing resistance setting.

Substituting Eq. (7-2) in Eq. (7-1') gives:

$$S_{bal} = 1/R_0 = 1/(R_D + R_x + R'). \tag{7-3}$$

Thus, the sensitivity of the balancing circuit monotonously increases as all of these resistances decrease. This is the reason why it was stated earlier that low-resistance potentiometers have a greater sensitivity than high-resistance potentiometers. In each particular case R_D and R_x are constant. As for R', it varies, depending on the settings of the contact arms in the balancing circuit, i. e., depending on the value of the unknown voltage. The character of this change depends on the circuit arrangement employed, and also on the operating principle underlying a given potentiometer.

We shall take up some of the possible arrangements separately, beginning with normal double decades. A diagram of this connection is given by Fig. 7-16. Obviously, R' is the total resistance of the two parallel branches R''_x and $R - R''_x$. The resistance of the supply battery is very small and may be neglected. Then:

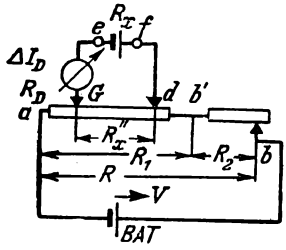


Fig. 7-16

$$R' = R''_x(R - R''_x)/(R''_x + R - R''_x) =$$
$$= R''_x(1 - R''_x/R) = R''_x(1 - E_x/V), \quad (7\text{-}4)$$

where E_x is the unknown e. m. f., and V is the voltage of the supply battery (since $R''_x/R = E_x/V$).

From Eq. (7-4) it follows that R' is always less than R''_x, for which reason it is wrong to assume $R' = R''_x$, as is sometimes done in sensitivity calculations. Practically, it may be taken that $R''_x = R'$ only when very small e. m. f. s are measured, i. e., when $E_x \ll V$.

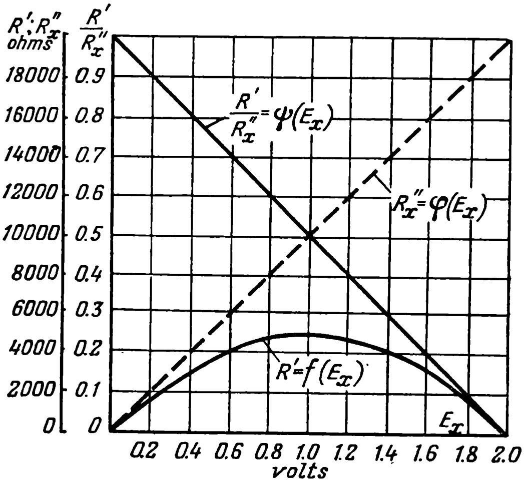


Fig. 7-17

Differentiating Eq. (7-4) for R''_x and equating the derivative to zero, we find that when R' is a maximum, $R''_x = 0.5\,R$ (the maximum being $0.25\,R$). Figure 7-17 gives the relationships: $R' = f(E_x)$, $R''_x = \varphi(E_x)$ and $(R'/R''_x) = \psi\,(E_x)$ when $E_{x\,max} = V = 2$ volts

and $R = 20{,}000$ ohms. It is not difficult to see that the difference between R' and R''_x is appreciable.

Figure 7-18 relates the sensitivity S_{bal} to E_x for the same conditions ($R = 20{,}000$ ohms, $E_{x\,max} = 2$ volts). The curve a is when $R_D = 50$ ohms; the curve b when $R_D = 1{,}000$ ohms, while the curves a' and b' apply to cases when the sensitivity is erroneously calculated on the assumption that $R' = R''_x$ (when R_D is equal to 50 ohms and 1,000 ohms, respectively).

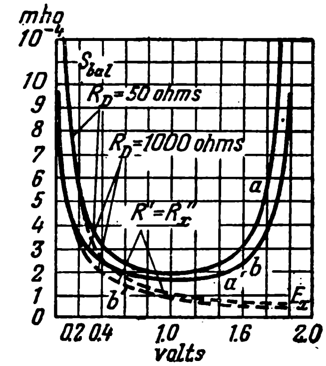


Fig. 7-18

From the above curves one may conclude that the absolute sensitivity of a potentiometer with Feussner decades changes quietly, rising at the beginning and the end of the range.

By the same reasoning, in the case of a potentiometer with reverse-connected double decades (such as at a or b in Fig. 7-5) we obtain the following expression for R' (with the resistance of the supply battery again neglected):

$$R' = R - R''_x + R''_x(R - R''_x)/(R''_x + R - R''_x) =$$
$$= R - R''_x + R''_x(1 - R''_x/R) = R[1 - (R''_x/R)^2] = R[1 - (E_x/V)^2].$$

The notation used is the same as in the previous case.

Figure 7-19 gives the relationships $R' = f(E_x)$ and $S_{bal} = \varphi(E_x)$ for two values of R_D and for the same other conditions ($R = 20{,}000$ ohms, $E_{x\,max} = 2$ volts). From the curves given, it follows that the sensitivity of a potentiometer with reverse-connected decades changes as quietly as in the case of one with normally connected decades. However, it increases only at the end, and monotonously decreases at the beginning, of the range. Thus, for greater sensitivity, E_x should be made as nearly equal to V as possible.

The quietly changing absolute sensitivity of potentiometers with both normal and reverse connected double decades advantageously distinguishes them from potentiometers using shunting decades we are going to discuss. A suitable example is supplied by the early practical potentiometer with shunting decades (see Sec. 7-2) we have not yet taken up. A simplified diagram of connections is given by Fig. 7-20 (where the groups of resistances of 10 and 0.1 ohm are omitted, since they are of minor significance to our discussion).

The arrangement of Fig. 7-20 may be rearranged into the circuit of Fig. 7-21*a*. By means of a mesh-star conversion, we obtain the circuit of Fig. 7-21*b* where the equivalent resistances of the star branches

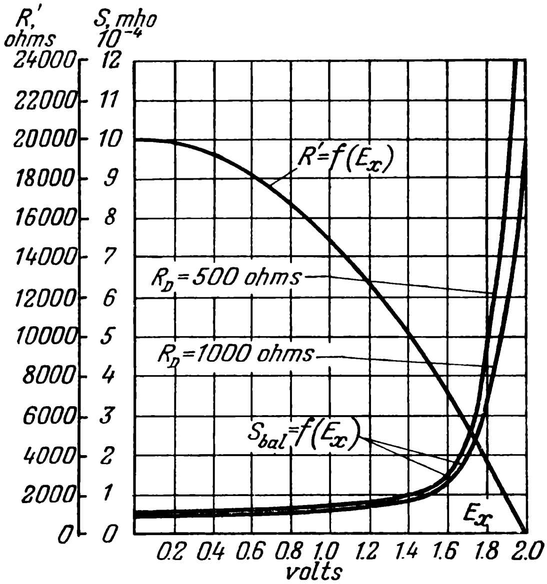


Fig. 7-19

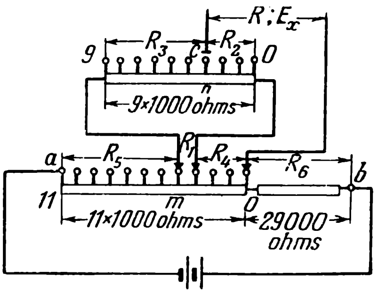


Fig. 7-20

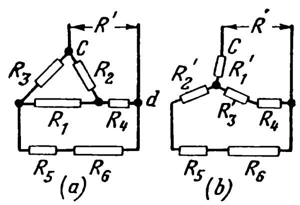


Fig. 7-21

are designated by R_1', R_2' and R_3'. From the converted circuit it follows that

$$R' = \frac{(R_4 + R_3')(R_2' + R_5 + R_6)}{R_3' + R_4 + R_2' + R_5 + R_6} + R_1'. \tag{7-5}$$

Designating the settings of the double and single decades by m and n, respectively, the various terms of Eq. (7-5) can be found thus:

$$R_1' = \frac{R_2R_3}{R_1+R_2+R_3} = \frac{n(9-n)}{10} 1,000;$$

$$R_2' = \frac{R_1R_3}{R_1+R_2+R_3} = \frac{9-n}{10} 1,000;$$

$$R_3' = \frac{R_1R_2}{R_1+R_2+R_3} = \frac{n}{10} 1,000;$$

$$R_4 = m\times 1,000;$$

$$R_5 = (10-m)\,1,000;$$

$$R_6 = 29\times 1,000;$$

$$R_5 + R_6 = (39-m)\,1,000.$$

Substituting the above expressions in Eq. (7-5) we finally get:

$$R' = (m + 0.02505m^2 - 0.10025n^2 + 0.99499mn)\,1,000.$$

Graphically this expression is shown in Fig. 7-22 as $R' = f(E_x)$, while Fig. 7-23 gives the sensitivity ΔI_g as a function of the unknown voltage when $\Delta E_x = 10\mu V$.

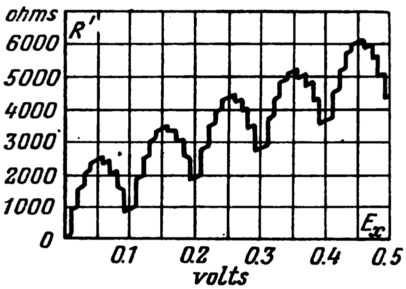


Fig. 7-22

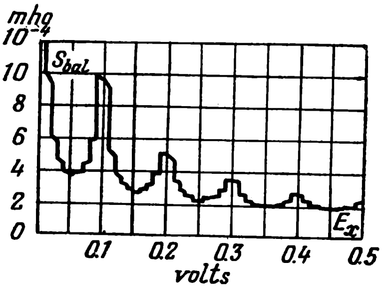


Fig. 7-23

From the curves shown it follows that the resistance of the balancing circuit and the sensitivity of the arrangements employing shunted decades change in increments which may be fairly large. This feature must be borne in mind in critical measurements, especially when the last decimal place is found by interpolation.

The above comparison shows that potentiometers with normal double decades operate under steadier conditions, both in terms of sensitivity and resistance, than those with reverse decades. Consequently, where a problem of choice arises, preference should be given to the former.

Speaking of bridge-type and superposition potentiometers, one above all refers to the fact that the resistance of the detector circuit and, consequently, the sensitivity of the potentiometer remain practically constant over the entire range. This, however, is due to different causes in each case. In the bridge type, as was mentioned *in passim* in Sec 7-1, the value of R' is maintained constant mainly by the use of complementary resistances placed in the two supply current circuits with the objective of producing the requisite current ratios in these circuits. Since the current ratios practically required are 1 : 9, 1 : 10 or anything close to this and never near unity (for there would be little sense in such a potentiometer), the values of the complementary resistances are at least by an order greater than the total resistance of each of the double decades used in the potentiometer. Obviously, the zero value of the balancing voltage can be obtained by placing the bulk of the complementary resistances in the supply current circuit, in a portion not included in the detector circuit (i. e., in the substitution decade circuit or as permanent shunts, if and when such are employed). For this reason, the detector circuit is not shunted in any degree at any setting of the decades—a feature which maintains its resistance constant.

In the superposition type of potentiometer, the resistance of the detector circuit is maintained constant due to the fact that the resistance of the current-distribution circuit is many times the resistance of the measuring decades. Again, the detector circuit is not shunted to any degree, and the sensitivity of the potentiometer remains practically constant.

Finally, a few words must be said about the selection of a galvanometer for use as a null detector in potentiometers. In low-resistance potentiometers the output resistance varies but little with the indication and may be regarded as practically constant. Therefore, it is not difficult to specify a galvanometer for sensitivity and damping.

On the other hand, the output resistance of high-resistance potentiometers may vary within broad limits, depending on the magnitude of the unknown quantity. Therefore, before a galvanometer can be selected for a high-resistance potentiometer, it is required to determine the maximum and minimum values of the output resistance of the potentiometer. Then, with the internal resistance of the unknown source specified, the total resistance and its limits are found. If the total resistance is expected to vary by more than 25 to 30 per cent, a high-resistance galvanometer may be recommended, or a resistance may be placed in series with a low-resistance galvanometer. As a result, variations in the resistance of the detector circuit will be minimized and the requisite damping of the galvanometer ensured.

The output voltage is usually assumed to be equal to the voltage drop across one coil of the last decade in a potentiometer. Where,

however, the limit of readibility of a potentiometer is not fully utilized, the output voltage may be equal to $V_\delta = \delta_x V_x$, where V_x is the unknown voltage and δ_x is the specified fractional (or percentage) error.

The requisite sensitivity of the galvanometer is computed as before, with the difference that for a potentiometer the output resistance is the total resistance across the galvanometer terminals (i. e., the output resistance of the potentiometer plus the internal resistance of the source of the unknown e. m. f. or voltage).

7-4. Applications of D. C. Potentiometers

As noted already, d. c. potentiometers can be used for the measurement of not only e. m. f. s and potential difference, but also of current and resistance, although by indirect methods. These indirect methods yield very accurate results and are widely used in the testing of precision instruments. The basis of these methods is the measurement of the voltage drop across a known resistance.

The measurement of current by these methods is experimentally very simple and requires no special explanations. The basic idea is that a known standard resistance is placed in the circuit whose current is to be measured. The voltage drop across this resistance is measured on a potentiometer, and the current is obtained by Ohm's Law. Since standard resistance coils are manufactured with a resistance of 10^n ohms, where n may have any value from -5 to $+5$, all computation boils down to placing the decimal point in the value of the resultant voltage drop. The magnitude of the resistance is of no importance. Practically, it is chosen, depending on the magnitude of the unknown current, and as small as possible, but so that the resultant voltage drop can be easily measured on a potentiometer. For better accuracy, the standard resistance coil must be such that the voltage drop across it due to the unknown current is close to the range of the potentiometer (then all the decades of the potentiometer will be utilized). The standard resistance coil must be chosen so that its heating will not exceed the maximum safe temperature. In general, this method is similar to the measurement of a current in terms of the voltage drop across a shunt, with all the specific features of the latter. It is widely employed in the testing of ammeters. The standard resistance is connected in series with the instrument under test.

A similar method is employed for measuring (or, rather, comparing) resistances (resistance standards, shunts, resistance boxes, etc.). A variety of arrangements, all based on Ohm's Law, are possi-

ble here, but generally a very simple arrangement is employed. The unknown resistance and an accurately known standard are placed in series in one and the same circuit—a feature which ensures that the same current flows through them. Obviously, the voltage drops across the two will be proportional to their resistances. These voltage drops are measured on a potentiometer, which fact requires two measurements to be taken. Since the two measurements are separated in time, it is essential to make sure that the current does not vary. Incidentally, variations in the supply current are a very common source of errors in this method. The value of the standard resistance and supply current is of no consequence. For better accuracy, however, it is a good plan to make the two readings as nearly equal to each other as possible. For this reason, the standard is chosen of the same order of magnitude as the unknown resistance, and the supply current is adjusted so as to obtain voltage drops convenient for measurement.

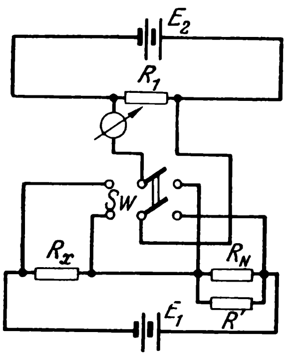


Fig. 7-24

The accuracy of the above methods can be greatly enhanced by the use of special techniques and arrangements. One of them, diagrammatically shown in Fig. 7-24, is due to Malikov. This method, intended for the comparison of resistances nearly equal, is especially advantageous for high-precision work (such as the comparison of standard resistances, etc.).

The main circuit consists of a storage battery E_1, the unknown resistance R_x, a standard resistance R_N, and a supplementary resistance box R' of up to 100,000 ohms, connected in parallel with the greater resistance, R_x or R_N (in Fig. 7-24, it is R_N). The auxiliary circuit includes a second battery E_2 and a standard resistance equal to R_x and R_N. The switch Sw makes it possible to place in the detector circuit either R_x or R_N. In the circuit of Fig. 7-24, ancillary components, such as current selector switches, standardizing resistances, etc., are omitted for simplicity.

If the three resistances (R_x, R_N and R_1) were precisely the same, then the galvanometer would give zero deflection with the switch Sw in any position. This is not so actually. Suppose that when the shunting resistance is R' the galvanometer deflects through α'_x and α'_N. Their difference, other than zero, will be $\alpha'_N - \alpha'_x = \alpha'$. Now let the shunting resistance be R''; the respective difference of deflection will then be $\alpha''_N - \alpha''_x = \alpha''$. For very small galvanometer

deflection it may be written that

$$\left.\begin{aligned} I_1\left(\frac{R'R_N}{R'+R_N}-R_x\right)&=c\alpha'; \\ I_1\left(\frac{R''R_N}{R''+R_N}-R_x\right)&=c\alpha'', \end{aligned}\right\} \tag{7-6}$$

where c is the smallest division of the galvanometer in volts.

For simplicity, let

$$R'R_N/(R'+R_N)=a;$$
$$R''R_N/(R''+R_N)=b.$$

Dividing Eqs. (7-6), we obtain:

$$(a-R_x)/(b-R_x)=\alpha'/\alpha'',$$
$$a\alpha''-R_x\alpha''=b\alpha'-R_x\alpha',$$
$$R_x(\alpha'-\alpha'')=b\alpha'-a\alpha''. \tag{7-7}$$

Adding and subtracting $a\alpha'$ to and from the right-hand side of Eq. (7-7) gives:

$$\left.\begin{aligned} R_x(\alpha'-\alpha'')&=a\alpha'-a\alpha''-a\alpha+b\alpha'; \\ R_x(\alpha'-\alpha'')&=a(\alpha'-\alpha'')-\alpha'(a-b); \\ R_x&=a-\frac{\alpha'}{\alpha'-\alpha''}(a-b). \end{aligned}\right\} \tag{7-8}$$

Substituting for a and b in Eq. (7-8), we finally obtain when $R_x<R_N$:

$$R_x=R'R_N/(R'+R_N)-\frac{\alpha'}{\alpha'-\alpha''}[R'R_N/(R'+R_N)-R''R_N/(R''+R_N)]. \tag{7-9}$$

When $R_x>R_N$, the resistances will interchange.

Since usually $R'\gg R_x$ or $R'\gg R_N$, on the basis of the relationship $(1/1+\varepsilon)=1-\varepsilon$, where ε is a very small quantity, Eq. (7-9) may be given the form:

$$R_x=R_N-R_N^2/R'-\frac{\alpha'}{\alpha'-\alpha''}R_N^2(1/R''-1/R').$$

The values of R' and R'' are so chosen that the galvanometer will deflect in opposite directions in each case. Sometimes (when R_N and R_x are nearly equal) the sign of galvanometer deflection can be reversed by inserting a shunt during one measurement and removing it during the other ($R'=\infty$).

In practice, the method is complicated by the necessity of taking measurements with the current flowing first in one and then in the opposite direction so as to eliminate the effect of thermal e. m. f. s. With appropriate precautions and care, however, the error can be

reduced to an extremely low magnitude of the order of 0.0001 per cent. Further reduction by a factor of 5 to 10 is attainable in the particularly precise comparisons of resistances of one ohm. Another and most important disadvantage of Malikov's method is that satisfactory results can only be obtained with a very stable supply battery. Another method, due to Levin, is more or less free from this drawback. Intended for the comparison of resistances, it is shown diagrammatically in Fig. 7-25. Since the two resistances are placed in a common circuit drawing current from two sources at a time, variations in their voltage have a smaller effect. However, the method is less convenient experimentally and involves the use of two galvanometers.

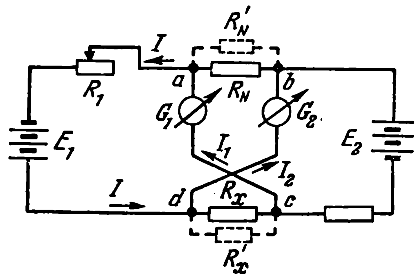


Fig. 7-25

The procedure is this: the circuit of the galvanometer G_2 is interrupted, and the resistance R_1 is varied until the points a and c are at the same potential, which fact will be indicated by the zero deflection of the galvanometer G_1. Then, the circuit of G_2 is completed, and the parallel-connected resistances R_N' and R_x' are adjusted (or only one of them, connected to the greater out of R_N and R_x) until the points b and d are at the same potential, and G_2 gives zero deflection. When the two galvanometers show zero deflection simultaneously, this is an indication of the complete equality between the voltage drops across the respective resistances, or

$$R_N R_N' /(R_N + R_N') = R_x R_x'/(R_x + R_x').$$

It is obvious that at complete balance the disconnection of one of the galvanometers will not in any way tell on the deflection of the other. Therefore, one of the galvanometers may be replaced by a shorting strap and a switch. Then the circuit must be adjusted until the galvanometer shows zero deflection with the switch both closed and opened.

To complete the picture of the applicability of the d. c. potentiometer method, a brief account of wattmeter testing should be added.

Since the testing of a wattmeter boils down to measuring the current in and the voltage across the respective circuits of the instrument, the feasibility of such tests with a potentiometer is quite obvious.

A wattmeter is usually tested at the rated voltage across its potential (shunt) circuit, and its readings, for various measured values of the current, are noted. The current (series) and the potential circuits are isolated and are supplied from independent sources. A check on the voltage can be maintained and the current measured either on two potentiometers (in which case the voltage can be monitored continuously) or on a single potentiometer by connecting it to the respective wattmeter circuits in turn. Ordinarily, the rated voltage of wattmeters exceeds the range of potentiometers, and use has to be made of a volt-box. The procedure of a test is as follows. Using a potentiometer, the rated voltage is set across the potential circuit of the wattmeter being tested (it should be monitored either continuously or intermittently, as qualified above, during the test). Then the current is varied in the series circuit of the wattmeter so as to obtain readings at all the numbered divisions of its scale. The power is obtained as the product of the rated voltage and the various measured values of the current. A comparison of the product with the respective reading of the wattmeter will give the accuracy of the instrument.

Chapter Eight

THE ALTERNATING-CURRENT POTENTIOMETER METHOD

8-1. Special Features of the A. C. Potentiometer Method

The potentiometer method, like the bridge method, can be successfully employed for the measurement of alternating voltages and currents. Of necessity, the balance condition has to be expressed in a complex form, and so the balancing voltage (or the slide-wire voltage drop) has to be adjusted for both magnitude and phase so as to obtain a balance.

Unlike the bridge method, the application of the potentiometer method to a. c. measurements requires the fulfilment of certain specific conditions. Failure to satisfy them will introduce complimentary errors or make a measurement entirely impossible. The point is that in the case of two (generally, independent) supply sources, they may differ in frequency and, especially, in waveform. The difference in frequency is usually eliminated by supplying both the potentiometer circuit and the circuit under test from a single source, with the two circuits isolated where necessary, by the use of transformers. Naturally, any difference in frequency will then be avoided. As an alternative, the two circuits can be supplied from two generators mounted on a common shaft driven by a common motor. One way or another the necessity of maintaining the frequency of the two voltages (across the slide-wire and the test circuit) is only too obvious, since otherwise balance (i. e., a zero difference between the vectors of the two voltages) would not be obtained.

Differences in the waveform are more difficult to eliminate for the simple reason that they may be brought about locally, due to the presence of nonlinear components (such as iron-cored coils) in the circuit. In order to be able to secure a balance in the presence of higher harmonics, the null detector is ordinarily either a vibration galvanometer or a frequency-selective valve detector (for measure-

ments at higher frequencies) *. Yet, the likelihood of an error is not eliminated, because a frequency-selective detector tuned to the fundamental frequency (the first harmonic) leaves out the higher harmonics. Therefore, the null detector will give zero deflection for a balance at the fundamental frequency and not at the instantaneous values of the resultant waveform.

To obtain an idea about the effect of the waveform on the measurement with a. c. potentiometers, we shall determine the magnitude of the associated error.

The r. m. s. value of the nonsinusoidal unknown voltage is given by:

$$V=\sqrt{V_1^2+V_2^2+V_3^2+\ldots}=V_1\sqrt{1+(V_2/V_1)^2+(V_3/V_1)^2+\ldots}, \quad (8\text{-}1)$$

where $V_1, V_2, V_3, \ldots$ are the r. m. s. voltages of the respective harmonics.

Similarly for the balancing voltage:

$$V'=V_1'\sqrt{1+(V_2'/V_1')^2+(V_3'/V_1')^2+\ldots} \quad (8\text{-}2)$$

At balance, the voltages of the first harmonics are made equal, or $V_1=V_1'$.

Substituting them from Eqs. (8-1) and (8-2) gives:

$$V=V'\sqrt{\frac{1+(V_2/V_1)^2+(V_3/V_1)^2+\ldots+(V_n/V_1)^2}{1+(V_2'/V_1')^2+(V_1'/V_1')^2+\ldots+(V_n'/V_1')^2}}. \quad (8\text{-}3)$$

Since, however, the r. m. s. values of the higher harmonics are small in comparison with the fundamental frequency and their squared ratios are small in comparison with unity, the radical in Eq. (8-3) may be rewritten using the relationships:

$$\sqrt{\frac{1+\alpha}{1+\beta}}\cong\sqrt{1+\alpha-\beta}\cong 1+\frac{1}{2}(\alpha-\beta),$$

where $\alpha\ll 1$ and $\beta\ll 1$.

Assuming $V'\cong V_1$, we obtain from Eq. (8-3):

$$V\cong V'\left[1+\frac{1}{2(V')^2}(V_2^2-V_2'^2+V_3^2-V_3'^2+\ldots+V_n^2-V_n'^2)\right]\cong$$

$$\cong V'\left[1+\frac{\sum\limits_{n=2}^{n=\infty}V_n^2-\sum\limits_{n=2}^{n=\infty}V_n'^2}{2(V')^2}\right]. \quad (8\text{-}4)$$

* It should be noted that if the null detector is not frequency-selective (or tunable), exact balance cannot be practically secured for a distorted waveform, since while the fundamental frequency may be accurately balanced, there will be higher harmonics left unbalanced.

As follows from Eq. (8-4), there will be no error ($V = V'$) in two cases:

(1) when the voltage waveforms are sinusoidal, i. e., there are no higher harmonics, i. e.:

$$\sum_{n=2}^{n=\infty} V_n^2 = \sum_{n=2}^{n=\infty} V_n'^2 = 0;$$

(2) when the two voltages are identical in waveform, i. e., have the same harmonics, and consequently:

$$\sum_{n=2}^{n=\infty} V_n^2 - \sum_{n=2}^{n=\infty} V_n'^2 = 0.$$

Ordinarily, the slide-wire voltage is closer to the sine waveform, i. e., $\Sigma V_n^2 - \Sigma V_n'^2 > 0$.

Consequently, if the waveform error is not allowed for, the measured voltage, V', will be somewhat lower than the true voltage V.

The percentage error will be

$$\frac{\Delta V}{V}\,\% = \left[\frac{\sum\limits_{n=2}^{n=\infty} V_n^2 - \sum\limits_{n=2}^{n=\infty} V_n'^2}{2V'^2}\right] 100\%. \tag{8-5}$$

The third harmonic is the most likely to be present. Let us determine its r. m. s. value in any one of the two voltages for which the error will not exceed 0.1 per cent (on the assumption that the other harmonics are absent):

$$V_3' = \sqrt{0.02 \times 0.1 V'^2} = 0.045 V'.$$

Thus, an error of 0.1 per cent due to waveform distortions will be introduced when the 3rd harmonic accounts for as little as 4.5 per cent of the total voltage. Similarly it can be shown that when $V_3' = 0.14\, V'$, the error will be one per cent. So, the waveform error is not one to be easily dismissed; every effort must be made in critical measurements to eliminate it by maintaining the voltage waveform as close to sinusoidal as possible. Still, there are many cases where waveform distortions are inevitable. Because of this, the potentiometer method loses in accuracy (and, consequently, in value) when applied to a. c. measurements.

A third disadvantage of the a. c. potentiometer method, also contributing an error, is the rather imperfect standardization procedure. While in the case of d. c. potentiometers this is done directly with a standard cell, which is a standard of utmost precision, and the

accuracy of measurement is very high, nothing of the kind is feasible with a. c. potentiometers: there exists no such thing as a standard of alternating e. m. f. Therefore, a. c. potentiometers are standardized by measuring either their supply current with a direct-reading ammeter of high precision, or the applied voltage with a voltmeter. In any case, however, the accuracy of standardization is limited by the accuracy of the indicating instrument used.

Ordinarily, precision type electrodynamometer instruments are employed for standardization purposes. Yet, their accuracy cannot be better than 0.2 to 0.3 per cent in the best of cases. In effect, complimentary errors due to the existence of higher harmonics, imperfect adjustment of the circuit components, etc., reduce the total accuracy of a. c. potentiometers to 0.5 per cent—a performance below that of direct-reading instruments of high accuracy.

On the other hand, the fundamental property of the potentiometer method in general—the absence of current in the detector circuit at balance*—fully holds in the measurement of alternating currents and voltages as well. Owing to this, a. c. potentiometers enable actual e. m. f. s to be measured and low-power circuits and sources investigated where the load due even to the indicator would be damaging. This feature is of special value in a. c. measurements, since all other instruments suitable for the purpose** draw much energy (hundreds of times as much as moving-coil instruments). Also, a. c. potentiometers are more sensitive than a. c. indicating instruments.

Furthermore, a. c. potentiometers prove extremely useful where it is essential to determine not only the magnitude but also the phase of the potential difference measured. Consequently, the results can be presented in vector form (which is required in investigations of electric circuits and networks, for example).

The advantages and disadvantages of a. c. potentiometers examined above determine the principal field of application for them: laboratory investigations of low-power electrical systems, especially where vector quantities have to be determined. They have been successfully employed for magnetic tests and measurements and also for the measurement of nonelectrical quantities. On the whole, they have not found wide use, although they are fundamentally promising and even indispensable in certain applications.***

* At balance, the internal resistance of a potentiometer may be assumed to be equal to infinity.

** We leave out very specific and promising electronic instruments the input impedance of which can be made very high.

*** The potentiometer method is widely employed for testing current and voltage instrument transformers. The special-purpose sets employed to this end are not discussed in this book.

In conclusion, it may be noted that many investigators have tried to avoid the use of an indicating instrument for standardizing a. c. potentiometers. In general, three approaches have been tried. In the first, use is made of a standard of alternating e. m. f. in the form of a miniature generator excited by permanent magnets. In the second, the alternating supply current is compared with direct current for the thermal effect with the aid of a differential thermo-cell or any other suitable device (based on the thermal effect). The direct current is measured conventionally with a potentiometer, or with a simple thermo-cell whose thermo-electric e. m. f. is determined by the balance method. The heater is connected in turn in the a. c. and the d. c. circuit, and the direct current is adjusted until it produces the same thermo-electric e. m. f. as the alternating current. Then, obviously, the r. m. s. value of the alternating current and the direct current are equal.

In the third, and more recent, approach, use is made of various voltage regulators, notably bridge-type stabilizers employing non-linear resistances. Indeed, if two resistances whose values depend on the current flowing in them are placed in opposite arms of a bridge, the balance will be conditioned by the value of the supply current. The nonlinear resistances can be metal filaments of the barretter type or semiconductor devices. Obviously, the sensitivity of the entire network will increase with increasing positive temperature coefficient of resistivity of the device.

All of the above devices somewhat improve the performance of potentiometers. However, a satisfactory and sufficiently simple solution still remains to be found. The scope of work under way gives enough reason to hope that the solution will be found before long.

8-2. Principles of Construction and Practical A. C. Potentiometers

Before an examination of practical a. c. potentiometers, the principles on the basis of which they can be classed must first be established.

Since in an a. c. potentiometer the balance is sought between two vector quantities, provision must be made for adjusting both the magnitude and phase of the balancing voltage. This objective can in principle be achieved by two methods.

By one of them, the unknown voltage is balanced by means of a single voltage which is variable in both magnitude and phase, each capable of independent adjustment; formally, both voltages are presented in the polar notation.* By the other, the balancing voltage is obtained by varying the magnitudes of two alternating voltages

* As in the Drysdale instrument.—*Tr.*

in quadrature with each other; formally the unknown voltage is presented in a system of rectangular coordinates.*

Both methods are practically employed on a fairly large scale. The first is more demonstrative and gives the requisite result immediately, while the second is simpler as far as the construction of the potentiometer is concerned, but involves the computation of magnitude and phase in terms of the two components.

Potentiometers in the first group are called polar, and in the second group coordinate.

A polar potentiometer consists in effect of two independent units: a potentiometer of the ordinary direct-current type to adjust the magnitude of the balancing voltage, and a phase-shifter.

The potentiometer proper is standardized by a simplified method, and the requisite balancing voltage is obtained by any of the methods discussed in Chapter 7, most commonly with a slide-wire arrangement or with shunting or double decades. The results thus obtained are quite satisfactory, since the voltage is to be adjusted to the third decimal place only.

The phase-shifter may be either of the induction type (based on a rotating magnetic field) or of the parametric type (based on nonlinear circuit elements). The former consists of a ring-shaped stator within which, fitting closely inside it, is a rotor which carries a winding supplying the potentiometer slide-wire circuit. A rotating field is produced when currents flow in the stator winding, and the phase of the rotor current can be changed, relative to the stator supply voltage, by rotating the rotor through any desired angle with a worm-gear transmission (within 360°). The phase displacement of the secondary e. m. f. is equal to the angle through which the rotor is moved from its zero position.

A parametric phase-shifter is, in a general case, a network so arranged that due to the constants of its components the phase of the output voltage is displaced with respect to the phase of the input voltage. The objective sought (but practically unattainable) is to be able to obtain a phase displacement of 360° without any reconnections in the network, so that the alteration of phase is not accompanied by alteration of the magnitude of the output voltage. Most often, parametric phase-shifters are various a. c. bridges whose arms are so chosen that the condition of phase balance:

$$\varphi_1 + \varphi_3 = \varphi_2 + \varphi_4,$$

is never satisfied. Most commonly, this will be a bridge consisting of two resistances and two capacitances. If the resistances are placed in opposite arms (say, the first and third), and the capacitances in the other pair of opposite arms (the second and fourth), the bridge

* As in the Gall or Campbell-Larsen instrument.—*Tr.*

will never be balanced ($\varphi_1 = \varphi_3 = 0$, and $\varphi_2 = \varphi_4 = -90°$). With the impedance of one of the arms being varied from zero to infinity, the phase of the output voltage will be displaced only through 180°. Since adjustment within the limits $0 < Z < \infty$ is unfeasible, the actual phase displacement of the output voltage will be somewhat smaller than 180°. Theoretically, the output voltage will be equal in magnitude to the input voltage only if there is no load at the output, which is never the case in practice. Use may also be made of bridges using one capacitance or inductance and three resistances; the phase displacement will then be smaller, and an additional reconnections have to be made in the bridge, if a phase shift of 180° is to be obtained.

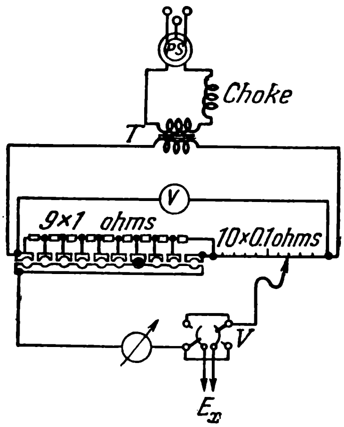


Fig. 8-1

Figure 8-1 gives the connections of a polar potentiometer used at one time in laboratory work. Referring to the figure, the three-phase induction-type phase-shifter *PS* supplies via an isolating transformer *T* the slide-wire circuit of the potentiometer consisting of a plug-type resistance box (of nine resistance coils, each of 1 ohm resistance) and a slide-wire 1 m long and of 1 ohm resistance. The potentiometer is standardized against a voltmeter. There is also a supplementary (standardizing) d. c. circuit, not shown in the diagram, making it possible to check the voltmeter against a standard-cell at any moment. The purpose of the choke is to suppress, at least in part, the higher harmonics. The range of the potentiometer is 5 volts.

In an improved modification of laboratory polar potentiometer with a three-phase induction-type phase-shifter, there are four adjustable resistances: one in the form of a single dial, two double dials, and a slide-wire. The range can be either 1.5 volts or 15 volts, depending on the set value of the supply current. The accuracy of the potentiometer is of the order of 0.1 per cent when the voltage is practically sinusoidal. The phase displacement is measured accurate to within 2 per cent (for phase angles of 1 to 2°) and about 0.5-1.0 per cent (for phase angles greater than that).

In both potentiometers examined above use is made of a three-phase shifter, which is a certain inconvenience, since a three-phase supply is required. Although the asymmetry of the supply voltage distorts the rotating field, the operation of the device is independent of the supply frequency. Of course, a single-phase shifter can be

constructed in which the rotating field would be produced by splitting the phase by means of a capacitor and a resistor. However, such a device would be frequency-dependent. A polar potentiometer using a phase-splitting transformer is shown in Fig. 8-2. The balance-securing resistances of this potentiometer comprise a single shunting dial and a slide-wire. The standard supply current (50 mA) is set by means of an adjusting resistance. The potentiometer has a shunt with which the supply current and, consequently, the range of the instrument can be varied (giving 1.815 and 0.181 volt, respectively). The accuracy of the potentiometer is about 0.2 per cent.

The connections of a potentiometer using a parametric phase-shifter are given by Fig. 8-3. The potentiometer is designed specifically for the measurement of impedances and has no devices for adjusting the supply current which is common to both the potentiometer

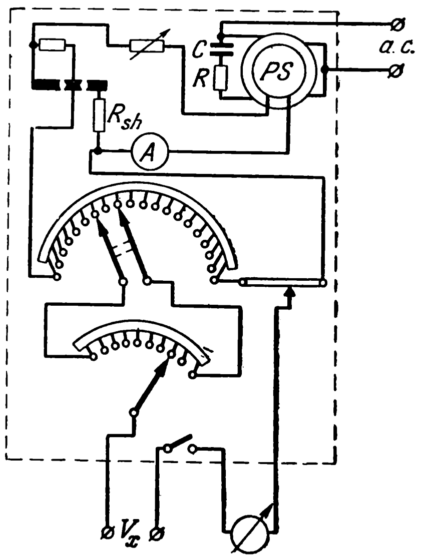


Fig. 8-2

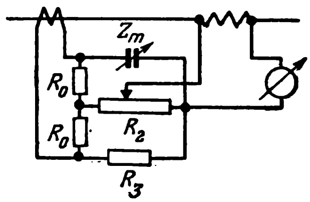


Fig. 8-3

and the circuit under test. Since the phase-shifter does not produce a phase displacement of 180°, a three-pole switch is added to make the requisite reconnections in the device. Also there is a two-pole ("sign-changing") switch in the unknown voltage circuit which reverses the flow of current, thereby extending the range of phase displacement to 360°. The accuracy of impedance measurements is rather low, being 0.5 to 0.7 per cent. The design formulas are rather cumbersome.

A parametric phase-shifter is also employed on a polar potentiometer designed for magnetic tests. In this potentiometer, the phase displacement need not be calculated in terms of the network constants since it is indicated directly by a suitably connected wattmeter.

A major disadvantage of potentiometers using phase-shifting transformers is low accuracy in the measurement of phase angles

(which are read from the dial of the phase-shifter). The accuracy is usually 0.5 to 1 degree, which often proves inadequate. The low accuracy is due to a number of factors: difficulties in making a phase-shifter which would produce a precisely circular rotating field, unbalanced load on the stator windings, variations in the air gap all the way round the circle, etc.

Parametric phase-shifters suffer from low accuracy in the measurement of both phase and magnitude. Because of these drawbacks,

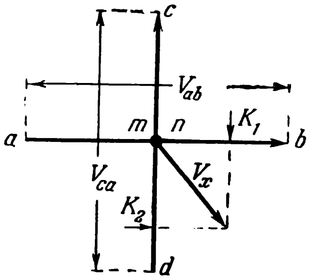


Fig. 8-4

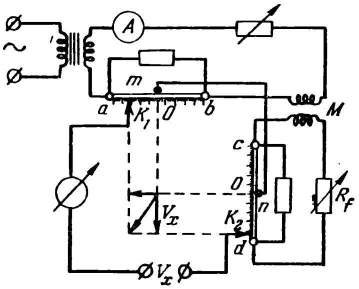


Fig. 8-5

polar potentiometers are no longer manufactured in the Soviet Union.

In coordinate potentiometers (and, indeed, in all a. c. potentiometers), use is most commonly made of the arrangement employing a phase-splitting device. This device may be either a coreless transformer or a coreless mutual inductor. The idea of the arrangement is this: two voltages in quadrature with each other are applied to two slide-wires whose centre points are electrically connected. The detector and the unknown voltage are connected between the contact arms of the slide-wires, so the centre points of the slide-wires serve as the origin of rectangular coordinates, as shown in Fig. 8-4. Obviously, when the two contact arms are set in the middle of the respective slide-wires, the voltage difference between them is zero. If one of the contact arms is left stationary and the other is shifted, a voltage difference appears between them, in phase with the supply voltage of the slide-wire when the contact arm is moved in one direction and in anti-phase when shifted in the opposite direction. The same will take place if the first contact arm is moved and the second is left stationary, with the only difference that the supply voltage of the second slide-wire is in quadrature with that of the first slide-wire.

Now assume that both contact arms have been moved from their zero position. Obviously, two voltages now act in the detector cir-

cuit, proportional to the displacement of the contact arms from the middle of the slide-wires, and in quadrature with each other. The resultant voltage will be the vector sum of the component voltages. If both contact arms are shifted in the same direction, the resultant vector will lie in one quadrant, in the first, say. If both are shifted in the opposite direction, the resultant vector will be reversed in phase and will lie in the third quadrant. When the contact arms are moved in different directions from their zero positions, the resultant vector will lie either in the second or in the fourth quadrant.

Naturally, the slide-wires should be electrically connected at certain points, since otherwise the detector circuit will be interrupted. On the other hand, if they were connected at the ends instead of at the middle, the resultant vector would only move within 90° in a single quadrant (depending on which points were connected). It is this, seemingly unimportant, feature—connection of the slide-wires at the middle—that enables the experimentor to obtain a resultant vector of any magnitude (within the range of the potentiometer, of course) and phase without any additional reconnections in the circuit.

The connections of a coordinate potentiometer are given by Fig. 8-5. The notation is the same as in the diagram of Fig. 8-4. The portion between a and b is called the *in-phase* slide-wire since the voltage across it is practically in phase with the current which supplies it. The portion between c and d is called the *quadrature* slide-wire since the voltage across it has a phase difference of 90° with that across the in-phase slide-wire due to the quadrature device M, which is a suitably constructed air-core transformer. The standard current is set by an ammeter simultaneously in both slide-wires. Since the secondary e. m. f. of the air-core transformer is proportional to the supply frequency, the supplementary adjusting resistance R_f must be changed when the supply frequency is changed.

The total voltage drop across each slide-wire is 40 mV (for a current of 0.5 A). This type of potentiometer is very simple and convenient to use. Unfortunately, its accuracy is rather low.

Figures 8-6 and 8-7 give the connections of two practical coordinate potentiometers intended for work in the audio frequency range. Referring to the diagrams, each of the potentiometers employs a phase-shifting method of its own, differing from the one examined earlier. In the circuit of Fig. 8-6, one branch consists of a resistance and an inductance, and the other of a resistance and a capacitance. For the currents I_1 and I_2 in branches I and II to be in quadrature, one must satisfy the condition $R_1 = R_2 = R = \sqrt{L/C}$, where R_1 and R_2 are the resistance of branches I and II, respectively. Since the left-and right-hand sides of each branch are symmetrical, the potential difference between the mid-points of the slide-wires (the

points O_1 and O_2) is zero. Thus, the unknown voltage can be balanced in any of the four quadrants of the cordinate plane. When the condition $\omega = 1/CR$ is fulfilled, the currents in the two branches are equal. Such potentiometers find use in wire-communications systems. A drawback of the arrangement is that in addition to the parameters of the circuit elements the determination of the components of the unknown voltage calls for knowledge of the supply frequency. Also, variations in the supply frequency change currents in the branches and, consequently, affect the calibration of the slide-wires.

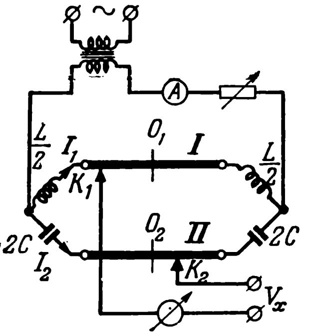


Fig. 8-6

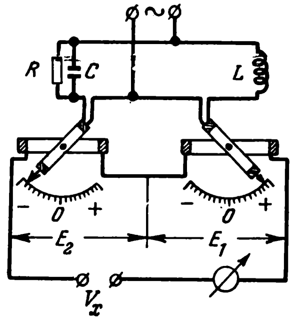


Fig. 8-7

The potentiometer of Fig. 8-7 intended for work in the frequency range 500-5,000 c/s, is advantageous in that capacitive currents have but a negligible effect on the results, since the potentiometer and test circuits are not connected electrically. A phase difference of 90° between the currents in the primaries of the two mutual inductors and, consequently, between the e. m. f. s induced in the secondaries, i. e., in the potentiometer circuit, is provided by connecting the primaries in two parallel branches supplied from a common source, in series with an inductance L, resistance R and capacitance C. The magnitude and sign of the induced e. m. f. s E_1 and E_2 depend on the deflection of the secondaries with respect to the primaries. As a result, the unknown voltage V_x can be balanced in any of the four quadrants of the coordinate plane, i. e., $V_x = \pm E_1 \pm jE_2$. The values of E_1 and E_2 can be directly read from the scales.

A disadvantage of this scheme and, indeed, of any potentiometer using mutual inductors as a quadrature device is that special measures have to be taken in order to shut out extraneous magnetic fields which may appreciably falsify the reading.

The commercial types of Soviet-made potentiometers in current production are based on the coordinate scheme using a single mutual inductor as a quadrature device. This arrangement is convenient

to use and has a fairly high accuracy. The discussion of one such potentiometer follows.

Figure 8-8 gives the connections of the Type P-56 coordinate potentiometer manufactured by the Tochelektropribor Factory in Kiev. The two slide-wire circuits are supplied with currents I_1 and I_2 which have a phase difference of 90° due to an air-core transformer

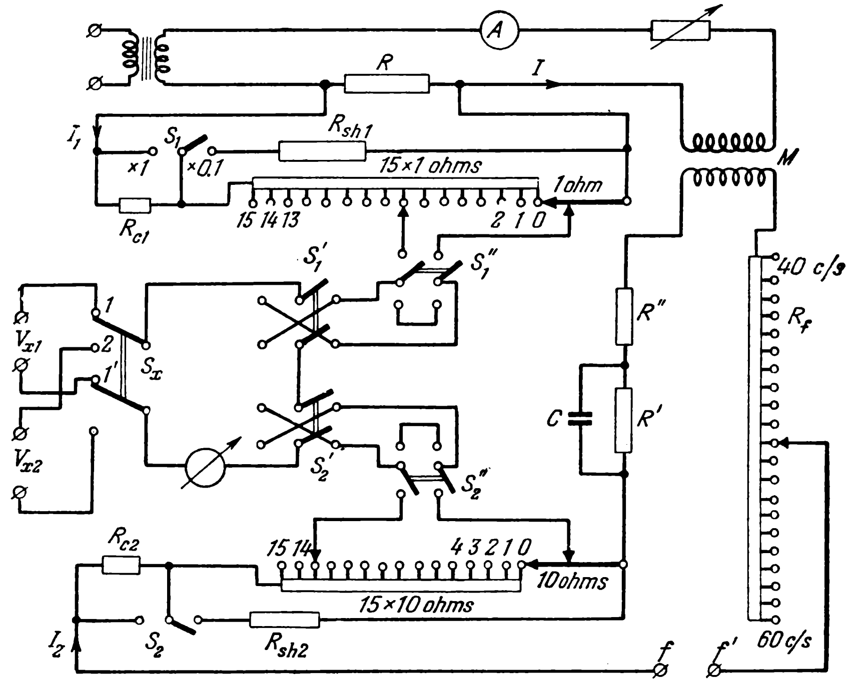


Fig. 8-8

M, similar to the one used in the circuit of Fig. 8-5. The standard current I of the potentiometer is 50 mA; it is adjusted by a class 0.2 electrodynamometer ammeter A supplied with the potentiometer. The in-phase slide-wire circuit includes the primary of the transformer M and a resistance R. The voltage drop appearing across R is fed to a slide-wire of 1 ohm and a series-connected decade having 15 coils of 1 ohm each. The quadrature slide-wire circuit contains the secondary of the transformer M, a slide-wire of 10 ohms, a decade having 15 coils of 10 ohms each, and a resistance box R_f serving to maintain the current I_2 constant at 40 to 60 c/s, and also resistances R' and R'' and a capacitance C, included for adjustment of exact quadrature. The null detector is a Type ВГ vibration galvanometer.

The constants of the circuit elements have been so chosen that the voltage drop across each coil in the decades and across the full length of the slide-wire is 0.1 volt. Thus, the range of the potentiometer along each of the axes is 1.6 volts. The range of the potentiometer can be reduced by a factor of ten (independently on either axis) by means of switches S_1 and S_2, current shunts R_{sh1} and R_{sh2} and compensating resistances R_{c1} and R_{c2} which serve to maintain the input impedance of the circuit when a shunt is brought in.

The potentiometer has two ranges: from 1.6 volts down to 0.001 volt and from 0.16 volt down to 0.0001 volt.

S_1' and S_2' are two "sign-changing" switches which may be necessary to reverse the direction of each of the two components of the slide-wire voltage. Therefore, the potentiometer can balance the voltage in any of the four quadrants of the coordinate plane within 360 electrical degrees. S_1'' and S_2'' are selector switches with which any of the two circuits can be disconnected without interrupting the balancing-voltage circuit. Owing to this, the operator can balance and measure the unknown voltages separately or, if they are in phase with any of the coordinate axes, to adjust the initial phase displacement of the supply current so that the vector of a certain voltage coincides with one of the axes. S_x is a selector switch by which the unknown voltages are placed in circuit, thereby making it possible to balance them in turn for comparison. The terminals f-f' receive the leads from an external resistor which may be necessary to extend the frequency range of the potentiometer to above 60 c/s. Ordinarily, these terminals are made common by a shorting link.

The potentiometer is furnished complete with a Type P-501 volt-box which gives the ranges of 3,7.5, 15, 30, 75, 150 and 300 volts. The potentiometer current is set to its standard value by means of a class 0.2 instrument (a Type ЭЛМА milliammeter for 500-1,000 mA).

8-3. Applications of A. C. Potentiometers

As follows from Section 8-2, the potentiometer method can be used to measure both the magnitude and phase of the unknown voltage. It should be remembered that the phase of the unknown voltage is given in relative terms, and so there must be a reference vector. Usually, this is the vector of the supply, coinciding with the voltage vector of the in-phase circuit. In some cases, the phase of the supply is made by a phase-regulating device to coincide with the phase of some other voltage (or current) vector, which may prove more convenient sometimes (for example, when the vector diagram of a complex network is to be obtained).

Apart from this fundamental application, a. c. potentiometers, like d. c. potentiometers, can be used to measure current and resist-

ance. What has been said in this connection about d. c. potentiometers (see Sections 7-1 and 7-4) fully applies to a. c. potentiometers. The range of potentiometers can be extended by the use of volt-boxes; currents are measured by determining the voltage drop due to the unknown current across an accurately known resistance; resistances are measured by comparing the voltage drops across the unknown and a standard resistance; the same considerations govern the selection of the values of standard resistances. Yet, a. c. work has certain specific features of its own. Thus, volt-boxes must be non-inductive; on the other hand, capacitive volt-boxes may be employed (especially for work at elevated frequencies); currents may only be measured with noninductive standard resistance coils.

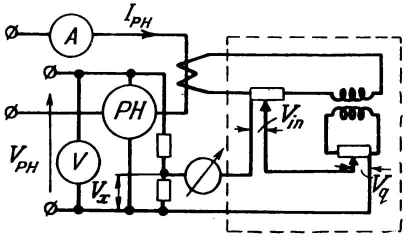


Fig. 8-9

Since a. c. potentiometers measure both the magnitude and relative phase of the unknown, they can handle a much wider range of problems: determine any components of alternating voltages and current as projections of one vector on the other, measure impedances, active power, the resistive and reactive components of impedances and admittances, the transfer factors of four-terminal networks, the magnetic characteristics of materials, and some other quantities. In many of these applications a. c. potentiometers ensure a higher accuracy than any other instruments.

In some cases, an a. c. potentiometer can provide an easy solution to a problem which is more difficult to tackle by other methods. This is true, for example, of measuring the impedance of iron-cored inductors. Since an inductor is a nonlinear element (its inductance is a function of the current flowing through it), it is essential that the measurement can be made at any desired value of current. Obviously, this can be done with an a. c. potentiometer. To this end, incidentally, the potentiometer should draw its supply from the secondary of the current transformer whose primary is connected in the supply circuit in series with the inductor under test.

Last but not least, a. c. potentiometers are exceedingly useful as instruments of comparison in the study of magnetic circuits. In a typical magnetic test, the magnetizing coil of the specimen is connected in series with a noninductive standard resistance, the voltage drop across the latter is measured with a potentiometer, then the magnetizing current is determined in terms of this voltage drop, and finally the strength of the magnetic field is found on the basis of the magnetizing current. The same potentiometer, suitably reconnected, measures the e. m. f. in the search coil of the specimen, and the magnetic induction in the specimen is then found. The magnetizing current of the specimen and the supply current of the potentiometer are drawn from a common source via an isolating transformer with separate secondaries and adjusted by autotransformers. Finally, the total loss in the specimen is determined on the basis of the active component of the magnetizing current.

In conclusion, it may be added that a. c. potentiometers can be used for the calibration and testing of electrical measuring instruments. Figure 8-9 gives the circuit for testing power factor meters with a coordinate potentiometer (such as the Type P-56). For its operation, the test circuit depends on the balancing of part of the unknown voltage V_x applied to the shunt circuit of the power factor meter with the vectorial sum of two voltage drops one of which, V_{in}, is in phase with the current in the series circuit of the power factor meter, and the other, V_q, is in quadrature with it. In Fig. 8-9, the in-phase and quadrature circuits are represented by slide-wires for simplicity. Obviously, at balance the phase displacement will be given by $\varphi = \tan^{-1}(V_q/V_{in})$. When the circuit components are suitably selected, the test can yield fairly accurate results.

Chapter Nine

CONTROL OF STRAY EFFECTS

9-1. Principal Sources of Stray Effects

The results obtained with any bridge or potentiometer will always be subject to errors. One of the causes of errors are electromagnetic effects and also internal thermal and contact e.m.f.s. They may all give rise to unwanted currents and voltages in the system (i.e., other than those taken from the supplies or deliberately introduced by the passive elements of the circuit), thereby distorting the expected distribution of the voltages or currents in it. Because of this, the operator will be misled as to the actual condition of the system.

The character of errors depends to a great extent on the kind of current used in a given measuring system.

In systems operating on direct current, these effects manifest themselves mainly in the form of thermal and contact e.m.f.s and as leakage currents.

Thermal e.m.f.s may be, above all, due to the heating of junctions and contacts of dissimilar metals in the system by the supply current or by extraneous sources of heat (radiators, direct sunlight, etc.). Contact e.m.f.s are generated by the rubbing of contacts made of dissimilar metals (in switches, keys, and slide-wire moving arms). This effect is especially noticeable when the mating contacts are made of metals which generate considerable thermal e.m.f.s against each other (for example, when the moving arm is made of copper, and the slide-wire of constantan).

In manual balancing, the generation of e.m.f.s due to friction between contacts made of dissimilar metals is of minor consequence, since these e.m.f.s rapidly drop to zero as soon as the moving contact is stopped (still, they handicap the balancing procedure and make the operator nervous). In automatic balancing, unwanted e.m.f.s may set the system hunting, thereby slowing down the whole procedure. In fact, if the quantity being measured is subject to rapid and considerable variations, these e.m.f.s may render operation of the system unsatisfactory altogether.

Another cause of unwanted e.m.f.s may be the heating of the knobs, studs and buttons by the operator's hand. This is particularly noticeable when working at low ambient temperature either indoors or outdoors.

Leakage between adjacent parts of a measuring circuit is due to poor insulation. It is especially dangerous in d.c. measuring circuits containing high resistances (over 10^5 ohms) and often operating on considerable voltages (50 to 100 volts and more) or using galvanometers of high sensitivity. Poor insulation in sections of a high-resistance circuit, coupled with a considerable potential difference be-

tween them, gives rise to leakage currents which distort the distribution of the main currents in the system. Thus, a conducting path due to poor insulation may shunt the unknown resistance and also facilitate leakage from the terminals of the galvanometer, which is particularly undesirable in the case of null methods.

As for a.c. measuring systems, they are mainly prone to electromagnetic effects. Other interfering factors are of minor importance for a number of reasons, above all because of the character of alternating current, and also because lower accuracy is expected of such systems, although they may be present.

The principal sources of electromagnetic effects in a.c. systems are the electric and magnetic fields set up between the various parts of the system and between the system and nearby objects. Also, the system may be subject to extraneous magnetic fields. As a consequence, the accuracy and reproducibility of the system may be impaired; readings may prove dependent on the position of the operator relative to the system and of the system relative to nearby objects.

Generally, the electric and magnetic fields between the various parts of the system and between the system and the surroundings act through unwanted coupling due to poor insulation, the capacitance and inductance of the circuit elements with respect to one another and to earth or surrounding objects (such as the operator's body). Therefore, examination of electromagnetic effects on measuring circuits naturally resolves into the examination of two factors: stray coupling and extraneous electromagnetic fields.

Stray coupling may be electrostatic and magnetic. It may be divided into two forms:

(1) coupling between circuit elements (internal coupling);

(2) coupling with surrounding objects, earth or other current-carrying circuits (external coupling).

Stray electrostatic coupling in an a.c. measuring circuit, both internal and external, is generally due to distributed conductances and susceptances. Owing to the fact, however, that a.c. measuring systems use high-quality insulation, and in quantities not impairing their mechanical strength for that matter, unwanted conductances are usually negligibly small in comparison with unwanted susceptances, especially due to capacitances. In other words, stray electrostatic coupling in measuring circuits operating on alternating current may be regarded as being mainly capacitive. Capacitive coupling is a function of the geometrical dimensions and relative position of the circuit components.

Capacitive coupling manifests itself above all in the shunting effect on circuit components. This affects the balance condition due to changes in the impedance and phase angles of the circuit elements, so the distribution of currents and voltages in the system is also changed. Changes in impedances and phase angles due to the shunting effect of capacitances is usually insignificant at low frequencies and in low-resistance circuits. However, they may introduce appreciable errors in, say, systems employed for the measurement of low reactances and loss angles or high impedances.

If a circuit is capacitively coupled to earth (surrounding objects), the potentials of the various points in the circuit with respect to earth will depend on the aggregate effect of these capacitances. The latter are not constant, and variations in any one of them (due to a changed relative position of the operator, say) will change the potentials at various points in the circuit, which may have a far-reaching effect on the operation of the circuit as a whole. The accompanying errors may be due both to the shunting effect of the stray admittances on the circuit elements and to the effect of the leakage currents in the null detector (because of variations in the branch currents).

The capacitance from the detector circuit to earth is especially undesirable. Because of the high sensitivity of the null detector, even low unwanted capacitances at low frequencies may introduce considerable errors (because the stray currents appearing in the detector circuit become comparable with the unbalance

current). A good proportion of this effect may be due to the capacitance of the connecting leads.

External capacitive coupling may produce unwanted voltages in the system. The frequency of the induced voltage may be either equal to or differ from that of the supply voltage. In the latter case, if the null detector is not frequency-selective, the operator may be misled as to the exact balance of the circuit (due to a "blurred zero" common with oscilloscopes), or the detector, if it is of the deflectional type, will indicate balance, i.e., zero deflection, when balance conditions have not been really obtained. On the other hand, this condition has no effect on frequency-selective, or tuned, detectors. The effect is more adverse when the induced voltage has the frequency of the supply voltage. This usually occurs in measuring systems taking their supply from an a.c. mains at commercial frequency via a transformer with high intercapacitances.

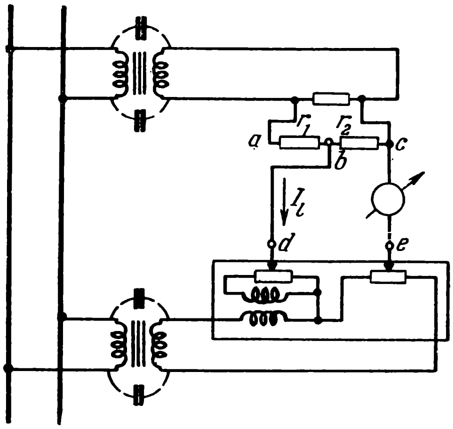


Fig. 9-1

Finally, several circuits in a measuring system may take their supply from a common source via isolating transformers. The capacitive coupling between the transformer windings may introduce errors due to the superposition of the leakage currents on the supply current and to the passage of these currents through the null detector.

As an example, Fig. 9-1 shows diagrammatically the effect of the intercapacitances in the isolation transformers of a coordinate potentiometer: the intercircuit leakage current I_l may produce a false balance in the potentiometer.

The effect of magnetic intercoupling in measuring systems operating on alternating current manifests itself in the e.m.f.s induced by a stray magnetic field in the system. This effect is especially pronounced when the system contains elements with a considerable magnetic leakage (self- and mutual inductors, transformers) or when the measuring circuit has a considerable time constant (L/R).

Internal magnetic coupling is due to the mutual inductance between the various parts of the circuit. Therefore, in circuits with capacitive elements this effect is insignificant and is limited to the interaction between the source and the detector.

External magnetic coupling affects any type of measuring circuit, if the latter contains considerable intercapacities and mutual inductances due to combinations of circuit elements and connecting leads. Nonastatic inductors and mutual

inductors, if any are used in the measuring system, are especially dangerous in this respect, since they are very sensitive to extraneous magnetic fields.

Extraneous magnetic fields have a particularly noticeable effect on a.c. measuring circuits operating at high frequencies. This effect manifests itself mainly in unwanted voltages induced in the measuring circuit and the detector. Also, it may cause changes in the resistive and reactive components of the circuit impedances.

9-2. Control of Stray Effects in D. C. Measuring Circuits

Obviously, the best way to control a disturbance is to eliminate its cause. As applied to the control of thermal e.m.f.s in d.c. systems, the above statement has found an embodiment in the fact that manganin, which has a very low thermo-electric e.m.f. with copper, is usually chosen as the material for the resistance coils and slide-wire. Yet, in critical measurements the circuit parts which may be heated by the current flowing through them or by any extraneous sources of heat should be additionally cooled by placing them in oil baths.

In order to minimize unwanted e.m.f.s, it is essential that the rubbing contacts of switches be made of metals which have very low thermal e.m.f.s with one another. In portable instruments intended for plant use, the knobs of the switches and keys must be so constructed as to protect the contacts from the heat of the operator's hand.

One of the methods to reduce the effect of unwanted e.m.f.s in precise instruments intended for the measurement of small e.m.f.s is to include the contacts of the switches with the supply circuit. Since the e.m.f. of the source is appreciably greater than the e.m.f. produced due to the rubbing of the contacts, the latter will affect the supply current very little. This method is employed in the bridge-type potentiometer of Fig. 7-3.

Also the effect of unwanted e.m.f.s can be guarded against by the use of one of the special measuring techniques described earlier, namely, by making the measurements of the same unknown so as to make the error neutralize itself, and by the substitution method.

Now we shall examine leakage-current control in d.c. measuring circuits.

One of the methods is to insulate the circuit as thoroughly as possible. In most cases, where the null detector is a galvanometer of normal sensitivity, it will suffice to use a high-quality insulating material for some of the current-carrying parts and to isolate the detector circuit from the supply circuit.

In all other cases, where thorough and high-quality insulation proves of little help, the only effective way is to use special potential shields in the form of suitably placed conductors or supports. The idea of shielding is that the conductor placed between earth and the parts of the circuit where leakage is especially undesirable reduces the potential difference between them and reroutes the leakage paths. Where several shielding conductors are employed in a circuit, they should be connected to one another and to one of the wires of the circuit so that none of the possible interconductances may shunt the highest resistance of the circuit.

The simplest example of the idea is provided by the measurement of high resistances with a galvanometer and a voltmeter. Leakage is possible through the insulation resistances R and R'. The current I_1 that can flow through them may sometimes be comparable with the current I_x flowing through the unknown resistance. If the insulator R be divided into two parts by a metal sheet electrically connected to a battery terminal by a conductor (the dotted line in Fig. 9-2), the voltage across one half of the insulator (R_2) will be equal only to the voltage drop across the galvanometer, and practically no leakage current will flow through R_2 because of the negligibly small difference of potentials. The current will continue

to flow through the other half of the insulator (R_1), without reaching the galvanometer, however. On this account, it will not affect the reading.

Physically, the idea of a divided insulator can be realized in several ways. For example, the support can be fitted with legs on double insulators (of hard rubber or any other dielectric) separated by metal sheets; the latter should be connected to one another and to one of the galvanometer terminals.

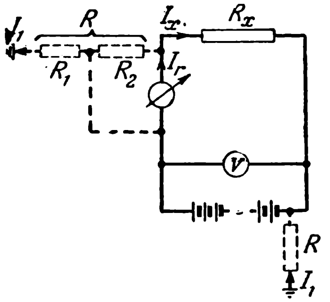


Fig 9-2

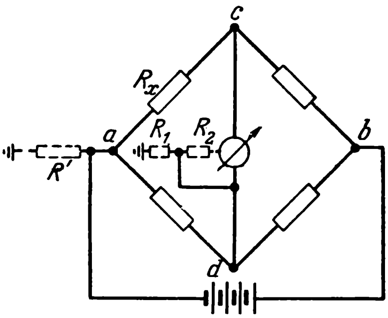


Fig. 9-3

The best result with the circuit of Fig. 9-2 is obtained when all the circuit components are set up on such supports, the separating conductors of which are connected to one another and to the common galvanometer-battery terminal.

This method can be employed in any other circuit arrangement, including bridges and potentiometers. For example, in the measurement of high resistances with a Wheatstone four-arm bridge in which the resistance of the ratio arms

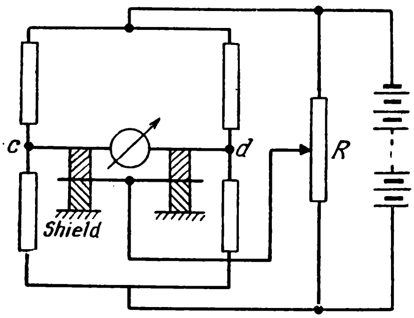


Fig. 9-4

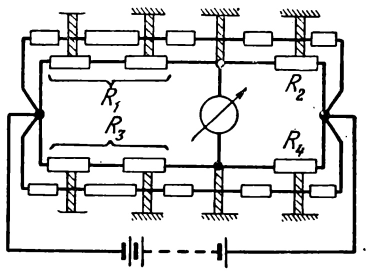


Fig. 9-5

is relatively low, the leakage from the galvanometer terminal c, electrically connected to one of the terminals of the unknown resistance, and also from the terminal a is most undesirable. In this case, it will be a good plan to protect the galvanometer as shown in the circuit of Fig. 9-3 where the leakage from the terminal c to earth is prevented by the shield connecting the point a of the circuit and the mid-point of the double insulator.

A similar effect can be obtained with the circuit of Fig. 9-4 in which an auxiliary voltage divider maintains the galvanometer shield GS at a potential very close to that of the galvanometer circuit.

As will be noted, this idea is carried a step further in the circuit of Fig. 9-5 where the shielding circuit reproduces the potential distribution of the main circuit. In this arrangement, each component of the main bridge is insulated with respect to a point in the shielding circuit at about the same potential, and not to earth. On this account, the likely leakage paths (the upper halves of the insulators) will show but insignificant differences of potential which cannot give rise to any appreciable leakage currents, even though the insulation may be of poor quality. In fact, the effect of leakage currents is practically eliminated. Because of the complexity of the circuit arrangement, however, it is resorted to only in the precise measurement of high resistances, of the order of 10^6 ohms and higher. As for the simpler arrangements—the shielding of only the detector circuit as suggested in the diagrams of Figs. 9-3 and 9-4 (the final choice depends on prevailing conditions), they may be used for the precise measurements of lower resistances as well.

In measurements of high voltages with a potentiometer, leakage at points A and B (Fig. 9-6) will give rise to an additional voltage drop due to the leakage

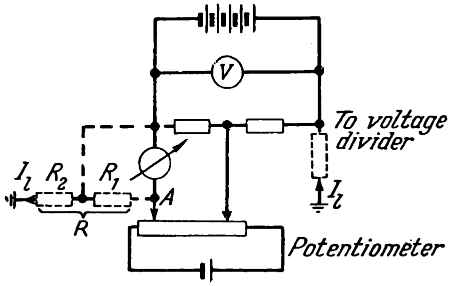


Fig. 9-6

current flowing through the galvanometer circuit, and the reading will be falsified. To eliminate the cause of the error, the circuit components should be set up on double insulators the separating conductors of which are connected by a common conductor to the galvanometer terminal in turn connected to a voltage divider. With this arrangement, the leakage currents will flow through the lower portions of the double insulators, by-passing the galvanometer. It may be added that this device gives the best possible results with the arrangement in question.

As in the case of bridge circuits, potentiometers can also use an auxiliary voltage divider in order to maintain the separating conductors at a potential close to that of the detector circuit.

Finally, potentiometers can likewise employ an auxiliary protective circuit reproducing the potential distribution of the main circuit.

9-3. Control of Stray Effects in A. C. Measuring Circuits

As was stated in Sec. 9-1, a.c. measuring circuits are mainly prone to electromagnetic effects. As a rule, electromagnetic effects are extremely changeable, varying, as they do, during a measurement. Therefore, they cannot sometimes be compensated by, say, the substitution method. For this reason, the control of electromagnetic effects in the case of a.c. measuring circuits has two objectives: elimination of the effects (where possible) or stabilization of their magnitude (if they cannot be eliminated). This is usually done by the use of electrostatic and magnetic shields, by careful layout of the circuit components with respect to one

another, by the use of a variety of auxiliary circuits, to mention but a few approaches.

Since electromagnetic effects can in the main be due to either an electric or magnetic field, we shall discuss two types of shielding: electrostatic and magnetic.

In accordance with Sec. 9-1, by electrostatic shielding will be meant the control of stray electrostatic coupling.

One of the objectives of electrostatic shielding is to suppress the cause (or causes) of the associated error, i.e., the electrostatic coupling itself and the voltage of the measuring circuit with respect to the surrounding objects, since they are the principal factors determining the magnitude of the leakage currents. Also, electrostatic shielding seeks to obtain a voltage distribution in the measuring circuit with respect to earth, such as would be independent of the variations in the coupling so as to eliminate the most significant interadmittances, or to provide leakage paths such as would reduce the effect of leakage currents on the operation of the measuring circuit to a minimum. Accordingly, the underlying principles of electrostatic shielding for a.c. measuring circuits may be stated thus:

(1) Reduction of the admittances due to electrostatic coupling, i.e., any conductances and capacitive susceptances that may exist between the circuit components and also from the circuit components to surrounding objects or earth.

(2) Reduction of the potential difference between the whole or part of the measuring circuit and surrounding objects. The ideal realization of this principle is equipotential shielding when any point in the measuring circuit or on its component is surrounded by objects at the same potential as the point itself.

(3) Establishment of leakage paths which would route leakage currents round the circuit components most sensitive to leakage effects (the null detector, high resistances, components of low reactance, and the like) and placement of capacitive coupling where its effect is a minimum.

These principles are realized by means of electrostatic shielding and earthing (or their combination). Also the reduction of admittances due to electrostatic coupling can be effected by the use of improved insulation (to reduce interconductances) and by careful arrangement of the measuring circuit (to reduce unwanted capacitive susceptances).

Standing somewhat apart from the three above principles is a fourth principle: the self-cancellation of leakage currents. It is mainly realized by the use of balanced arrangements.

In some cases, the effect of leakage currents can be minimized by increasing the supply current of the measuring circuit (while keeping the supply voltage constant), since the smaller the leakage current in comparison with the supply current, the less its relative effect. This technique can be employed where the constants of the measuring circuit are not governed rigidly by some other considerations.

Now we shall examine in more detail the principal methods of electrostatic control as applied to a.c. measuring circuits: electrostatic shielding, earthing, and balanced arrangements.

Electrostatic shielding serves to stabilize the magnitude of electrostatic coupling and also to control its effect on the characteristics of the components in a given circuit. As a rule, this type of shielding is accomplished by means of metal shields made from foil, a wire mesh, or a material of lower electrical conductivity such as sheet zinc, or by means of a metal coat applied to wood or some insulating material. Sometimes, a shield may take the form of several layers of fine wire wound around the circuit component to be protected (a resistor, an inductor or a transformer coil).

In the presence of a shield, all capacitance of the component to surrounding objects or to other circuit components is replaced by its capacitance to the shield.

Since the shield is always connected to some point on the component or circuit, the stray capacitance is thus concentrated at this point. In this way, all stray capacitances in a circuit may be concentrated at a number of points whose location should be so chosen as to reduce their effect on the operation of the circuit.

Circuit components can be electrostatically shielded by two fundamentally different methods. The simplest and most commonly used method is single-terminal electrostatic shielding, where the shield is connected to one of the terminals of the shielded component, say terminal A in Fig. 9-7. In this case, terminal A is a branching point for leakage currents. Then arranging the point A to be at earth potential or the unwanted admittance Y_{sh} to be connected in parallel either with the detector or supply circuit (in the case of a balanced bridge) or to an arm of an auxiliary branch (if any is used), the effect of the admittance Y_{sh} on the impedance of the shielded component will be fully eliminated.

More complicated, yet more advantageous in certain cases is three-terminal electrostatic shielding in which the shield is connected to a point on the main or

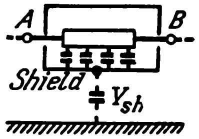


Fig. 9-7

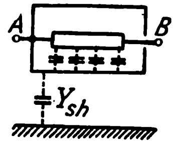


Fig. 9-8

an auxiliary circuit rather than to the shielded component (in Fig. 9-8, it is shown connected to a point on an auxiliary branch). This point is selected so that the admittance Y_{sh} either shunts low-resistance components or auxiliary elements, or is connected between equipotential points. In both cases, the impedance of the shielded component will be made practically independent of the variable admittance Y_{sh}.

The choice between the two methods of electrostatic shielding is governed (alongside with the different complexity of realization) by the relative effect of the distributed unwanted admittance between the shielded component and the shield on the impedance of the shielded component. Single-terminal shielding is usually accompanied by the appearance of an additional capacitive reactance in the impedance of the component being shielded. In three-terminal shielding, in which the shielded component actually becomes a three-terminal network, part of the current flowing from the circuit to the shielded component through one of its main terminals can be routed back into the circuit through the third terminal (the shield), by-passing the other main terminal. With this method, the properties of circuit elements can be revealed with purity unattainable without shielding or with single-terminal shielding. Among other things, it is possible to obtain a pure capacitance, i.e., a capacitance free from energy losses, or to give a component such properties as it would have only in the absence of any unwanted coupling. Most often, the shield is maintained at the potential of one of the terminals of the shielded element.

In most cases, it is required that the shield have the least possible effect on the shielded component. The ideal case would be equipotential shielding in which any small area of a component is enclosed by a shield at the same potential as the shielded area. The most perfect and simplest equipotential shielding can be obtained with the detector circuit. Since, at balance, all the points in the detector circuit are at one and the same potential, it will suffice to give the shield the potential of one of the terminals in order to shield the whole circuit equipotentially. This potential may be applied not only by connecting the shield to a

certain point on an auxiliary branch, but also by connecting the shield directly to a detector terminal (if this does not affect the operation of the main circuit).

Where different points on a circuit element are at different potentials, a single shield at a certain single potential proves insufficient for equipotential shielding, so that use has to be made of a several shields with the potentials on them distributed exactly as they are on the element being shielded. Obviously, ideal equipotential shielding in such a case will only be possible with an infinitely great number of shields, i.e., with the distributed potential of the shield. Practically, such shields can be constructed similarly to the element to be shielded, for example, in the form of coils wound exactly as the resistances used in the circuit (see below). Ordinarily, however, use is made of a limited number of shields,

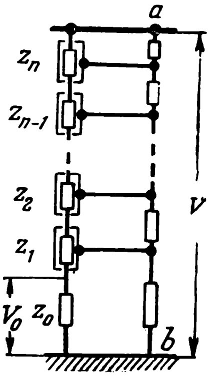


Fig. 9-9

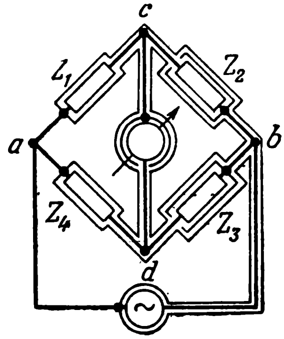


Fig. 9-10

since good results are obtained with as few as six to eight sections. A very important application of sectionalized equipotential shielding is the shielding of high-voltage dividers. Figure 9-9 shows diagrammatically one such divider, Z_0, Z_1, Z_2, ..., Z_n. The sections of the divider are enclosed in shields connected to an auxiliary divider *ab*. Since the auxiliary divider is also subject to unwanted coupling which may affect the voltage distribution across it, it has sometimes to be also shielded, using either single-terminal shielding or, again, sectionalized shielding by a second auxiliary divider.

In practical circuits, use may be made of either single-terminal or three-terminal shielding or of their combination. In the latter case, so-called double shielding can be employed, where a circuit element with a single-terminal shield has additionally a three-terminal shield whose point of connection is so located that the intershield capacitance is connected in parallel with circuit elements insensitive to the shunting effect. The four-arm bridge of Fig. 9-10, for example, uses double shielding for the detector circuit and for the arms Z_2 and Z_3. The three-terminal shield is connected to the junction *a*, for which reason the intershield capacitances shunt only the supply source and the arm Z_2. Obviously, this is done on the assumption that Z_2 is an element little sensitive to capacitive shunting (it may be a high capacitance, a low resistance, etc.). If this bridge used only single-terminal shields, the intershield capacitances would shunt also (in addition to the supply source and Z_2) the arm Z_1, or the unknown impedance, thereby introducing a considerable error (especially if Z_1 is a low capacitance, a high resistance, etc.).

It should be added that the use of double shielding not only eliminates unwanted shunting, but also renders the intershield capacitances definite in magnitude and independent of external conditions.

Where the requirements for the measuring system are especially stringent or measurements have to be made under some special conditions, the three shielding methods examined above may be supplemented by an additional shield enclosing all of the system and connected to a certain point on the main or auxiliary circuit or earthed (depending on the circumstances). By replacing the earth capacitances of the inner shield with those to the outer shield, such an arrangement completely eliminates external electrostatic effects on the measuring system. Sometimes, where the requirements for the measuring system are not particularly exacting, use may be made of an outer electrostatic shield alone.

Now we shall consider the use of earthing and its effects on the operation of a.c. measuring circuits.

By earthing, as a method of controlling stray effects on a.c. measuring systems, is meant the electrical connection of a certain point on the system to objects around the system, having a certain potential (in a particular case, it will be earth potential). This is done where the capacitances to these objects may introduce errors in the measurements with a given system.

When a point on a circuit is earthed, this eliminates the effect due to the earth capacitance lumped at this point, and the point acquires earth potential. At the same time, the distribution of potentials with respect to earth in the circuit is rendered more definite in magnitude and less dependent on variations in the remaining coupling (should the impedance of the circuit elements be considerably lower than the stray impedances). In addition to this, earthing can often appreciably reduce errors by bringing down the voltages due to the leakage currents flowing through the stray capacitances and also by short-circuiting the leakage currents outside the circuit elements most sensitive to them.

A measuring system should be earthed at a point where the basic principles of stray-effect control will be fulfilled as far as possible. The selection of the earthing point can also be affected by some other considerations, such as the conditions of measurement, safety to personnel, operation of the various circuit elements (symmetrical or asymmetrical connection), etc. The selection of the earthing point may be materially affected by the presence, or otherwise, of shields in the system. In any of such cases, a preliminary analysis of the possible effects of stray capacitances is essential.

The earthing of a point completely eliminates the capacitance of this point to earth. Therefore, it is essential to determine the relative values of the unwanted capacitances so as to eliminate the highest or most undesirable of them. Because of this, it is a good plan to earth one of the terminals of the adjustable circuit element (i.e., the terminal which will be touched by hands, thereby introducing a high and indefinite earth capacitance).

Where the earthing of any point on the main circuit cannot materially improve its operation, it will be advisable to earth a certain point on the auxiliary circuit (or branch). A classical example is provided by the earthing of the point E on the earth branch Y_5Y_6 connected in parallel with the supply source of the bridge of Fig. 9-11.* If the impedances of the bridge arms and the auxiliary branch are in such a relationship that

$$Y_1/Y_2 = Y_4/Y_3 = (Y_5 + Y_A)/(Y_6 + Y_B) \tag{9-1}$$

then the points c and d of the detector branch will be at earth (or zero) potential, as if they were permanently earthed. However, as distinct from the permanent earthing of the points c and d, no effect is produced by either the admittances Y_C and Y_D since they are connected between equipotential points, or the admit-

* Known as the Wagner earth branch or circuit.

tances Y_A and Y_B, since they are connected in parallel with the auxiliary arms Y_5 and Y_6 and not with the bridge arms (which would be the case if the points c and d were permanently earthed). In bridges incorporating an earth branch, Eq. (9-1) is fulfilled by balancing in turn the bridge made up of Y_1, Y_2, Y_3 and Y_4 and the bridge consisting of Y_1, Y_2, Y_5 and Y_6 (or Y_3, Y_4, Y_5 and Y_6) through adjustment of Y_5 and Y_6, with the null detector placed either between c and d or between c and E (or d and E), until the detector gives zero deflection with the switch in either position.

Earthing is not a universal means of controlling the effects due to leakage currents and produces limited results. Yet, with an adequately chosen earthing point, it can reduce unwanted coupling and improve the accuracy and reproducibility of the system. Indeed, in some cases, earthing may prove the main means of controlling stray effects on a.c. measuring systems (such as in the case of inductive-ratio bridges).

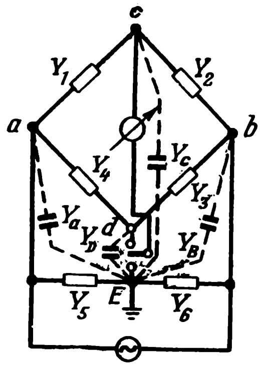


Fig. 9-11

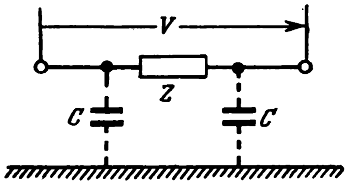


Fig. 9-12

Still another method of controlling the effects due to leakage currents is their cancellation by the use of balanced circuit arrangements.

A circuit element, as viewed from certain two of its terminals, is called balanced if these terminals present the same impedance to earth. In measurements, balanced elements are employed because of the advantages they offer over unbalanced. Thus, if a voltage is applied to the terminals of a balanced element (Fig. 9-12), their respective potentials to earth will be equal to half the applied voltage. Therefore, the impedances of the terminals to earth are halved. The most valuable feature about balanced elements is that they are insensitive to leakage currents to earth, since those flowing in one half balance out those in the other.

The symmetry of an element can be obtained either through an internal circuit arrangement and construction or through external means. In the latter case, it is customary to place between an unbalanced element and a balanced measuring circuit an intermediate four-terminal network whose terminals connected to the measuring circuit are balanced. Most simply, such a four-terminal network is realised in the form of a transformer. With such a transformer, an element with any degree of asymmetry can be used as balanced. At the same time, impedance matching will be accomplished.

In a.c. measuring systems, this technique is mostly applied to the detector and source circuits. For this purpose, a balancing transformer is placed between the detector or source, which may be unbalanced, and the measuring system proper. The windings of this transformer connected to the measuring circuit are balanced with respect to earth. With balancing transformers, it is possible to use a source or a detector irrespective of the symmetry of its output or input terminals in any measuring system, without any danger of introducing

a complementary error. A balancing transformer is usually required to have a balanced primary where a measuring circuit with a balanced output uses a null detector with an unbalanced input (input balancing transformers). A balanced secondary will be required where a balanced measuring circuit is supplied from an unbalanced source. The high and stable symmetry of balancing transformers, coupled with their efficiency as isolating elements, is ensured by a combination of a balanced winding, suitable shields, and elimination of any capacitive and electrostatic coupling between the windings. Where, in spite of all the measures, full symmetry cannot be obtained, the terminals of the winding supposed to be balanced are balanced by means of differential capacitors.

Where a balanced winding is not essential and the only objective sought is good isolation, use is made of so-called isolating transformers which differ from the balancing variety by simpler windings and shields.

Both balancing and, especially, isolating transformers should have the least possible leakage magnetic fields. Jumping a little ahead, it may be noted that this requirement can be satisfied by the use of magnetic shielding, magnetic materials of high permeability, and magnetic circuits of the shape that keeps magnetic leakage to a minimum.

One of the objectives of balancing and isolating transformers is to reduce intercircuit leakage currents. Incidentally, an isolating transformer with negligibly small intercapacitances may be used in potentiometers. This, however, entails additional difficulties in work on elevated frequencies. Also, there are potentiometers in which the requisite tight coupling between the two circuits is obtained by supplying one of them from the secondary of a current transformer (such as in the measurement of resistances). This coupling is provided by the capacitance between the transformer windings, which cannot be very small. Thus, practical potentiometers have often to be content with relatively heavy intercircuit capacitive currents.

Where the intercircuit capacitive currents increase to a point that the error due to them cannot be neglected, a radical way out is to balance them out by means of a voltage, adjustable both in magnitude and phase, taken from an auxiliary source. Such an auxiliary source is, for example, employed in the potentiometer manufactured by the Tochelektropribor Factory for high-frequency work. A simplified diagram of its connections is given by Fig. 9-13. Referring to the diagram, the intercircuit capacitive current in the neutral wire *ac* is balanced out by the current flowing in the same wire due to an auxiliary voltage $\Delta\dot{V}$. Balance is obtained by adjusting the magnitude and phase of $\Delta\dot{V}$ and the settings of the contact arms on the voltage dividers of the potentiometer until the detector shows zero deflection with the key K both opened and closed.

Finally, there is magnetic shielding of a.c. measuring networks. What is meant here is mainly shielding against magnetic intercoupling and extraneous high-frequency electromagnetic fields.

Magnetic intercoupling comes in for discussion first.

Since the e.m.f.s and currents due to magnetic coupling between adjacent elements cannot be calculated with sufficient accuracy (and so, no corrections can be computed in advance), errors due to magnetic intercoupling should above all be controlled by reducing the mutual inductance between, and the size of, the circuit components. This can be accomplished by suitably disposing the circuit elements, connecting them with twisted leads, and arranging inductors and mutual inductors at some distance from one another so that their axes are at right angles.

Wherever possible (but not to the detriment of electrostatic shielding), care must be taken not to place the auxiliaries, especially the source, in a common box with the bridge or potentiometer, since this increases magnetic coupling.

Magnetic coupling can also be reduced by the use of circuit elements so constructed as to limit magnetic leakage, such as toroidal inductance coils and transformers with cores of high-permeability ferromagnetic materials.

In precise work, however, these measures often prove inadequate. Then resort should be made to magnetic shielding: either ferromagnetic (sometimes called magnetostatic) based on the use of high-permeability materials, and electromagnetic based on the demagnetizing effect of eddy currents in the shield.

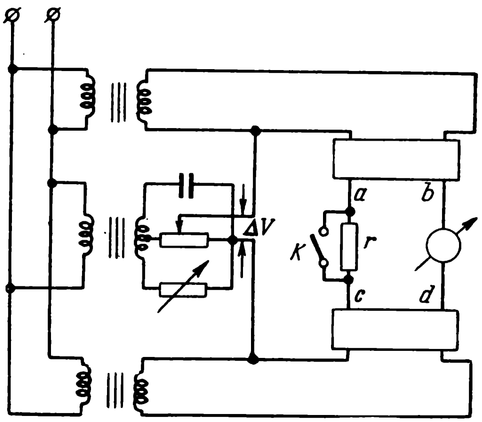


Fig. 9-13

The effectiveness of ferromagnetic shielding, usually employed to shut out d.c. and low-frequency magnetic fields, is approximately proportional to the permeability of the material. Therefore, shields for this application are made from high-permeability ferromagnetic materials such as Permalloy, Mumetal and the like. The effectiveness of shielding can be enhanced by the use of multiple shields. The spacing between the individual shields in a multiple unit should be greater than the thickness of a single shield, i.e., equal to the distance between the shielded component and the innermost shield. All the shields in a unit must be of the same thickness (0.5 to 1 mm). The effectiveness of multiple shields is worth the expensive ferromagnetic materials they are made from.

As the frequency of the supply or unknown voltage increases, electromagnetic shielding accounts for an ever greater share in the total shielding effect (i.e., due to magnetostatic and electromagnetic methods). Its effectiveness increases with both increasing permeability and conductivity of the shield material and with increasing frequency. Therefore in radio-frequency work, shields need not be made from ferromagnetic materials. Instead, use is made of good conductors such as copper or aluminium. In radio-frequency circuits, especially popular are clad-iron shields (the cladding is usually copper) which combine high effectiveness with low consumption of nonferrous metals.

To sum up, the intelligent arrangement of the circuit and the correct use of shields can reduce magnetic intercoupling practically to zero in most cases.

The effects of outside magnetic fields are generally controlled by the same methods as magnetic coupling within a circuit: by the use of bifilar windings and shields, by arranging the circuit at a sufficient distance from sources of magnetic fields, and by suitable relative disposition of the circuit components. Also, it is advisable to use astatically wound standard inductors and mutual inductors

since, except when a circuit is placed closely to a source of a magnetic field, external magnetic fields are remarkably uniform.

Where strong and nonuniform outside fields are involved, it will suffice in many cases to supply the circuit from a source with a frequency other than that of the outside field (provided the null detector used is of the frequency-selective type).

Outside r.f. fields are best shut out by a common metal shield enclosing the whole of the system.

A very simple and efficient way of reducing magnetic coupling in low-frequency bridge networks is to arrange the circuit elements at some distance (up to 1 metre) from the junctions which are built into a common unit and placed closely to one another. The bridge arms are then located in a common plane, with the null detector and the source placed on either side. The wires run to the detector and source at right angles to the plane of the network, while the leads connecting up the circuit elements are of the coaxial type, i.e., a lead for one direction is placed within a metal tube which serves as a conductor for the opposite

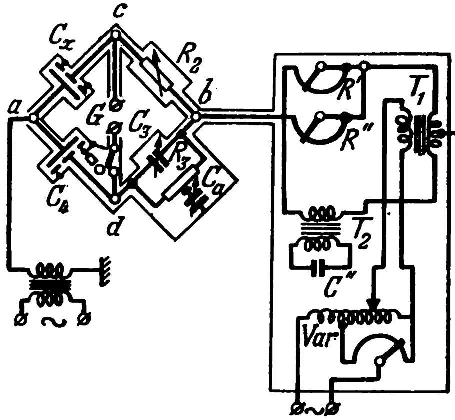


Fig. 9-14

direction. The central conductor is insulated from the outer tube by washers or beads made of a high-quality dielectric (porcelain, quartz, pyrex, polystyrene, etc.). Of course, this method can be used where the size of the measuring system is not critical.

Some of these methods may also improve the accuracy of measurement. Thus, astatically wound mutual inductors, in addition to reducing the effects of outside magnetic fields, also reduce errors due to capacitive leakage currents in comparison with conventional mutual inductors of the same inductance value. Coaxial leads, apart from reduced induction and lower external field effects, are advantageous in that their capacitance is more constant, let alone that it may be reduced by increasing the inside diameter of the tube. An earthed common shield, in addition to shutting out outside magnetic and electromagnetic fields, renders the earth capacitances of the network elements more definite.

A further example of successful shielding is provided by the resistive ratio arms of a.c. bridges, connected in parallel with the source branch and made in the form of coils with a double-layer opposing winding (mentioned earlier). The lower layers make the main arms of the bridge, while the upper layers constitute an

auxiliary branch providing an equipotential shield for the main bridge arms. Since the layers are wound in opposite senses, the magnetic fluxes due to the different layers cancel out each other, thereby reducing the residual inductance of the arms and the effects of outside magnetic fields. At the same time, the own capacitance of the coils is very low owing to the unifilar winding of each layer.

It should be stressed that whether or not a given method of stray-effect control should be employed depends on the actual conditions of measurement, above all on the requisite accuracy and the conditions under which the component under test operates. In many cases it will suffice to incorporate in the measuring circuit a reasonable minimum of control features. Nor should it be sought to employ the best possible method of stray effect control, the more so that protective devices bring about certain changes in the constants of the measuring circuit. The unwarranted use of too refined features (let alone the incorrect employment of some of them) may impair the performance of the system (resulting in reduced sensitivity, reduced accuracy, etc.).

An example of an adequately protected a.c. measuring system is provided by the Type МДП shielded bridge manufactured in the Soviet Union. Its connections are given by Fig. 9-14. The bridge is made for a supply voltage of 3 to 10 kV at 50 c/s and is designed to measure capacitances from 40 to 20,000 pF and loss angles from 1×10^{-4} to 1.0. A salient feature of the bridge is the series connection of two supply sources: an adjustable low-voltage source and a fixed high-voltage source. The bridge is enclosed in an earthed common shield which is connected to the common terminal of both sources and is equipotential with respect to the detector branch. The shield and detector branches are equipotential when

$$\dot{V}_h/\dot{V}_l = Z_1/Z_2 = Z_4/Z_3,$$

where $\dot{V}_h$ is the voltage of the high-voltage source and $\dot{V}_l$ is the voltage of the low-voltage source. The equipotential condition is obtained by adjusting the low voltage $\dot{V}_l$ both in magnitude and phase.*

This arrangement completely eliminates any unwanted coupling and provides equipotential protection for the detector. The guard ring of the high-voltage standard capacitor is maintained at the potential of the main electrode, for which reason the capacitance C_4 is rendered definite in magnitude. The capacitances of the bridge arms to the shield do not affect the balance condition, since they either act between equipotential surfaces (the capacitances lumped at the points *c* and *d*) or shunt the two sources (the capacitances lumped at the points *a* and *b*).

The variable capacitor C_a, which is equivalent to an inductance connected in parallel with R_3, serves to balance the phase components of the arms R_2 and R_3. To obtain such balance prior to a series of measurements, the capacitances C_x and C_4 are replaced by two identical impedances $Z_1 = Z_4$, then C_3 is set to zero, R_2 is made equal to R_3, and C_a and the low voltage are varied until the bridge is brought to balance. At balance, $Z_1Z_3 = Z_2Z_4$. Since $Z_1 = Z_4$, then $Z_2 = Z_3$ or $R_2 + j\omega L_2 = R_3 + j\omega L_3$, where L_2 and L_3 are the effective inductances of the arms.

The low voltage is regulated by a circuit comprising a continuously adjustable autotransformer (*AT*) and a parametric phase-shifter using a transformer T_1, a capacitor C'' connected via another transformer T_2 (in order to increase its effective capacitance) and variable resistors R' and R''. The continuously adjustable autotransformer and the voltage divider at its input vary the magnitude of the low voltage. Its phase is adjusted by variations of R' and R''.

Balance in the МДП bridge is obtained exactly as in a bridge incorporating a Wagner earth branch, i.e., by adjusting in turn the low voltage (with the detector

* In the Manufacturer's description of the bridge this voltage is termed "protective".

connected to the shield) and the elements of the main network (with the detector connected in the bridge). Balance is obtained when the detector gives zero deflection both when connected to the shield and to the bridge. The null detector is a vibration galvanometer with a valve amplifier. The bridge measures capacitances accurate to within $\pm$ (0.5 per cent $+$ 5 pF) and the tangent of the loss angle accurate to within $\pm$(1.5 per cent $+ 6\times 10^{-5}$), on a supply voltage of not lower than 3 kV.

In conclusion it should be noted that in many cases the selection of any methods for control of stray effects is governed by the component under test. For example, the conditions under which, say, the unknown resistance has to operate during a measurement will limit in advance the number of possible arrangements for any a.c. bridge, determining which of the points in the bridge can and which cannot be earthed.

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TO THE READER

Peace Publishers would be grateful for your comments on the content, translation and design of this book. We would also be pleased to receive any other suggestions you may wish to make. Our address is: 2, Pervy Rizhsky Pereulok, Moscow, U.S.S.R.

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